

# 行政院國家科學委員會專題研究計畫 期中進度報告

## 傳輸線之面積、時間延遲、功率及雜訊最佳化(1/2)

計畫類別：個別型計畫

計畫編號：NSC91-2215-E-002-036-

執行期間：91年08月01日至92年07月31日

執行單位：國立臺灣大學電子工程學研究所

計畫主持人：張耀文

報告類型：精簡報告

處理方式：本計畫可公開查詢

中 華 民 國 92 年 6 月 3 日

# 傳輸線之面積、時間延遲、功率及雜訊最佳化 Area, Delay, Power, and Noise Optimization for Transmission Lines

計畫編號：NSC 91-2215-E-002-036

執行期限：91 年 8 月 1 日至 92 年 7 月 31 日

主持人：張耀文副教授 臺灣大學電子工程學研究所

## 一、中文摘要

對於深次微米，高效能電路，當在決定電路延遲時間，電感的效應伴演著一個非常重要的角色。在本計畫中，我們推導出一個準確的公式來模擬晶片階層之高速電路延遲模型 (buffered RLY/RLC wires and trees)。我們的公式可以處理平衡與非平衡的電路 (balanced and un-balanced trees) 且考慮加入緩衝器 (buffer insertion)，並根據 180nm 的製程技術 (technology)。

**關鍵詞：**平衡與非平衡的電路，緩衝器

## 二、英文摘要(Abstract)

For deep-submicron, high-performance circuits, the inductive effect plays a very important role in determining the circuit delay. In this project, we derive accurate formulae for modeling the delays of buffered RLY/RLC wires and trees. Our formulae can handle balanced and un-balanced trees and consider buffer insertion based on the 180 nm technology.

**Keywords:** Balanced and un-balanced trees, buffer

## 三、背景和目的

### 1. Background

As technology advances into the very deep-submicron era, interconnect delay dominates overall circuit performance. Therefore, accurately modeling the interconnect delay becomes a major challenge in high performance IC design. For deep-submicron, high-performance circuits, ignoring inductance effects may incur a large amount of error, since an RC model as compared to an RLC model may create errors of up to 30% in the total propagation delay of a repeater system [9]. As technology improves and die size increases, short rise/fall times of signals and long wires make inductive effects more significant than before [15]. Therefore, it is very important to consider the effects of inductance.

Timing is a crucial concern in the design of high-performance circuits. Arunachalam et al. in [3] proposed accurate CMOS gate delay models for general RLC loads. The waveform resulted from their delay model excellently agrees with SPICE results; however, they do not present any formula for propagation delay. Many delay models have been proposed to calculate delay (e.g., [4, 5, 9, 14, 18]); however, these models cannot apply to tree structures. Modeling and analysis techniques for timing calculation under tree structures have been studied extensively in the literature [1, 2, 7, 8, 10, 11, 12]. Previous work in [1] proposed a method (Fitted Elmore Delay) for delay estimation by using the curve fitting technique. However, their work does not consider inductance. The work in [2] only considered the RC delay model, and did not include the inductance effect. The works in [7, 8] extended the Elmore delay to include the inductance effect, but they did not consider buffer insertion/sizing. Ismail and Friedman in [10] proposed an algorithm for buffer insertion/sizing in an RLC tree. However, if the tree is unbalanced, as pointed out in the paper, the delay estimation may incur significantly larger errors. The works in [11, 12] adopted two-pole simulation of interconnect trees via the moment matching technique, and used non-uniform lumped segments to model the distributed lines. However, they did not apply buffer insertion/sizing to reduce the delay. Kahng and Muddu in [13] provided an analytic delay model for interconnection lines under the step input, and extended their model to estimate the delay in arbitrary interconnect trees. However, their model does not consider buffer insertion/sizing, and cannot calculate for any percentage of delay time. Ismail and Friedman in [10] presented an algorithm to insert and size buffers in an RLC tree for minimizing the delay. However, their empirical formulae obtained by curve-fitting with circuit simulation were only for the 50% propagation delay and the 10%--90% rise time. Therefore, their works cannot treat any percentage of delay time. Banerjee in [4] considered buffer insertion/sizing for an RLC interconnection line and

did not handle the problem with the tree structure. Table 1 compares the features of important related works.

	Buffer Insertion/Sizing	Interconnection Trees	Calculation for any percentage delay
[4]	✓		✓
[10]	✓	✓	
[13]		✓	
Our Work	✓	✓	✓

Table 1: Comparison of features with the related previous works.

## 2. Objective

In this project, we derive accurate formulae for modeling the delays of buffered RLY/RLC wires and trees. The RLY model not only can model RLC interconnect, but also can consider off-path subtree effects. Our formulae can handle balanced and un-balanced trees and consider buffer insertion based on the 180 nm technology.

## 四、研究方法

We shall in this form discuss the problem formulation, notations, accurate formulae for modeling the delay of RLY wires, buffered RLC load, and buffered RLC trees.

### 1. Problem Formulation

In this project, we shall develop accurate formulae for modeling the delays of buffered RLY/RLC wires and trees. Our formulae need to handle balanced and un-balanced trees and consider buffer insertion. We formulate this problem as follows:

- *Input: Buffered RLY/RLC wires and trees.*
- *Objective: Determine the path delay estimation for buffered RLY/RLC wires and trees.*

### 2. Notations

We use the following notations throughout this project.

- ◆  $h_i$ : the length of wire  $i$ .
- ◆  $r$ : the unit-length resistance of a wire.
- ◆  $l$ : the unit-length inductance of a wire.
- ◆  $c$ : the unit-length capacitance of a wire.
- ◆  $c_b$ : the input capacitance of a minimum sized buffer.
- ◆  $r_b$ : the output resistance of a minimum sized buffer.
- ◆  $c_p$ : the output parasitic capacitance of a minimum sized buffer.
- ◆  $k$ : the size of a buffer.
- ◆  $R_S$ : the resistance of the driver.
- ◆  $C_L$ : the capacitance of the load.

### 3. Accurate Delay Model

#### 3.1. Delay Model for RLY Wires

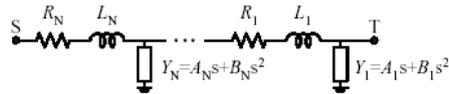


Figure 1: An equivalent single "RLY" line for an  $RLC$  tree.

Sriram and Kang in [16] developed an equivalent single RLY line (see Figure 1) for an RLC tree, where  $N$  is the number of RLY sections in an RLC tree. As shown in Figure 2, we can model a distributed RLC line of length  $h_i$  as a single RLY segment.

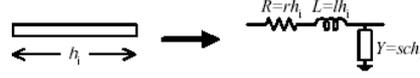


Figure 2: An interconnect wire can be modeled as a  $RLY$  segment.

The transfer function for the structure of Figure 1 is given by

$$H(s) = \frac{V_O(s)}{V_I(s)} = \frac{1}{1 + b_1 s + b_2 s^2 + \dots} \quad (1)$$

By the approximation method proposed by Gao et al. in [7], we can approximate Equation (1) as follows:

$$H(s) \approx \frac{1}{1 + b_1 s + b_2 s^2},$$

where

$$\begin{aligned} b_1 &= \sum_{j=1}^N A_j \sum_{i=j}^N R_i, \\ b_2 &= \sum_{j=1}^N A_j \sum_{l=j}^N L_l + \sum_{j=1}^N B_j \sum_{l=j}^N R_l \\ &\quad + \sum_{j=2}^N A_j \sum_{l=j}^N R_l \sum_{i=1}^{j-1} A_i \sum_{d=i}^{j-1} R_d. \end{aligned}$$

The first and second moments of the transfer function from Equation (1) can be obtained by the coefficients  $b_1$  and  $b_2$ , i.e.,  $M_1 = b_1$  and  $M_2 = b_1^2 - b_2$ . The two poles  $s_1$  and  $s_2$  of the transfer function could be real or complex depending on the sign of  $(b_1^2 - 4b_2)$ . Thus, we separately discuss the results from two poles response for each of these cases classified in [13].

**Case I. Real Poles:** The condition for this case is  $(b_1^2 - 4b_2) > 0$ . The step response, which is the inverse Laplace transform of  $\frac{1}{s} H(s)$ , is given by

$$v(t) = V_O \left( 1 - \frac{s_2}{s_2 - s_1} e^{s_1 t} + \frac{s_1}{s_2 - s_1} e^{s_2 t} \right),$$

where

$$s_{1,2} = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_2}}{2b_2}.$$

For a step input, the  $f \times 100\%$ , (where  $0 \leq f < 1$ ) delay,  $t_f$ , (i.e.,  $v(t_f) = fV_O$ ) is the solution of the following equation [4]:

$$1 - f - \frac{s_2}{s_2 - s_1} e^{s_1 t_f} + \frac{s_1}{s_2 - s_1} e^{s_2 t_f} = 0. \quad (2)$$

Therefore, we can use the Newton-Raphson method to numerically solve the delay that was calculated by Equation

(2).

**Case II. Complex Poles:** The condition for this case is

$(b_1^2 - 4b_2) < 0$ . The time-domain response for this case is given by

$$v(t) = V_O \left( 1 - \sqrt{1 + \left(\frac{\alpha}{\beta}\right)^2} e^{-\alpha t} \sin(\beta t + \rho) \right),$$

where

$$\begin{aligned} \alpha &= \frac{M_1}{2(M_1^2 - M_2)}, \\ \beta &= \frac{\sqrt{3M_1^2 - 4M_2}}{2(M_1^2 - M_2)}, \\ \rho &= \tan^{-1}\left(\frac{\beta}{\alpha}\right). \end{aligned}$$

We consider a step input. Thus,  $v(\neq) = fV_O$  is the solution of the following equation.

$$1 - f - \sqrt{1 + \left(\frac{\alpha}{\beta}\right)^2} e^{-\alpha t} \sin(\beta t + \rho) = 0. \quad (3)$$

Similarly, we also use the Newton-Raphson method to solve the delay that was calculated by Equation (3).

**Case III. Double Poles:** The condition for this case is

$(b_1^2 - 4b_2) = 0$ . The time-domain response is given by

$$v(t) = V_O \left( 1 - e^{s_1 t} - \frac{2t}{b_1} e^{s_1 t} \right), \quad (4)$$

where

$$s_1 = -\frac{b_1}{2b_2}.$$

Similarly, the Newton-Raphson method can be applied to calculate the delay that was calculated by Equation (4).

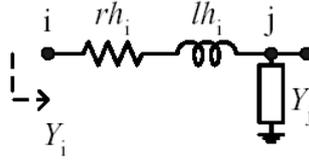


Figure 3: Computation of Subtree Admittance

For a series section of RLC segments, Kahng and Muddu in [12] presented an expression for the coefficient of  $s$  and  $s^2$  of admittance. As shown in Figure 3, the admittance at node  $i$  can be expressed in terms of the admittance at node  $j$ .

$$Y_i = \frac{1}{rh_i + slh_i + \frac{1}{Y_j}} = Y_j - Y_j^2 rh_i - slh_i Y_j^2 + \dots \quad (5)$$

Using the above recursive equation, the admittance of the off-path subtrees can be computed.

### 3.2. Delay Model for Buffered RLC Load

A CMOS inverter driving an RLC load is shown in Figure 4. For an interconnect wire of length  $h_i$ , its total resistance is  $R = rh_i$ , total inductance is  $L = lh_i$ , and total capacitance is  $C = ch_i$ , where  $r$ ,  $l$ , and  $c$  are the resistance, inductance, and capacitance per unit length of the interconnect, respectively. To consider the velocity saturation effects in short-channel devices, a CMOS inverter is modeled by using the alpha power law [17].  $V_o$  and  $V_1$  are the output voltage of the CMOS inverter and the output voltage at the end of the interconnect wire, respectively.

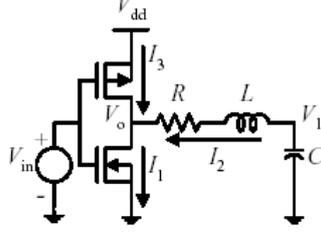


Figure 4: A CMOS inverter drives an RLC load.

The input voltage  $V_{in}$  is a fast ramp signal that can be approximated by a step signal:

$$V_{in}(t) = \frac{t}{\tau_r} V_{dd} \quad \text{for } 0 \leq t \leq \tau_r,$$

where  $\tau_r$  is the input transition time.

Because  $V_o$  and  $V_1$  depend on  $V_{in}$  and the operation region of NMOS transistor, we separately discuss three different conditions in the following [18]:

**Case I.**  $t_n \leq t \leq \tau_r$ : The NMOS transistor is ON and operates in the saturation region. We have the following equations:

$$\begin{aligned} V_1(t) &= L \frac{dI_{DS}}{dt} + RI_{DS} + V_o(t), \\ C \frac{dV_1(t)}{dt} &= -I_{DS} = -B_n \left( \frac{t}{\tau_r} V_{dd} - V_T \right). \end{aligned}$$

Therefore, the solution of  $V_o(t)$  is

$$\begin{aligned} V_o(t) &= V_{dd} - V_c(t) - V_r(t) - V_l(t), \\ V_r(t) &= RB_n \left( \frac{t}{\tau_r} V_{dd} - V_T \right), \\ V_l(t) &= LB_n \frac{V_{dd}}{\tau_r}, \\ V_c(t) &= \frac{B_n \tau_r}{2CV_{dd}} \left( \frac{t}{\tau_r} V_{dd} - V_T \right)^2, \end{aligned}$$

where  $V_T$  is the switching threshold voltage and  $t_n$  is the time for  $V_{in}$  to reach  $V_T$ .

**Case II.**  $t_n \leq t \leq \tau_r$ :  $V_{in}$  is fixed at  $V_{dd}$  and the NMOS transistor continues to operate in the saturation region. The discharge current is equal to the saturated drain-to-source current of the NMOS transistor:

$$I_{DS} = I_{nsat} = B_n(V_{dd} - V_T) = \text{constant}.$$

Therefore,

$$\begin{aligned} V_o(t) &= V_1(\tau_r) - RB_n(V_{dd} - V_T) \\ &\quad - \frac{B_n}{C}(V_{dd} - V_T)(t - \tau_r), \end{aligned} \quad (6)$$

where

$$V_1(\tau_r) = V_{dd} - \frac{B_n \tau_r}{2CV_{dd}} (V_{dd} - V_T)^2,$$

$t_{nsat}$  is the time when the NMOS transistor leaves the saturation region,  $V_{nsat}$  is the drain saturation voltage and is usually around  $0.7V_{dd}$  in short-channel devices [14]. As  $V_o = V_{nsat}$ ,  $t = t_{nsat}$ , where  $t_{nsat}$  is determined from Equation (6). Therefore, we have

$$\begin{aligned} \tau_{nsat} &= \tau_r \\ &+ \frac{C}{I_{D0}} \left( V_{dd} - V_{nsat} - \frac{\tau_r I_{D0}}{2C V_{dd}} (V_{dd} - V_T) - R I_{D0} \right). \end{aligned}$$

**Case III.**  $t \geq t_{nsat}$ : After  $V_o$  drops below  $V_{nsat}$ , the NMOS transistor enters the linear region:

$$\begin{aligned} C \frac{dV_1(t)}{dt} &= -\gamma_n V_{DS} = -\gamma_n V_o, \\ V_1(t) &= L \frac{dI_{DS}}{dt} + R I_{DS} + V_o, \end{aligned}$$

where  $\chi_n$  is the effective output conductance.

Therefore,

$$V_o = K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t},$$

where  $\alpha_{1,2} = \frac{\frac{1+R\gamma_n}{Lr_n} \pm \sqrt{\left(\frac{1+R\gamma_n}{Lr_n}\right)^2 - \frac{4}{LC}}}{2},$

$k_1$  and  $k_2$  can be determined from  $V_o(t_{nsat})$  and  $V_o'(t_{nsat})$ . Because  $r_1$  is typically much greater than  $r_2$ , we have

$$V_o = V_{nsat} e^{-\alpha_2(t - \tau_{nsat})}.$$

Therefore, the propagation delay time (50%) of a CMOS inverter is

$$t_{0.5} = \frac{1}{\alpha_2} \ln \frac{2V_{nsat}}{V_{dd}} + \tau_{nsat} - \frac{\tau_r}{2}.$$

### 3.3. Delay Model for Buffered RLC Trees

In this section, we extend our delay model to handle arbitrary balanced and un-balanced buffered RLC trees. For instance, consider an un-balanced buffered RLC tree with a root (or a source) and a set of leaves (or sinks) as shown in Figure 5. The buffer is inserted in an arbitrary location of the tree. Our delay model not only can handle different wire lengths but also can compute any percentage of delay time.

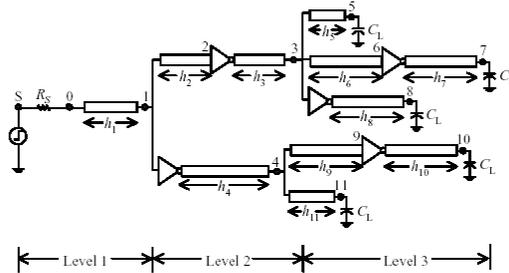


Figure 5: An un-balanced buffered RLC tree.

Suppose we are to compute the delay from the source S to node 7 (critical

path) in Figure 5. Buffer insertion divides the path into three stages. The path can be represented by the equivalent circuit shown in Figure 6.

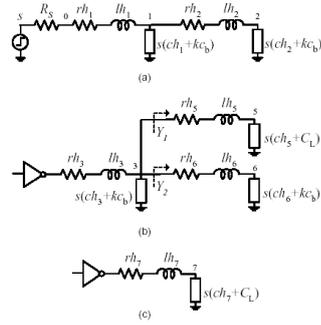


Figure 6: Representation of the path between the source  $S$  and node 7. (a) Stage 1 of the path; (b) Stage 2 of the path; (c) Stage 3 of the path.

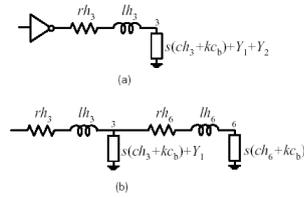


Figure 7: The equivalent circuits of Figure 6(b). (a) The equivalent RLY circuit for calculating gate delay; (b) The equivalent RLY circuit for calculating wire delay.

In order to calculate the delay time of stage 2, we show the equivalent circuits of Figure 6 (b) in Figure 7. We apply the method presented in Sections 3.1 and 3.2. Assume that the delay times of stage 1, stage 2, and stage 3 are  $t_1$ ,  $t_2$ , and  $t_3$ , respectively. The total delay between the source  $S$  and node 7 is

$$\tau_t = \sum_{i=1}^3 \tau_i, \quad (7)$$

## 五、成果 (Publications)

1. T.-C. Chen, S.-R. Pan, and Y.-W. Chang, "Timing modeling and optimization under the transmission line model," accepted and to appear in *IEEE Trans. VLSI Systems*, 2003.
2. S.-L. Wang, T.-C. Chen, and Y.-W. Chang, "Accurate Delay Modeling for Buffered RLY/RLC Trees," submitted to *Proc. of IEEE/ACM International Conference on Computer-Aided Design*, 2003.

## 六、參考文獻

1. A. I. Abou-Seido, B. Nowak, and C. Chu, "Fitted Elmore Delay: A Simple and Accurate Interconnect Delay Model," *Proc. ICCD*, pp. 422--217, Sep. 2002.
2. V. Adler and E. G. Friedman, "Uniform Repeater Insertion in RC Trees," *IEEE Trans. on CAS-IV*, vol. 47, no. 10, pp. 1515--1523, Oct. 2000.
3. R. Arunachalam, F. Dartu, and L. T. Pileggi, "CMOS Gate Delay Models for General RLC Loading," *Proc. ICCD*, pp. 224--229, Oct. 1997.
4. K. Banerjee and A. Mehrotra, "Analysis of On-Chip Inductance Effects using a Novel Performance Optimization Methodology for Distributed RLC," *Proc. DAC*, pp. 798--803, June 2001.
5. K. Banerjee and A. Mehrotra, "Analysis of On-Chip Inductance Effects for Distributed RLC Interconnects," *IEEE Trans. on CAD*, vol. 21, no. 8, pp.904--915, Aug. 2002.
6. W. C. Elmore, "The Transient Response of Damped Linear Networks with Particular Regard to Wide Band Amplifiers," *J. Applied Physics*,

- Vol. 19, No. 1, 1948.
7. D. S. Gao and D. Zhou, "Propagation Delay in RLC Interconnection Networks," Int. Symposium on Circuit and Systems, pp. 2125--2128, May 1993.
  8. Y. I. Ismail and E. G. Friedman, "Equivalent Elmore Delay for RLC Trees," IEEE Trans. on CAD, vol. 19, no. 1, pp. 83--97, Jan. 2000.
  9. Y. I. Ismail and E. G. Friedman, "Effects of Inductance on the Propagation Delay and Repeater Insertion in VLSI Circuits," IEEE Trans. on VLSI, vol. 8, no. 2, pp. 195--206, Apr. 2000.
  10. Y. I. Ismail and E. G. Friedman, "Repeater Insertion in Trees Structured Inductive Interconnect," IEEE Trans. on CAS-II, vol. 48, no. 5, pp. 471--481, May. 2001.
  11. A. B. Kahng and S. Muddu, "Optimal Equivalent Circuits for Interconnect Delay Calculations Using Moments," Proc. European Design Automation Conf., pp. 164--169, Sep. 1994.
  12. A. B. Kahng and S. Muddu, "Two-pole Analysis of Interconnection Trees," Proc. MCMC Conf., pp. 105--110, Jan. 1995.
  13. A. B. Kahng and S. Muddu, "An Analytical Delay Model for RLC Interconnects," IEEE Trans. on CAD, vol. 16, no. 12, pp. 1507--1514, Dec. 1997.
  14. Ankireddy Nalamalpu and Wayne Burleson, "Repeater Insertion in deep sub-micron CMOS: Ramp-based Analytical Model and Placement Sensitivity Analysis," Proceedings of International Symposium on Circuits and Systems, vol.3, pp. 766 -769, June. 2000.
  15. Semiconductor Industry Association, International Technology Roadmap for Semiconductors 1999 Edition, 1999.
  16. M. Sriram and S. M. Kang, "Performance Driven MCM Routing Using a Second Order RLC Tree Delay Model," Proc. IEEE Intl. Conf. on Wafer Scale Integration, pp. 262-267, Jan. 1993.
  17. T. Sakurai and A. R. Newton, "Alpha-Power Law MOSFET Model and its Applications to CMOS Inverter Delay and Other Formulas," IEEE Journal of Solid-State Circuits, Vol. 25, No. 2, pp. 584-594, Apr. 1990.
  18. K. T. Tang and E. G. Friedman, "Delay and Power Expressions Characterizing a CMOS Inverter Driving an RLC Load," Proceedings of the IEEE International Symposium on Circuits and Systems, pp. III.283-III.286, May 2000.