行政院國家科學委員會專題研究計畫 成果報告

分散效應存在下最適 2-變級設計之研究
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 95-2118-M-002-008-
執 行 期 間 ：95 年 08 月 01 日至 96 年 07 月 31 日
執 行 單 位 ：國立臺灣大學農藝學系暨研究所

計畫主持人：廖振鐸

計畫參與人員：博士班研究生-兼任助理：李其融、陳嘉瑩
碩士班研究生-兼任助理：王愷忻

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 96 年 10 月 15 日

行政院國家科學委員會專題研究計畫成果報告

分散效應存在下之最適2-變級設計之研究

Optimal Two-level Fractional Factorial Designs in the Presence of Dispersion Effects

計畫類別: 個別型計畫

計畫編號: NSC-95-2118-M-002-008

執行期間: 95 年 8 月 1 日 至 96 年 7 月 31 日

計畫主持人: 廖 振 鐸

處理方式: 可立即對外提供參考

執行單位: 國立台灣大學農藝學研究所試驗統計組

中華民國 96 年 10 月 12 日

中文摘要

在實驗初期通常實驗資料的變因有很多, 因此 2-變級複因子設計被廣泛用於估計實驗中重要的位置效應。而同質變方是變方分析中最基本的假設, 在此假設成立下, 最適2^{n-p}部分複因子設計的研究已相當完善。但是當實驗的反應變數之變異程度, 會因某些因子在不同變級而有顯著的差異時, 我們稱這些因子爲分散因子。然而在分散效應存在下, 如何決定最適設計是一個在文獻上少見的研究議題, 若能找出合適的設計, 不但可以減低實驗的成本, 也可以提高實驗的效率, 因此這是個值得深入探討的研究議題。本研究是針對在已知具有分散因子存在, 而欲估計的位置效應未知的情況下, 找尋最適的 2^{n-p}部分複因子設計。所以利用最小偏差準則的概念, 建構出最適設計。首先從最簡單的模式: 僅有一個實驗因子會影響分散效應, 開始探討。由於最適設計的決定會受分散效應的影響, 所以字元長度需要作調整以建立新的最小偏差準則。然後藉由 R-電腦程式做完整的搜尋, 找出最適的2^{n-p}部分複因子設計以及在此設計哪些因子適合被指定爲分散因子, 並將一些實用的設計列表以供查詢使用。進一步我們探討當有兩個實驗因子會影響分散效應的模式, 並將一些實用的設計列表以供查詢使用。

關鍵詞: 部分複因子設計; 分散因子; 位置效應; 最小偏差準則.

Abstract

During the initial stages of experimentation, two-level regular fractional factorial designs (FFDs) are commonly used to identify important factors which may significantly affect the response(s) of the experiment. The homogeneity of variance is a basic assumption in the ANOVA analysis for location effects. The design issue of optimal 2^{n-p} regular FFDs based on the homogenous variance assumption has been studied extensively. However, when the variance of the response variable changes as some specific factors change from one setting to another, these factors affecting the variation of the response are called as *dispersion factors* in this study. Interestingly, to the best of our knowledge, the issue addressing the minimum aberration designs for location effects in the presence of dispersion factors has not been found in the literature. In this study, we shall investigate the minimum aberration 2^{n-p} regular FFDs under the assumption that there are some specific factors responsible for the dispersion of the response. The dispersion effects may violate the usual assumption of variance homogeneity in ANOVA. Therefore, the aberration criterion needs to be modified in order to discuss this issue. It is anticipated that the choice of minimum aberration designs may depend upon the prior information on the dispersion effects. Specific attention will first be given to the simplest situation that there is exactly one factor responsible for the dispersion effects. After a thorough investigation on this, we extend the results to the situation that two factors involve the dispersion effects.

Keywords: Fractional factorial design; Dispersion factor; Location effect; Minimum aberration criterion.

1 Introduction

During the initial stages of experimentation, two-level fractional factorial designs (FFDs) are commonly used to screen out important factorial effects. The issue regarding the optimal 2^{n-p} regular FFDs based on the aberration criterion has been studied extensively, see Wu and Hamada (2000). The minimum aberration designs are basically derived from ANOVA models with homogenous variance assumption. Recently, the study on the dispersion effects in factorial designs has drawn considerable attention, see Box and Meyer (1986), Wang (1989), Bergman and Hynên (1997), Pan (1999), Brenneman and Nair (2001), McGrath and Lin (2001a, 2001b), among others. However, almost all of the works cited above focus on the analysis method of identifying important location effects and dispersion effects. On the other hand, Liao and Iyer (2000) discuss the choice of two-level designs for estimating all location main effects and the dispersion effects affected by one specific factor. Furthermore, Liao (2006) studies the choice of two-level designs for estimating all location main effects and identifying the factor responsible for the dispersion effects among all possible candidate factors. Interestingly, to the best of our knowledge, the issue of addressing the minimum aberration designs for location effects in the presence of dispersion factors has not been found in the literature.

In this study, we shall investigate the minimum aberration 2^{n-p} regular FFDs under the assumption that there are some specific factors responsible for the dispersion of the response. The dispersion effects may violate the usual assumption of variance homogeneity in ANOVA. Therefore, the aberration criterion needs to be modified in order to discuss this issue. It is anticipated that the choice of minimum aberration designs may depend upon the prior information on the dispersion effects. Specific attention will first be given to the simplest situation that there is exactly one factor responsible for the dispersion effects. After a thorough investigation on this, we extend the results to the situation that two factors involve the dispersion effects.

The rest of the article is organized as follows. In the next section, we discuss the minimum aberration 2^{n-p} FFDs with one dispersion factor and present the table designs of run sizes equal to 16 and 32. Section 3 contains the results for the two-dispersion-factor situation. Discussion and final remarks are given in the final section.

2 Minimum Aberration 2^{n-p} Fractional Factorial Designs with One Dispersion Factor

The use of minimum aberration designs is mainly based on some practical principles such as the effect sparse principle and the hierarchical ordering principle (Wu and Hamada, 2000). The user would obtain all possible lower-order estimable location effects using such designs under the homogenous variance assumption. However, when the dispersion of response changes with the distinct levels of a specific factor, this may impact the construction of designs according to the minimum aberration criterion.

In a regular 2^{n-p} FFD, we consider the situation that there is exactly one specific factor responsible for the dispersion effects, this factor is called as the dispersion factor. Also the dispersion factor is known a prior and, without loss of generality, it is assigned to the first factor A, where factor A is also labeled as factor 1, i.e. A $\rightarrow$ 1. Similarly, the other $n-1$ factors are labeled as 2, 3, $\cdots$, n, i.e. B $\rightarrow$ 2, C $\rightarrow$ 3 and so on. Let σ_H^2 be the variance of the response when the dispersion factor is set at its high level and σ_L^2 be the variance of the response when it is set at its low level. Both σ_H^2 and σ_L^2 are positive numbers.

Let $\boldsymbol{y}$ be the response vector corresponding to the design used in the experiment. Then the linear model can be written in the following matrix form.

$$\mathrm{E}(\boldsymbol{y}) = \boldsymbol{X}\boldsymbol{\beta}, \quad \mathrm{V}(\boldsymbol{y}) = \gamma_0 \boldsymbol{I}_N + \gamma_1 \boldsymbol{D}_1 = \gamma_0(\boldsymbol{I}_N + \rho \boldsymbol{D}_1) \tag{2.1}$$

where $\boldsymbol{\beta}$ is an unknown $v \times 1$ vector consisting of the maximum number of possibly estimable lower-order effects, indicating that $\boldsymbol{\beta}$ is composed of exactly one effect from each of the 2^{n-p} alias sets determined by a regular 2^{n-p} FFD. That is, $v = N = 2^{n-p}$. $\boldsymbol{X}$ is the $N \times N$ model matrix with elements $+1$ or -1 determined by the used design; γ_0 is the dispersion mean, i.e. $\gamma_0 = (\sigma_H^2 + \sigma_L^2)/2$; γ_1 is the dispersion main effect of factor 1, i.e. $\gamma_1 = (\sigma_H^2 - \sigma_L^2)/2$; $\rho = \gamma_1/\gamma_0$ and $-1 < \rho < 1$; $\boldsymbol{I}_N$ is the identity matrix of order N; $\boldsymbol{D}_1$ is a diagonal matrix whose diagonal elements correspond to the levels of the dispersion factor in the treatment combinations. The j^{th} diagonal element of $\boldsymbol{D}_1$ is $+1$ or -1 according to whether the dispersion factor occurs at its high level or low level, respectively, in response j.

Based on (2.1) and the assumption that the dispersion parameters γ_0 and ρ are known, the normal equations of BLUE (Best Linear Unbiased Estimator) $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ are given by

$$\boldsymbol{X}'[\mathrm{V}(\boldsymbol{y})]^{-1}\boldsymbol{X}\hat{\boldsymbol{\beta}} = \boldsymbol{X}'[\mathrm{V}(\boldsymbol{y})]^{-1}\boldsymbol{y}, \tag{2.2}$$

where $[\mathrm{V}(\boldsymbol{y})]^{-1} = (\gamma_0)^{-1}(1-\rho^2)^{-1}(\boldsymbol{I}_N - \rho\boldsymbol{D}_1)$. Moreover, the variance-covariance matrix of $\hat{\boldsymbol{\beta}}$ is given by

$$Cov(\hat{\boldsymbol{\beta}}) = (\boldsymbol{X'}[\mathrm{V}(\boldsymbol{y})]^{-1}\boldsymbol{X})^{-1}.$$

The matrix $\boldsymbol{M} = \boldsymbol{X'}[\mathrm{V}(\boldsymbol{y})]^{-1}\boldsymbol{X}$ is the information matrix for $\boldsymbol{\beta}$. Let m_{ij} denote the $(i+1, j+1)th$ element of $\boldsymbol{M}$. Then m_{ij} is equivalently rewritten by

$$m_{ij} = (\gamma_0)^{-1}(1-\rho^2)^{-1}[(\boldsymbol{x}_i \circ \boldsymbol{x}_0 \circ \boldsymbol{x}_j) - \rho(\boldsymbol{x}_i \circ \boldsymbol{x}_1 \circ \boldsymbol{x}_j)],$$

where the operation $\circ$ represents the generalized inner product, i.e. $\boldsymbol{x}_i \circ \boldsymbol{x}_j \circ \boldsymbol{x}_k = \sum_{l=1}^{N} x_{il}x_{jl}x_{kl}$; $\boldsymbol{x}_0$ denotes the unit vector corresponding to the mean effect μ; $\boldsymbol{x}_1$ the second column of $\boldsymbol{X}$ corresponding to the main effect of factor 1; $\boldsymbol{x}_i$ and $\boldsymbol{x}_j$ correspond to the $(i+1)th$ and the $(j+1)th$ elements of $\boldsymbol{\beta}$, respectively.

There are three possible values of m_{ij} if the used design is restricted to the regular 2^{n-p} FFDs of resolution III or higher.

1. When $\boldsymbol{x}_i \circ \boldsymbol{x}_0 \circ \boldsymbol{x}_j = N$ and $\boldsymbol{x}_i \circ \boldsymbol{x}_1 \circ \boldsymbol{x}_j = 0$, the m_{ij} is equal to $m_0 = N(\gamma_0)^{-1}(1-\rho^2)^{-1}$. Clearly, when $i = j$, the m_{ij} is equal to m_0, so all of the diagonal elements of $\boldsymbol{M}$ are equal to m_0.
2. When $\boldsymbol{x}_i \circ \boldsymbol{x}_0 \circ \boldsymbol{x}_j = 0$ and $\boldsymbol{x}_i \circ \boldsymbol{x}_1 \circ \boldsymbol{x}_j = N$, the m_{ij} is equal to $m_1 = -N(\gamma_0)^{-1}(1-\rho^2)^{-1}\rho$.
3. When $\boldsymbol{x}_i \circ \boldsymbol{x}_0 \circ \boldsymbol{x}_j = 0$ and $\boldsymbol{x}_i \circ \boldsymbol{x}_1 \circ \boldsymbol{x}_j = 0$, the m_{ij} is equal to zero.

The regular 2^{n-p} FFDs in the current study are assumed to have resolution at least III, ensuring that all main effects are contained in $\boldsymbol{\beta}$. The r-letter words in the defining relations of the used design may be classified into two types according to whether they involve factor 1 or not. Those are $I = kw_iw_j$ and $I = 1w_iw_j$, where $k \neq 1$ and w_i and w_j denote any two distinct words excluding 1 and k. Without loss of generality, it is assumed that the two effects corresponding to w_i and w_j are contained in $\boldsymbol{\beta}$, then we have two possible submatrices, whose rows and columns are indexed with w_i and w_j, given by

$$\boldsymbol{M_0} = \begin{bmatrix} m_0 & 0 \\ 0 & m_0 \end{bmatrix}, \text{ if } I = kw_iw_j \text{ exists in the defining relations of the used design,}$$

and

$$\boldsymbol{M_1} = \begin{bmatrix} m_0 & m_1 \\ m_1 & m_0 \end{bmatrix}, \text{ if } I = 1w_iw_j \text{ exists in the defining relations of the used design.}$$

It is interesting to notice that each row or each column of $\boldsymbol{M}$ has at most one entry equal to m_1. A simple argument for this observation is as follows. Suppose there are two m_1 in the $(k+1)$th row which is indexed with effect w_k, i.e. we assume that $I = 1w_kw_g$ and $I = 1w_kw_h$. This must result in $I = (1w_kw_g)(1w_kw_h) = w_gw_h$, indicating that w_g and w_h must be in the same alias sets. This contradicts the assumption that $\boldsymbol{\beta}$ consists of exactly one effect of each alias set.

The submatrix $\boldsymbol{M_0}$ is non-singular and orthogonal. On the other hand, the submatrix $\boldsymbol{M_1}$ is non-singular but non-orthogonal. Hence, a 2^{n-p} FFD with the aliasing word of type kw_iw_j ensures that the effects w_i and w_j are estimated with the highest efficiency. The one with the aliasing word of type $1w_iw_j$ still has the effects w_i and w_j estimable but less efficiency. Therefore, it is requested to find the design with relatively smaller number of aliasing words of type $1w_iw_j$ for the same r-letter words. This will guide us to develop a new criterion of minimum aberration.

The minimum aberration criterion is originally derived based on the practical principle of effect hierarchy, from which, (i) the lower-order effects are more likely to be significant than higher-order effects, i.e., the lower-order effects are assumed to be more important than the higher-order effects; and (ii) effects of the same order are assumed to be equally important. However, when one of the factors is recognized as the dispersion factor and the difference between the values of σ_H^2 and σ_L^2 is significantly large, then the BLUEs of effects w_i and w_j which involve the type of submatrix $\boldsymbol{M_1}$ may become hardly reliable. Suppose that the absolute value of $\rho = \gamma_1/\gamma_0 = (\sigma_H^2 - \sigma_L^2)/(\sigma_H^2 + \sigma_L^2)$ is very close to 1, i.e., one of σ_H^2 and σ_L^2 is significantly larger than the other. And the submatrix $\boldsymbol{M_1}$ of $\boldsymbol{M}$ is explicitly expressed as

$$\boldsymbol{M_1} = \begin{bmatrix} m_0 & m_1 \\ m_1 & m_0 \end{bmatrix} = N(\gamma_0)^{-1}(1-\rho^2)^{-1} \begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}.$$

Hence, it is obvious to see that the two rows of $\boldsymbol{M_1}$ are close to completely dependent if $|\rho|$ approaches 1 (see (2.2)), i.e., the effects w_i and w_j can turn out completely aliased. Consequently, the existence of defining relation $I = 1w_iw_j$ in a design may make the estimation of effects w_i and w_j less efficient, or even unestimable. Taking this argument into account with the effect hierarchy principle, we propose a new minimum aberration criterion in the following.

Let the *wordlength pattern without the dispersion factor* involved be defined by

$$W_0(d) = (A_{3,0}(d), A_{4,0}(d), \cdots, A_{n,0}(d)),$$

where $A_{r,0}$ is the number of words without the dispersion factor of length r in the defining relations . Similarly, the *wordlength pattern with the dispersion factor* involved is defined by

$$W_1(d) = (A_{3,1}(d), A_{4,1}(d), \cdots, A_{n,1}(d)),$$

where $A_{r,1}$ is the number of words with the dispersion factor of length r in the defining relations. The wordlength pattern of a design in the presence of dispersion factor is defined by

$$W(d) = ((A_{3,1}(d), A_{3,0}(d)), (A_{4,1}(d), A_{4,0}(d)), \cdots, (A_{n,1}(d), A_{n,0}(d))), \qquad (2.3)$$

where $A_{r,0}$ is inserted between $A_{r,1}$ and $A_{r+1,1}$, for $r \geq 3$. Also let $A_r(d) = A_{r,1}(d) + A_{r,0}(d)$. Therefore, we may use (2.3) to construct the minimum aberration designs under the assumption that there exists a specific factor responsible for the dispersion of response. The definition of minimum aberration criterion under the current study is modified as follows.

Definition. For any two 2^{n-p} FFDs d_1 and d_2, let s be the smallest r such that $A_{s,1}(d_1) < A_{s,1}(d_2)$, or $A_{s,1}(d_1) = A_{s,1}(d_2)$ but $A_{s,0}(d_1) < A_{s,0}(d_2)$, then d_1 is said to have less aberration than d_2. If there is no other design with less aberration than d_1, then d_1 has minimum aberration.

For example, consider two 2^{7-3} FFDs. Let d_1 be determined by the defining relations $I = 1235 = 1246 = 1347 = 3456 = 2367 = 2457 = 1567$ and d_2 be determined by the defining relations $I = 125 = 136 = 247 = 2356 = 1457 = 34567 = 123467$. The wordlength patterns of d_1 and d_2 are given by

$$W(d_1) = ((0, 0), (4, 3), (0, 0), (0, 0), (0, 0))$$

and

$$W(d_2) = ((2, 1), (1, 1), (0, 1), (1, 0), (0, 0)).$$

$A_{3,1}(d_1) < A_{3,1}(d_2)$, so d_1 has less aberration than d_2.

We would take advantage of the algorithm provided in Chen, Sun and Wu (1993) to generate our designs with $N = 16$ and 32. The procedures are given below.

Step 1: For a fixed pair of (n, p) with $2^{n-p} = 16$ or 32, generate all possible non-isomorphic designs using the Chen-Sun-Wu algorithm.

Step 2: Screen out the ones with the maximum resolution from the resulting non-isomorphic designs.

Step 3: Calculate the wordlength pattern, defined by (2.3), for each maximum resolution design by treating factor i as the dispersion factor, for $i = 1, 2, \cdots, n$.

Step 4: Select the minimum aberration design among all possible candidate designs generated from Step 3.

The resulting 16-run and 32-run minimum aberration 2^{n-p} FFDs with one dispersion factor are displayed in Tables 1 and 2, respectively. In the tables, we also list the factors that can be assigned as the dispersion factor. Note that the first nine factors are labeled by 1, 2, $\cdots$, 9 as usual, and the $(10+i)$th factor is labeled by t_i, for $i = 0, 1, \cdots, 9$.

Table 1: *The 16-run minimum aberration 2^{n-p} FFDs with dispersion factor 1.*

Design	Generators
5-1	$5 = 1234$
6-2	$5 = 123,\ 6 = 124$
7-3	$5 = 123,\ 6 = 124,\ 7 = 134$
8-4	$5 = 123,\ 6 = 124,\ 7 = 134,\ 8 = 234$
9-5	$5 = 12,\ 6 = 23,\ 7 = 24,\ 8 = 134,\ 9 = 1234$
10-6	$5 = 12,\ 6 = 23,\ 7 = 13,\ 8 = 24,\ 9 = 134,\ t_0 = 1234$
11-7	$5 = 12,\ 6 = 13,\ 7 = 23,\ 8 = 14,\ 9 = 24,\ t_0 = 134,\ t_1 = 234$
12-8	$5 = 12,\ 6 = 13,\ 7 = 23,\ 8 = 14,\ 9 = 24,\ t_0 = 134,\ t_1 = 234,\ t_2 = 1234$
13-9	$5 = 12,\ 6 = 23,\ 7 = 13,\ 8 = 123,\ 9 = 24,\ t_0 = 14,\ t_1 = 124,\ t_2 = 34,\ t_3 = 234$
14-10	$5 = 12,\ 6 = 13,\ 7 = 23,\ 8 = 123,\ 9 = 14,\ t_0 = 24,\ t_1 = 124,\ t_2 = 34,\ t_3 = 134,\ t_4 = 234$
15-11	$5 = 12,\ 6 = 13,\ 7 = 23,\ 8 = 123,\ 9 = 14,\ t_0 = 24,\ t_1 = 124,\ t_2 = 34,\ t_3 = 134,\ t_4 = 234,\ t_5 = 1234$

Table 2: *The 32-run minimum aberration 2^{n-p} FFDs with dispersion factor 1.*

Design	Generators
6-1	$6 = 12345$
7-2	$6 = 234$, $7 = 1245$
8-3	$6 = 235$, $7 = 245$, $8 = 1345$
9-4	$6 = 239$, $7 = 249$, $8 = 259$, $9 = 1345$
10-5	$6 = 123$, $7 = 124$, $8 = 125$, $9 = 1345$, $t_0 = 2345$
11-6	$6 = 235$, $7 = 245$, $8 = 345$, $9 = 234$, $t_0 = 125$, $t_1 = 135$
12-7	$6 = 235$, $7 = 245$, $8 = 345$, $9 = 234$, $t_0 = 125$, $t_1 = 135$, $t_2 = 145$
13-8	$6 = 23t_3$, $7 = 24t_3$, $8 = 34t_3$, $9 = 234$, $t_0 = 25t_3$, $t_1 = 35t_3$, $t_2 = 235$, $t_3 = 145$
14-9	$6 = 123$, $7 = 124$, $8 = 134$, $9 = 234$, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$
15-10	$6 = 123$, $7 = 124$, $8 = 134$, $9 = 234$, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$, $t_5 = 345$

3 Minimum Aberration 2^{n-p} Fractional Factorial Designs with Two Dispersion Factors

In this section, we extend the results to the 2^{n-p} FFDs with two dispersion factors. Let the *wordlength pattern without the dispersion factor 1 or 2* involved be defined by

$$W_0(d) = (A_{3,0}(d), A_{4,0}(d), \cdots, A_{n,0}(d)),$$

where $A_{r,0}$ is the number of words without the dispersion factor 1 or 2 of length r in the defining relations. Similarly, the *wordlength pattern with the dispersion factor 1* involved is defined by

$$W_1(d) = (A_{3,1}(d), A_{4,1}(d), \cdots, A_{n,1}(d)),$$

where $A_{r,1}$ is the number of words with factor 1 of length r in the defining relations; the *wordlength pattern with the dispersion factor 2* involved is defined by

$$W_2(d) = (A_{3,2}(d), A_{4,2}(d), A_{5,2}(d), \cdots, A_{n,2}(d)),$$

where $A_{r,2}$ is the number of words with factor 2 of length r in the defining relations; the *wordlength pattern with the dispersion factor 1 and 2* involved is defined by

$$W_{12}(d) = (A_{3,12}(d), A_{4,12}(d), A_{5,12}(d), \cdots, A_{n,12}(d)),$$

where $A_{r,12}$ is the number of words with both factors 1 and 2 of length r in the defining relations. The wordlength pattern of a design in the presence of two dispersion factors is thus defined by

$$W(d) = ((A_{3,12}(d), A_{3,1}(d), A_{3,2}(d), A_{3,0}(d)), \cdots, (A_{n,12}(d), A_{n,1}(d), A_{n,2}(d), A_{n,0}(d))). \tag{3.1}$$

WLOG, it is assumed that the dispersion effect of factor 1 is larger than that of factor 2. Therefore, we may use (3.1) to construct the minimum aberration designs under the assumption that there exists two specific factors responsible for the dispersion of response. The definition of minimum aberration criterion under the current study is modified as follows.

Definition. For any two 2^{n-p} FFDs with two dispersion factors, d_1 and d_2, let s be the smallest r such that

1. $A_{s,12}(d_1) < A_{s,12}(d_2)$, or
2. $A_{s,12}(d_1) = A_{s,12}(d_2)$ but $A_{s,1}(d_1) < A_{s,1}(d_2)$, or
3. $A_{s,12}(d_1) = A_{s,12}(d_2)$ and $A_{s,1}(d_1) = A_{s,1}(d_2)$ but $A_{s,2}(d_1) < A_{s,2}(d_2)$, or
4. $A_{s,12}(d_1) = A_{s,12}(d_2)$, $A_{s,1}(d_1) = A_{s,1}(d_2)$ and $A_{s,2}(d_1) = A_{s,2}(d_2)$ but $A_{s,0}(d_1) < A_{s,0}(d_2)$,

then d_1 is said to have less aberration than d_2. If there is no other design with less aberration than d_1, then d_1 has minimum aberration.

For example, consider two 2^{7-3} FFDs. Let d_1 be determined by the defining relations $I = 1235 = 1246 = 1347 = 3456 = 2367 = 2457 = 1567$ and d_2 be determined by the defining relations $I = 125 = 136 = 247 = 2356 = 1457 = 34567 = 123467$. The wordlength pattern of d_1 is given by

$$W(d_1) = ((0,0,0,0), (2,2,2,1), (0,0,0,0), (0,0,0,0), (0,0,0,0))$$

and the wordlength pattern of d_2 is given by

$$W(d_2) = ((1,1,1,0), (0,1,1,0), (0,0,0,1), (1,0,0,0), (0,0,0,0)).$$

Because $A_{3,12}(d_1) < A_{3,12}(d_2)$, d_1 has less aberration than d_2.

We would take advantage of the algorithm provided in Chen, Sun and Wu (1993) to generate our designs with $N = 16$ and 32. The procedures are given below.

Step 1: For a fixed pair of (n, p) with $2^{n-p} = 16$ or 32, generate all possible non-isomorphic designs using the Chen-Sun-Wu algorithm.

Step 2: Screen out the ones with the maximum resolution from the resulting non-isomorphic designs.

Step 3: Calculate the wordlength pattern, defined by (3.1), for each maximum resolution design by treating factors i and j as the dispersion factors, for $1 \leq i \neq j \leq n$.

Step 4: Select the minimum aberration design among all possible candidate designs generated from Step 3.

The resulting 16-run and 32-run minimum aberration 2^{n-p} FFDs with two dispersion factors are displayed in Tables 3 and 4, respectively. In the tables, we also list the factors that can be assigned as the dispersion factors. Note that the first nine factors are denoted by 1, 2, $\cdots$, 9 as usual, and the $(10+i)$th factor is denoted by t_i, for $i = 0, 1, \cdots, 9$.

Table 3: *The 16-run minimum aberration 2^{n-p} FFDs with dispersion factor 1 and 2.*

Design	Generators
5-1	$5 = 1234$
6-2	$5 = 123$, $6 = 134$
7-3	$5 = 123$, $6 = 124$, $7 = 134$
8-4	$5 = 123$, $6 = 124$, $7 = 134$, $8 = 234$
9-5	$5 = 13$, $6 = 23$, $7 = 34$, $8 = 124$, $9 = 1234$
10-6	$5 = 79$, $6 = 37$, $1 = 39$, $8 = 47$, $9 = 24$, $t_0 = 347$
11-7	$5 = 1t_1$, $6 = 13$, $7 = 3t_1$, $8 = 14$, $9 = 4t_1$, $t_0 = 134$, $t_1 = 234$
12-8	$5 = 17$, $6 = 13$, $7 = 23$, $8 = 14$, $9 = 47$, $t_0 = 134$, $t_1 = 347$, $t_2 = 1347$
13-9	$5 = 1t_2$, $6 = 3t_2$, $7 = 13$, $8 = 13t_2$, $9 = 4t_2$, $t_0 = 14$, $t_1 = 14t_2$, $2 = 34$, $t_3 = 34t_2$
14-10	$5 = 1t_4$, $6 = 13$, $7 = 3t_4$, $8 = 13t_4$, $9 = 14$, $t_0 = 4t_4$, $t_1 = 14t_4$, $t_2 = 34$, $t_3 = 134$, $t_4 = 234$
15-11	$5 = 12$, $6 = 13$, $7 = 23$, $8 = 123$, $9 = 14$, $t_0 = 24$, $t_1 = 124$, $t_2 = 34$, $t_3 = 134$, $t_4 = 234$, $t_5 = 1234$

Table 4: *The 32-run minimum aberration 2^{n-p} FFDs with dispersion factor 1 and 2.*

Design	Generators
6-1	$6 = 12345$
7-2	$6 = 134$, $7 = 1245$
8-3	$6 = 135$, $7 = 145$, $8 = 2345$
9-4	$6 = 239$, $7 = 249$, $8 = 259$, $9 = 1345$
10-5	$6 = 123$, $7 = 134$, $8 = 135$, $9 = 1245$, $t_0 = 2345$
11-6	$6 = 135$, $7 = 145$, $8 = 345$, $9 = 134$, $t_0 = 125$, $t_1 = 235$
12-7	$6 = 135$, $7 = 145$, $8 = 345$, $9 = 134$, $t_0 = 125$, $t_1 = 235$, $t_2 = 245$
13-8	$6 = 13t_3$, $7 = 14t_3$, $8 = 34t_3$, $9 = 134$, $t_0 = 15t_3$, $t_1 = 35t_3$, $t_2 = 135$, $t_3 = 245$
14-9	$6 = 123$, $7 = 134$, $8 = 124$, $9 = 234$, $t_0 = 135$, $t_1 = 125$, $t_2 = 235$, $t_3 = 145$, $t_4 = 345$
15-10	$6 = 123$, $7 = 124$, $8 = 134$, $9 = 234$, $t_0 = 125$, $t_1 = 135$, $t_2 = 235$, $t_3 = 145$, $t_4 = 245$, $t_5 = 345$

4 Final Remarks

In this study, we focus on finding two-level minimum aberration FFDs when there exists one or two dispersion factors. We provide a series of designs with run sizes equal to 16 and 32, which should prove useful in practical use. In our experience, the number of dispersion factors in real experiments cannot be large, so the situations with one or two dispersion factors should be applicable in practice. However, the extension of results to the situation with three or more dispersion factors can be very complicated, this needs a further investigation.

In the present study, we don't provide a clear cut point of the dispersion effects to determine whether the dispersion factors may seriously affect the estimation of location effects or not. Because this may impact the selection of the designs, it is needed to study on this issue by analyzing some real data sets.

The dispersion factors are assumed to be known when choosing the designs in the current study, however, the dispersion factors can be unknown for most of experiments. Therefore, it may be of interest to investigate the construction of good designs for this situation.

*References Bergman, B. and Hynên, A. (1997). Dispersion effects from fractional factorial designs in the 2^{k-p} series. *Technometrics*, 39, 191-198.

Box, G. E. P. and Meyer, R. D. (1986). Dispersion effects from factorial designs. *Technometrics*, 28, 19-27.

Brenneman, W. A. and Nair, V. N. (2001). Methods for identifying dispersion effects in unreplicated factorial experimenters: A critical analysis and proposed strategies. *Technometrics*, 28, 388-405.

Chen J., Sun, D. X. and Wu, C. F. J. (1993). A catalogue of two-level and three-level fractional factorial designs with small runs. *International Statistical Review*, 61, 131-145.

Cheng, S. W. and Wu, C. F. J. (2002). Choice of optimal blocking schemes in two-level and three-level designs. *Technometrics*, 44, 269-277.

Liao, C. T. and Iyer, H. K. (2000). Optimal 2^{n-p} fractional factorial designs for dispersion effects under a location-dispersion model. *Commun. Statist.-Theory Meth.*, 29, 823-835.

Liao, C. T. (2006). Two-level factorial designs for searching dispersion factors and estimating location main effects. *Journal of Statistical Planning and Inference*, 136, 4071-4087.

McGrath, R. N. and Lin, D. K. J. (2001a). Confounding of location and dispersion effects in unreplicated fractional factorial designs. *Journal of Quality Technology*, 33, 129-139.

McGrath, R. N. and Lin, D. K. J. (2001b). Testing multiple dispersion effects in unreplicated fractional factorial designs. *Technometrics*, 43, 406-414.

Searle, S. R., Casella, G. and McCulloch, C. E. (1992). Variance components. New York: Wiely.

Wang, P. C. (1989). Tests for dispersion effects from orthogonal arrays. *Comput. Statist. Data Anal.*, 8, 109-117.

Wu, C. F. J. and Hamada, M. (2000). Experiments: planning, analysis, and parameter design Optimization. New York: Wiely.