ON THE MOTION COORDINATION AND PLANNING OF MULTI-AXIS MANIPULATORS WITH ORTHOGONAL REGIONAL STRUCTURES

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ABSTRACT

This paper presents a general theory on the generation of piecewise constant speed profile for coordinated motion of multi-axis manipulators with orthogonal regional structures. Motion with constant speed is important and required in many manufacturing processes, such as milling, welding, finishing and painting. In this paper, a piecewise constant speed profile is constructed by a sequence of Hermite cuves to form a composite Hermite curve in parametric domain. Due to the continuity of acceleration in the proposed speed profile, it generates relatively better product quality than traditional techniques. Besides, we also provide a method for the feasibility study of manufacture capability in terms of the given manipulator, the desired path, and the assigned speed. This includes the consideration of manipulator dynamics, actuator limitation, path geometry, jerk constraints and motion kinematics. The result is a general one and is applicable to all curves tracked by multi-axis manipulators with orthogonal regional structures.

1. INTRODUCTION

Motion with constant speed is required in many manufacturing processes, such as milling, cutting, welding, finishing and painting, etc. The importance of constant-speed motion is that in most cases it generates relatively better (or, say, more uniform) quality than non-constant speed cases. For example, moving a painting or welding gun at a varying speed will result in non-uniform paint or seam thickness. Similarly, varying feedrate in CNC machining causes fluctuant metal flow and machine vibration and consequently poorer surface quality. In this paper, we present a general theory on the generation of coordinated piecewise constant speed motion for multi-axis manipulators. Here, the "piecewise constant speed motion" is defined as a motion which contains several constant speed segments and the speeds for various segments can be different. This problem of course contains the case of single stage constant speed motion. Besides, we also provide a method for the feasibility study of actuator capability in terms of the given manipulator, the desired path, and the assigned speed. This includes the consideration of manipulator dynamics, actuator limitation, path geometry and motion kinematics. The problem is defined below.

Let $\underline{P}(u) = (P_x(u), P_y(u), P_z(u))$ be a parametric equation of a desired spatial path to be followed by a multi-axis manipulator as shown in Fig.1a. Also let the segments from u_2 to u_3 and from u_4 to u_5 need to be tracked in constant speeds V_2 and V_4 respectively (also refer to Fig.1b), where u is a parameter. How can the manipulator follow this path accurately as well as smoothly? Also how to check the feasibility of an assigned speed profile with the consideration of the actuator limitation and jerk constraint? Currently, to achieve constant speed motion, most controllers use the trapezoidal speed profile. This method is suitable for tracking of straight lines with single-stage constant speed. However, if the path is curved or a piecewise constant speed is required, tracking with trapezoidal speed profile suffers the discontinuity of acceleration, consequently jerk becomes a problem when high-speed and high-precision are desired [1-6]. In this paper, instead of using trapezoidal velocity profile, Hermite curves (parametric cubic curves in geometric form) are used for representing each segment of the velocity profile (as shown in Fig.1b). The overall profile is a composite Hermite curve.

2. PIECEWISE CONSTANT SPEED PROFILE AS IN COMPOSITE HERMITE CURVES

2.1 Hermite Curves- Geometric Form of Parametric Cubic Curves

Here, we first introduce Hermite curves. A parametric curve defined by two end points and their corresponding first derivatives is called a Hermite curve. The curve has the following form:

$$q = F_1\ q_0 + F_2\ q_1 + F_3\ \overset{v}{q}_0 + F_4\ \overset{v}{q}_1 \tag{1}$$

where v is the parameter and $v \in [0, 1]$; q_0 and q_1 are two end point and $\overset{v}{q}_0$ and $\overset{v}{q}_1$ are their corresponding derivatives; and F_1, F_2, F_3, and F_4 are the blending functions which are:

$$F_1(v) = 2v^3 - 3v^2 + 1 \qquad F_2(v) = -2v^3 + 3v^2$$
$$F_3(v) = v^3 - 2v^2 + v \qquad F_4(v) = v^3 - v^2 \tag{2}$$

Equation (1) can be rewritten in matrix form as $q = \underline{F}\,\underline{B}$, where $\underline{F} = [F_1\ F_2\ F_3\ F_4]$ and $\underline{B}$ $(= [q_0\ q_1\ \overset{v}{q}_0\ \overset{v}{q}_1]^T)$ denotes the boundary vector.

2.2 Construction of a Hermite Speed Profile in V^2

Based on our previous understanding on motion path to multi-axis machine kinematics conversion [7], we decide to choose $\hat{V}^2$ instead of $\hat{V}$ to construct the speed profile. By so doing some unnecessary singularities in later calculation can be avoided. The boundary vector for the Hermite curve of $\hat{V}^2$ can be obtained as:

$$\underline{B} = [\hat{V}_0^2\ \ \hat{V}_1^2\ \ (\frac{d\hat{V}^2}{dv})_0\ \ (\frac{d\hat{V}^2}{dv})_1]^T \tag{3}$$

For a piecewise constant speed profile, a sequence of such Hermite curves may be joined together end to end to form a composite speed curve. To do so, the joined Hermite curves need to be reparametrized.

2.3 Reparametrization

For reparametrization, a linear relationship is required to preserve the cubic form of the parametric equations. When u is a new parametric variable which is designated as the variable of the entire composite speed profile, and u has a range in [0, 1]. Now let the i^{th} Hermite curve segment of the original speed profile which is in v domain from 0 to 1 be parametrized. This means to change the profile from v domain to u domain and the ranges from [0, 1] to $[u_i, u_{i+1}]$. This requires that

$$v = \frac{u - u_i}{u_{i+1} - u_i} \tag{4}$$

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By substituting Eqs.(4) into Eq.(1), the i^{th} segment of the composite curve equals

$$
\begin{aligned}
V^2(u) = {} & [2(\frac{u-u_i}{u_{i+1}-u_i})^3 - 3(\frac{u-u_i}{u_{i+1}-u_i})^2 + 1]V_i^2 + [-2(\frac{u-u_i}{u_{i+1}-u_i})^3 \\
& + 3(\frac{u-u_i}{u_{i+1}-u_i})^2]V_{i+1}^2 + [(\frac{u-u_i}{u_{i+1}-u_i})^3 - 2(\frac{u-u_i}{u_{i+1}-u_i})^2 + \\
& \frac{u-u_i}{u_{i+1}-u_i}](\frac{dV^2(u)}{du})_i(u_{i+1}-u_i) + [(\frac{u-u_i}{u_{i+1}-u_i})^3 - (\frac{u-u_i}{u_{i+1}-u_i})^2] \\
& (\frac{dV^2(u)}{du})_{i+1}(u_{i+1}-u_i) \qquad u = [u_i \; u_{i+1}]
\end{aligned} \tag{5}
$$

The boundary vector of $V^2(u)$ for i^{th} segment can be found as

$$
\begin{aligned}
\underline{B}_i &= [V_i^2 \;\; V_{i+1}^2 \;\; (\frac{dV^2}{du})_i(u_{i+1}-u_i) \;\; (\frac{dV^2}{dv})_{i+1}(u_{i+1}-u_i)]^T \\
&= [\hat{V}_0^2 \;\; \hat{V}_1^2 \;\; (\frac{d\hat{V}^2}{dv})_0 \;\; (\frac{d\hat{V}^2}{dv})_1]^T
\end{aligned} \tag{6}
$$

3. THEORY ON THE GENERATION OF COORDINATED MOTION

3.1 Desired Velocity and Acceleration for Coordinated Motion

Formulas for the coordinated motion of different type of multi-axis manipulators with orthogonal regional structure have been derived in [7-8].

In summary, velocities for coordinated motion have the following common form:

$$\dot{l}(u) = t_l(u)\, V \qquad l = x, y, z, \theta_1, \theta_2 \tag{7}$$

where $\dot{l}$ indicates the desired velocity of axis l which can be either a linear axis x, y, or z, or a rotary axis θ_1 or θ_2; $t_l(u)$ is the effective tangent of the motion which is a function of the path position $\underline{P}(u)$, tool orientation $\underline{O}(u)$ and machine configuration; And V is the tangential speed along the path.

The desired accelerations for coordinated motion are:

$$\ddot{l} = t_l(u)\, A + \kappa_l(u)\, V^2 \tag{8}$$

where the effective curvature $\kappa_l(u)$ equals $\frac{dt_l(u)}{ds}$;represents the arc length; And A is the tangential acceleration along the path.

3.2 Space/Time Conversion in Coordinated Motion

Equations (7) and (8) provide the desired velocity and acceleration in terms of the parameter u. For the command generation, it is necessary to obtain the position, velocity or acceleration as a function of time. The time required to traverse the specified path from u_0 to u is

$$t = \int_{u_0}^{u} \frac{\sqrt{\overset{u}{P}{}_x^2(u) + \overset{u}{P}{}_y^2(u) + \overset{u}{P}{}_z^2(u)}}{[V^2(u_0) + 2\int_{u_0}^{u} A(u')\sqrt{\overset{u'}{P}{}_x^2(u') + \overset{u'}{P}{}_y^2(u') + \overset{u'}{P}{}_z^2(u')}\, du']^{1/2}}\, du \tag{9}$$

where $\overset{u}{P}$ denotes dP/du. Eqs.(7)-(9) provide desired reference values for velocity and acceleration control in multi-axis machine controller.

3.3 Resultant Jerk

In motion control, the discontinuity of acceleration causes machine structures vibration. The vibrations may lead to tracking inaccuracies and machine parts wear. The rate of change of acceleration is referred to as jerk. Control of the jerk can often be more cost-effective solution to these problems than redesigning mechanical structures. An analytical way to calculate the resultant jerk in the coordinated motion of multi-axis machine is presented in [8]. It has the common form of

$$J_l = t_l(u)\frac{dA}{dt} + 3\kappa_l(u) V A + \frac{d\kappa_l(u)}{ds} V^3 \tag{10}$$

where J_l indicates the resultant jerk of axis l in the coordinated motion.

4. GENERATION OF PIECEWISE CONSTANT SPEED MOTION

In order to generate coordinated motion in piecewise constant speed, i.e., to obtain $\dot{l}$, $\ddot{l}$, t and J of Eqs.(7), (8), (9), and (10), the values of V, A and dA/dt are needed. Speed value V can be obtained from the constructed speed profile based on the assigned values. The next step is to formulate A and dA/dt.

There are two types of Hermite segments involved in the composite speed profile, namely segments of constant speed and segments of transition, as shown in Fig.1b. The tangential acceleration of a path can be obtained by the time derivative of the tangential speed, i.e.,

$$
\begin{aligned}
A(u) &= \frac{dV(u)}{du}\frac{du}{dt} = \frac{dV(u)}{du}\frac{V(u)}{\sqrt{\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2}} \\
&= \frac{1}{2\sqrt{\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2}}\frac{dV^2(u)}{du}
\end{aligned}
$$

Referring to Fig.1b, for a segment of constant speed profile, the acceleration should be zero at both end points of each segment of Hermite curve, that is:

$$A_i = A_{i+1} = (\frac{dV^2(u)}{du})_i = (\frac{dV^2(u)}{du})_{i+1} = 0$$

Therefore, Eq.(5) becomes:

$$
\begin{aligned}
V^2(u) = {} & [2(\frac{u-u_i}{u_{i+1}-u_i})^3 - 3(\frac{u-u_i}{u_{i+1}-u_i})^2 + 1]V_i^2 + [-2(\frac{u-u_i}{u_{i+1}-u_i})^3 + \\
& 3(\frac{u-u_i}{u_{i+1}-u_i})^2]V_{i+1}^2 = F_1(u)V_i^2 + F_2(u)V_{i+1}^2 \qquad u = [u_i \; u_{i+1}]
\end{aligned} \tag{11}
$$

4.1 Segments of Constant Speed

For constant speed portions (e.g., $u_2 - u_3$ or $u_4 - u_5$ in Fig.1b), the acceleration and change rate of acceleration both vanish, that is

$$V^2(u) = V_i^2 = V_{i+1}^2 \quad and \quad A(u) = \frac{dA(u)}{dt} = 0 \tag{12}$$

4.2 Segments of Transition

For transition portions (e.g., $u_1 - u_2$, $u_3 - u_4$ or $u_5 - u_6$ in Fig.1b), the acceleration function can be found as follows:

$$
\begin{aligned}
A(u) &= \overset{u}{F}_1(u)\frac{1}{2\sqrt{\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2}}V_i^2 + \overset{u}{F}_2(u)\frac{1}{2\sqrt{\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2}}V_{i+1}^2 \\
&\equiv CA_i(u)\, V_i^2 + CA_{i+1}(u)\, V_{i+1}^2
\end{aligned} \tag{13}
$$

Similarly, we can obtain the change rate of acceleration as

$$\frac{dA(u)}{dt} = \Big(\Big[\frac{\overset{uu}{F}_1(u)}{2(\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2)} - \overset{u}{F}_1(u)\frac{(\overset{u}{P}_x\overset{uu}{P}_x + \overset{u}{P}_y\overset{uu}{P}_y + \overset{u}{P}_z\overset{uu}{P}_z)}{2(\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2)^2}\Big]V_i^2 + \Big[\frac{\overset{uu}{F}_2(u)}{2(\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2)} - \overset{u}{F}_2(u)\frac{(\overset{u}{P}_x\overset{uu}{P}_x + \overset{u}{P}_y\overset{uu}{P}_y + \overset{u}{P}_z\overset{uu}{P}_z)}{2(\overset{u}{P}{}_x^2 + \overset{u}{P}{}_y^2 + \overset{u}{P}{}_z^2)^2}\Big]V_{i+1}^2\Big)V(u) \equiv (CJ_i(u)V_i^2 + CJ_{i+1}(u)V_{i+1}^2)V(u) \quad (14)$$

5. FEASIBILITY STUDY OF THE ASSIGNED SPEED PROFILE

For production engineers, it is sometimes desirable to know whether a set of assigned speed profile is feasible for the designated manipulator to execute. In other words, we want to know what is the limitation of a manipulator in terms of its motion capability. To answer this question involves the consideration of manipulator dynamics.

5.1 Dynamic Equations

The dynamics equations for different type of manipulators with orthogonal regional structure can be expressed in the following common form:

$$\kappa_{Fl}(u)V^2 + t_{Fl}(u)A = \mathscr{H}_{ml}E_{ml} - G_l V \quad (15)$$

where κ_{Fl} and t_{Fl} are the effective centrifugal and tangential inertia, respectively; $\mathscr{H}_{ml}$ represents the actuator limit; G_l is a coefficient including the effect of friction force, contact force and back EMF of motor.

5.2 Feasibility of Speed Profile under Dynamic and Jerk Constraints

5.2.1 For Transition Segments

From Eq.(15), the driving force or torque of a motor $\mathscr{H}_{ml}E_{ml}$ has its maximum and minimum values, which cannot be exceeded, that is:

$$(\mathscr{H}_{ml}E_{ml})_{\min} \le t_{Fl}A(u) + \kappa_{Fl}V^2(u) + G_lV(u) \le (\mathscr{H}_{ml}E_{ml})_{\max} \quad (17)$$

Equation (17) contains two equations which have the form as:

$$(\mathscr{H}_{ml}E_{ml})_{ext} - (t_{Fl}A(u) + \kappa_{Fl}V^2(u)) = G_lV(u) \quad (18)$$

where $(\mathscr{H}_{ml}E_{ml})_{ext}$ is either $(\mathscr{H}_{ml}E_{ml})_{\max}$ or $(\mathscr{H}_{ml}E_{ml})_{\min}$. After we take square on both sides, Eq.(18) becomes:

$$[t_{Fl}A(u) + \kappa_{Fl}V^2(u)]^2 - [2(\mathscr{H}_{ml}E_{ml})_{ext}\kappa_{Fl} + G_l^2]V^2(u) - 2(\mathscr{H}_{ml}E_{ml})_{ext}t_{Fl}A(u) + [(\mathscr{H}_{ml}E_{ml})_{ext}]^2 = 0 \quad (19)$$

Substitute Eqs.(11) and (13) into Eq.(19), we obtain:

$$A(u)V_{i+1}^4 + B(u, V_i, (\mathscr{H}_{ml}E_{ml})_{ext})V_{i+1}^2 + C(u, V_i, (\mathscr{H}_{ml}E_{ml})_{ext}) = 0 \quad (20)$$

Equation (20) can be casted into a form of a quadratic equation. If the transition portion has been specified by users (i.e., u_i and u_{i+1} are given) and also the speed at u_i is known, the feasible range of V_{i+1} can be obtained by taking into consideration of actuator limitation.

From Eqs.(10), (11), (13) and (14), the jerk equation has the common form as:

$$J_l = t_l\frac{dA(u)}{dt} + 3\kappa_lV(u)A(u) + \frac{d\kappa_l}{ds}V^3(u) = (t_l[CJ_i(u)V_i^2 + CJ_{i+1}(u)V_{i+1}^2] + 3\kappa_l[CA_i(u)V_i^2 + CA_{i+1}(u)V_{i+1}^2] + \frac{d\kappa_l}{ds}[F_1(u)V_i^2 + F_2(u)V_{i+1}^2])V(u) \quad (21)$$

In general, the jerk is expected to be limited within the range of $[-(J_l)_{\max}, (J_l)_{\max}]$, that is

$$\Big|t_l\frac{dA(u)}{dt} + 3\kappa_lV(u)A(u) + \frac{d\kappa_l}{ds}V^3(u)\Big| \le (J_l)_{\max} \quad (22)$$

In order to solve the feasible range of V_{i+1}, we take the square on both sides of Eq.(22) and obtain Eq.(23).

$$(t_l[CJ_i(u)V_i^2 + CJ_{i+1}(u)V_{i+1}^2] + 3\kappa_l[CA_i(u)V_i^2 + CA_{i+1}(u)V_{i+1}^2] + \frac{d\kappa_l}{ds}[F_1(u)V_i^2 + F_2(u)V_{i+1}^2])^2[F_1(u)V_i^2 + F_2(u)V_{i+1}^2] \le [(J_l)_{\max}]^2 \quad (23)$$

Rearrange Eq.(23), we obtain

$$V_{i+1}^6 + P(u, V_i)V_{i+1}^4 + Q(u, V_i)V_{i+1}^2 + R(u, V_i) \le S(u, V_i)[(J_l)_{\max}]^2 \quad (24)$$

Equation (24) can be casted into a form of a cubic equation and the range of allowable V_{i+1} is being solved.

5.2.2 For Constant Speed Segments

In constant speed segments, *A(u)* and *dA(u)/dt* are zero. Therefore Eqs.(17) and (22) can be reduced to Eqs.(25) and (26), respectively.

$$(\mathscr{H}_{ml}E_{ml})_{\min} \le \kappa_{Fl}V^2(u) + G_lV(u) \le (\mathscr{H}_{ml}E_{ml})_{\max} \quad (25)$$

$$-(J_l)_{\max} \le \frac{d\kappa_l}{ds}V^3(u) \le (J_l)_{\max} \quad (26)$$

Based on the Eqs.(17), (22), (25) and (26), we can obtain the feasible range of speed *V(u)* with an overall consideration of the actuator limitation and jerk constraint on both transition and constant speed portions. Because *l* indicates *x*, *y*, *z*, joint 1 or joint 2, there are totally twenty inequalities. Therefore, the final range of the feasible speed will be the intersection of these twenty ranges, and can be identified numerically.

6. ILLUSTRATIVE EXAMPLE

An example is given here for the illustration of the theory developed in this investigation.

1. Description of the problem

A five-axis gantry type of machine with Euler-angle rotational structure (shown in Fig.2) is asked to track a path defined by the following parametric equations:

$$P_x(u) = 0.2881\,u^3 - 0.3841\,u^2 + 0.3841\,u + 0.1150\ m$$

$$P_y(u) = -1.6935\,u^2 + 1.6935\,u + 0.0976\ m$$

$$P_z(u) = -0.1481\,u^3 - 0.2722\,u^2 - 0.2772\,u - 0.1110\ m$$

The path can be originally as this analytical form or can be originally defined by a sequence of discrete positions and then transform into this parametric form by applying spline interpolation. For simplicity, the corresponding orientations are only defined at u =0, 0.5 and 1 as *(-0.7276, -0.4851, 0.4851), (0.2887, -0.4081, 0.8661)* and *(-0.6860, -0.5145, 0.5145)* respectively. The inspection path and orientation are depicted in Fig.3.

Specifications of the gantry type of manipulator with Euler-angle structure:

a) Mass of the moving tables and joints

$$M_x = 400\,kg;\; M_y = 300\,kg;\; M_z = 100\,kg;$$

$$M_{AB} = 4\,kg;\; M_{BC} = 3\,kg;$$

b) Moment of inertia for links AB and BC

$$(I_{AB})_{z1z1} = 0.1\,kg-m^2;\quad (I_{BC})_{z2z2} = 0.1\,kg-m^2;$$

c) Length of links AB and BC

$$d_1 = 0.2\,m;\quad d_2 = 0.2\,m$$

d) Coefficients of motors,

$$h_{mx} = 400\,N\sec/m;\; h_{my} = 300\,N\sec/m;\; h_{mz} = 200\,N\sec/m;$$
$$h_{m\theta_1} = 4\,N\sec/rad;\quad h_{m\theta_2} = 4\,N\sec/rad$$

e) Maximum driving force of motors for each axis,

$$\mathscr{H}_{mx}E_{mx} = \pm\,1050\,N;\; \mathscr{H}_{my}E_{my} = \pm\,600\,N;\; \mathscr{H}_{mz}E_{mz} = [-300\; -1280]\,N;\; \mathscr{H}_{m\theta_1}E_{m\theta_1} = \pm\,100\,N-m;\; \mathscr{H}_{m\theta_2}E_{m\theta_2} = \pm\,50\,N-m;$$

f) Maximum allowable jerk,

$$J_l = \pm\,20m/\sec^3 \;\; or \;\; \pm\,78rad/\sec^3;$$

If we expect the constant speed at the segment u = [0.2, 0.44] is 0.63 m/sec and 0.42 m/sec at the segment u= [0.55, 0.8]. A software package is generated based on the formulas derived in this investigation. Giving below are the results from solving the problem at hand.

2. **The piecewise constant speed profile:** The piecewise constant speed profile constructed by a composite Hermite curve is shown in Fig.4.

3. **Desired displacement for coordinated motion:** The desired displacements of each axis for coordinated motion, which can be used as the references for position control, are shown in Fig.5.

4. **Desired velocity for coordinated motion:** The desired velocities of x, y, z axes and joint 1 and joint 2 for a coordinated motion are plotted in Fig.6. These are informations for velocity control.

5. **Desired acceleration for coordinated motion:** Coordinated velocity is originated from coordinated acceleration. Figure 7 shows the acceleration components of each axis for the coordinated motion

6. **Conversion between parameter and time:** Then the relationship between parameter u and time t can be obtained by Eq.(9). Figure 8 shows the relationship between parameter and time for the piecewise constant speed profile.

7. **Resultant jerk in coordinated motion:** The resultant jerks in coordinated motion are shown in Fig.9.

8. **Required capacity of manipulator:** The required capacities of motors for each moving axis are shown in Fig.10. Figures 9 and 10 provide references for feasibility check of allowable jerk and actuator capability.

7. CONCLUDING REMARKS

This paper presents an analytical study on thegeneration of a piecewise constant speed profile for the coordinated motion of multi-axis manipulator with orthogonal regional structures. The optimal constant speed profile for a specified path and assigned manipulator has been analytically established by considering path geometry and manipulator dynamics. The result is a general one and is applicable to all parametric curves tracked by any cartesian type of five-axis manipulator. We believe that this work should be useful in the integration of automatic CAD/CAM processes.

ACKNOWLEDGMENT

The financial support of IMAR/NSF is greatly appreciated. IMAR stands for the Institute for Manufacturing and Automation Research which is financially supported by some local companies in the great Los Angeles area and by the government of the state of California.

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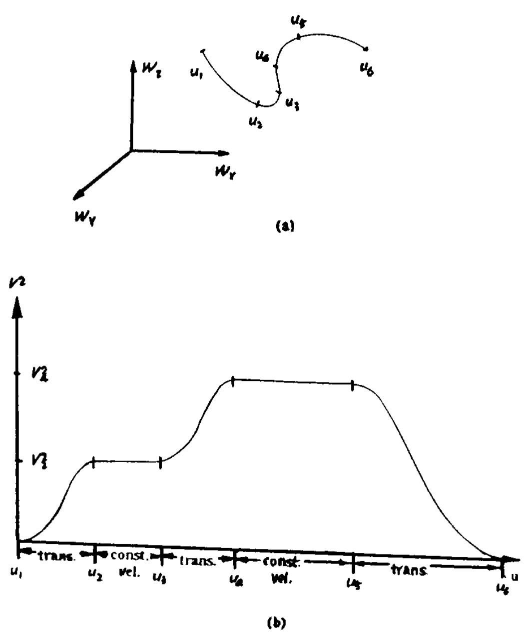

Figure 1: A Spatial Path and Piecewise Constant Speed Profile

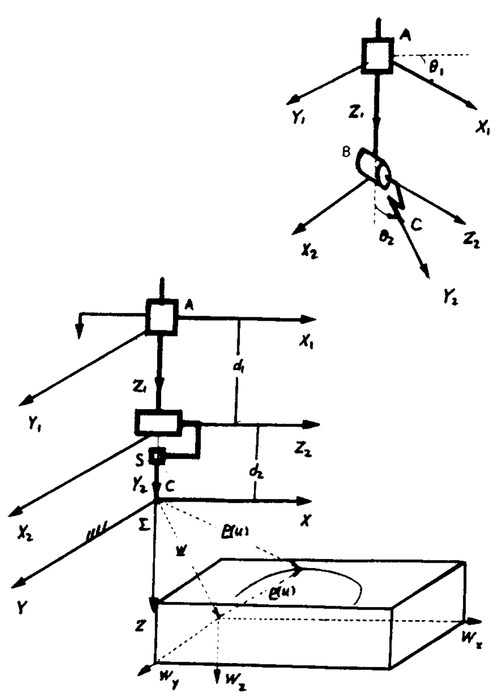

Figure 2: Euler-Angle Rotational Structure

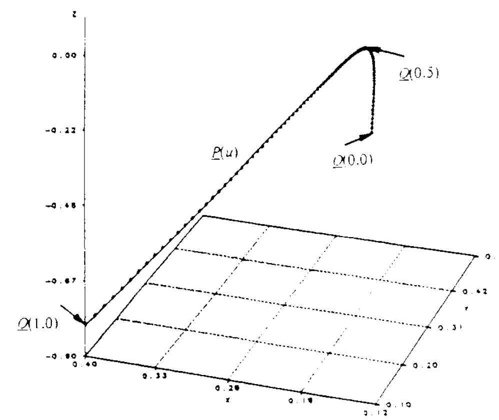

Figure 3: Specific Path and Orientation

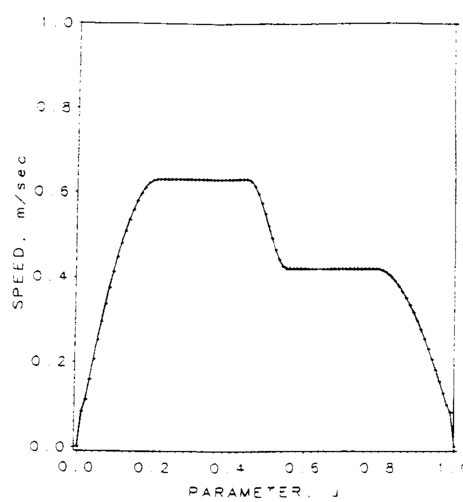

Figure 4: Piecewise Constant Speed Profile

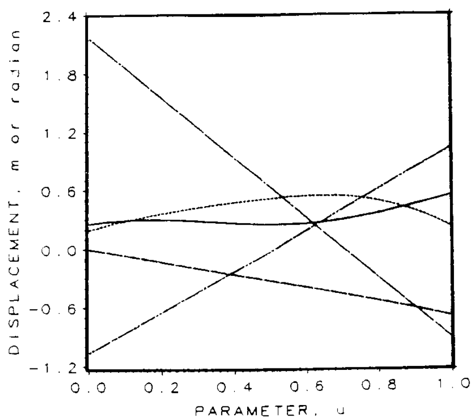

Figure 5: Desired Displacement for Coordinated Motion

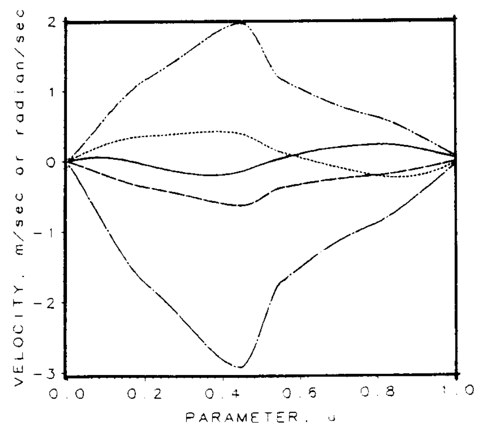

Figure 6: Desired Velocity for Coordinated Motion

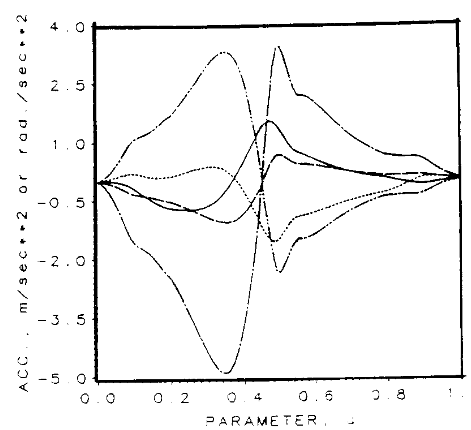

Figure 7: Desired Acceleration for Coordinated Motion

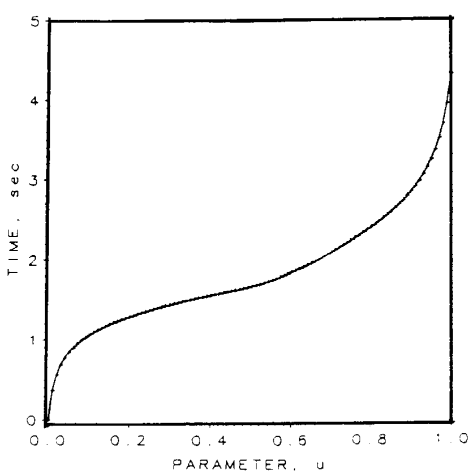

Figure 8: The Relationship Between Time and Parameter

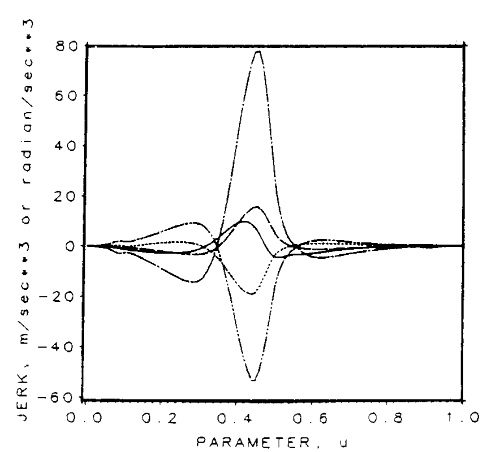

Figure 9: Resultant Jerk in Coordinated Motion

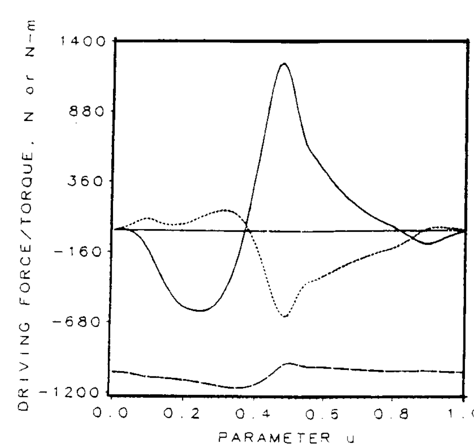

Figure 10: Required Capacity of Machines