

行政院國家科學委員會專題研究計畫 成果報告

失衡性單向隨機線型模式的容許區間之研究

計畫類別：個別型計畫

計畫編號：NSC93-2118-M-002-009-

執行期間：93年08月01日至94年07月31日

執行單位：國立臺灣大學農藝學系暨研究所

計畫主持人：廖振鐸

報告類型：精簡報告

處理方式：本計畫可公開查詢

中 華 民 國 94 年 10 月 19 日

行政院國家科學委員會專題研究計畫成果報告

失衡性單向隨機線型模式的容許區間之研究
**A Tolerance Interval for Unbalanced One-Way
Random Effects Models**

計畫類別: 個別型計畫

計畫編號: NSC-93-2118-M-002-009

執行期間: 93 年 8 月 1 日至 94 年 7 月 31 日

計畫主持人: 廖 振 鐸

處理方式: 可立即對外提供參考

執行單位: 國立台灣大學農藝學研究所試驗統計組

中華民國 93 年 10 月 20 日

中文摘要

容許區間 (tolerance interval) 被廣泛地運用於各個領域中，舉凡在工程方面、製藥業方面等等的應用。Hahn and Meeker (1991) 整理了許多容許區間的建構方法及其應用的文獻回顧。然而，其中大部份僅針對單一變量的問題做討論，並未介紹較複雜模型下的容許區間。其次，有別於單一變量的研究中，多數的文獻針對均衡單向隨機效應模型 (balanced one-way random effects model) 來討論的有 Lemon (1977), Mee and Owen (1983), Mee (1984) 以及 Vangel (1992) 等。最近，Liao and Iyer (2003) 建構了一套方法來解決所有均衡線性混合模型的容許區間問題。

本研究針對在實際應用上常遇到的一失衡性單向隨機效應模型 (unbalanced one-way random effects model) 一容許區間建構問題來討論。其關鍵主要在於失衡性資料在傳統的 ANOVA 平方和分解會產生非相互獨立 (mutually independent) 的卡方隨機變數 (chi-squared random variables)。本研究藉由 Weerahandi (1993) 所提出的廣義樞紐量 (generalized pivotal quantities)、廣義信賴區間 (generalized confidence intervals) 的觀念以及 LaMotte and McWhorter (1978) 的平方和分解 (canonical decomposition) 方式，建立一個確切(exact)的單尾容許區間及良好近似的(approximate)雙尾容許區間。並且利用統計模擬來驗證此方法之可行性。本研究主要成果已整理成英文論文，已發表在國際期刊 *Technometrics*, 2005, vol 47, pp 323-335. 。

關鍵詞：廣義檢定函數，廣義信賴區間，變異數成分，線型模式。

英文摘要

Statistical tolerance intervals are useful in practical applications in Engineering, Pharmaceutical industries, and many other areas. Construction of tolerance intervals for univariate distributions has been extensively studied, particularly for exponential family distributions such as normal, lognormal, exponential etc. An excellent review of these procedures and their applications are given in Hahn and Meeker (1991). However, tolerance intervals have not been explored in detail for more complex situations. Most of the papers on the tolerance interval problem have concentrated on the balanced one-way random effects model. The reader may refer to Lemon (1977), Mee and Owen (1983), Mee (1984), and Vangel (1992) among others. Recently, Liao and Iyer (2003) have developed a tolerance interval procedure applicable to all balanced linear mixed models.

In this study, we develop procedures for constructing an exact one-sided tolerance interval and an approximate two-sided tolerance interval for a general setting. The proposed methods can be applicable to the general balanced mixed models and one-way unbalanced random models. Our specific interest is given to the unbalanced one-way random model since it is a very common model used in various practical experiments. The proposed methods are derived from the concept of generalized pivotal quantities introduced by Weerahandi (1993). In addition, statistical simulation studies are conducted to verify that the proposed methods can be recommended for practical use. Partial results of this research have been published in *Technometrics*, vol 47, pp323-335, 2005.

Keywords: generalized p-values, generalized test variables, generalized confidence intervals, variance components.