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Environment, Asset Characteristics, and Optimal Effluent Fees

JYH-BANG JOU
Graduate Institute of National Development, College of Social Science, National Taiwan University, No. 1 Roosevelt Rd. Sec 4, Taipei 106, Taiwan, R.O.C. (E-mail: jbjou@ccms.ntu.edu.tw)

Abstract. This article investigates the issue of optimal effluent fees in a framework where waste emissions are abated by investing in capital of which the pay-off is uncertain and the cost is fully sunk. The stock of waste emissions harms an individual firm's production, but the firm will underestimate this external effect upon investing. Consequently, the firm will invest less capital, and thereby, pollute more than is socially desirable. The regulator, who can use effluent fees to correct this, should impose lower effluent fees on irreversible investments than on costlessly reversible ones when uncertainty arises.

Key words: effluent fees, environmental externality, irreversible investment, real options

JEL classification: G31, H21

1. Introduction

One central topic in environmental economics is how to design an optimal policy to correct market failures due to environmental externalities (Cropper and Oats 1992). Recently, a large number of studies have extended the "endogenous" growth models to address this topic (e.g., Bovenberg and de Mooij 1997; Bovenberg and Smulders 1995; Gradus and Smulders 1993; Mohtadi 1996). However, these studies typically ignore the effects of asset characteristics on the optimal policy. Two types of asset characteristics associated with developing a natural area are pointed out by Arrow and Fisher (1974): (i) irreversibility and (ii) uncertain costs and benefits. Later on, several authors argue that these two characteristics are also associated with capital investment that abates emissions (see, e.g., Roberts and Spence 1976; Saphores and Carr 1999). The purpose of this article is to investigate how these two characteristics affect the design of optimal effuent fees.

This article adds environmental externalities into the model commonly used in real options literature (e.g., Bertola and Caballero 1994; Dixit 1991). An industry, which is composed of many identical risk-neutral firms, faces uncertainty in the form of a technology-shift factor that evolves as a geometric Brownian motion. Each firm employs capital to abate waste emissions. The stock of waste emissions harms an individual firm. However, the firm will treat the other firms' investment strategies as given upon investing, and therefore, will underestimate the environmental externality. The policy to correct this includes a subsidy, deposit-refund schemes, an effluent standard, and an effluent fee (Cropper and Oats 1992; Farzin 1996). This article, which focuses on the last instrument, yields the following result: when uncertainty arises, the regulator should impose lower effluent fees on irreversible investments than on costlessly reversible ones; otherwise, the regulator should impose equal effluent fees on both types of investments.

This article is related to the environmental economics literature such as Weitzman (1974), Roberts and Spence (1976) and Mohtadi (1996). The first two studies investigate the conditions under which an effluent fee is more efficient than an effluent standard when polluted firms have better information about their own cost conditions than the regulator. In contrast, this article assumes that both firms and the regulator share the same information about these conditions. Mohtadi extends the endogenous growth models (e.g., Rebelo 1991) by assuming that the aggregate stock of capital reduces both the output of firms and the welfare of agents. Mohtadi then shows that a combination of an effluent fee and an effluent standard leads to a higher level of social welfare than an effluent fee alone. In contrast, this article derives the optimal effluent fees while assuming that capital exhibits positive externalities on production through abating emissions.[1]

This article is also closely related to the study that applies real options approach to investigate the optimal environmental policy including Pindyck (2000), Saphores and Carr (1999) and Xepapadeas (1999). While this article allows a regulator to implement price control policy, the former two articles allow a regulator to reduce stocks of environmental pollutants once-and-for-all. In Pindyck, the regulator controls a variable that is related to these stocks. In contrast, in Saphores and Carr, the regulator directly controls these stocks. Furthermore, Xepapadeas allows uncertainty demand, emission tax, and abatement technology. However, he does not explicitly model externality nor relate asset characteristics to optimal environmental policies.

2. The Model

This article adds environmental externalities from waste emissions into the model commonly used in real options literature (e.g., Bertola and Caballero 1994; Dixit 1991). An industry is composed of N identical risk-neutral firms, indexed by 1 to N, and faces a demand function with a constant-elasticity ε (≥ 0), i.e.,

$$Q(t) = P(t)^{-\varepsilon}, \tag{1}$$

where $Q(t)$ is quantity, and $P(t)$ is price. Each firm i produces final goods according to a Cobb-Douglas technology

$$q_i(t) = l_i(t)^{\gamma} x_a(t)^{-\lambda} Z(t), 1 > \gamma > 0, \ \lambda > 0, \tag{2}$$

where $q_i(t)$ is firm i's output, $l_i(t)$ is the amount of labor employed by firm i, and $x_a(t)$ is the industry's average level of waste emissions. The adverse effect of $x_a(t)$

on $q_i(t)$ is external with λ measuring its size. This external effect indicates that firm i's output will be lower not only when it emits more, but also when other firms in the industry emit more. For example, a firm's employees may suffer from the industry-wide waste emissions like air pollution. This kind of externality is supported by many empirical studies (e.g., Ballard and Medema 1993). In addition, $Z(t)$ is a technology-shift factor evolving the geometric Brownian motion with no drifts as

$$dZ(t) = \sigma Z(t) d\Omega(t), \tag{3}$$

where σ (>0) is the instantaneous volatility of the growth rate of $Z(t)$, and $d\Omega(t)$ is an increment to a standard Wiener process.[2]

An individual firm can employ capital to abate emission.[3] Waste emissions are assumed to be unrelated to either output or labor so as to yield a tractable solution for the firm's choice of output, and therefore, the optimal effluent fee. More specifically, firm i's waste emissions, denoted by $x_i(t)$, and its amount of abatement capital stock, denoted by $k_i(t)$, is given by

$$x_i(t) = k_i(t)^{-\theta}, \tag{4}$$

where θ (> 0) is used to measure the efficiency of abatement capital.

Equation (2) indicates that waste emissions exhibits negative externalities on production. Given the abatement technology shown by (4), this also implies that capital exhibits positive externalities on production through abating emissions.[4] As a result, market outcomes are inefficient, thus requiring governmental interventions. The instruments commonly introduced include a subsidy, deposit-refund schemes, an effluent standard, and an effluent fee (Farzin 1996; Cropper and Oats 1992). In the following, I will focus on the first instrument by assuming that the regulator collects effluent fees from firms, and then returns the proceeds to these firms in the form of lump-sum transfers.

Suppose that $q(t) = (q_1(t) \ldots, q_N(t))$ and $k(t) = (k_1(t) \ldots, k_N(t))$. The operating flow profit of firm i, denoted by $\pi_i(q(t), k(t), Z(t))$, is equal to its revenue, $P(t)q_i(t)$, minus the sum of its operating cost, denoted by $C_i(q(t), k(t), Z(t))$ in (6) below, and effluent fees, denoted by $F_i(k_i(t))$ in (7) below, thus yielding

$$\pi_i(q(t), k(t), Z(t)) = P(t)q_i(t) - C_i(q(t), k(t), Z(t)) - F_i(k_i(t)). \tag{5}$$

Denote w as a given wage rate. By (2), firm i's variable cost, which is equal to $wl_i(t)$, is thus given by

$$C_i(q(t), k(t), Z(t)) = wq_i(t)^g x_a(t)^{\lambda g} Z(t)^{-g}, \tag{6}$$

where $g = 1/\gamma > 1$ and $x_a(t) = (\sum_{j=1}^{N} k_j^{-\theta})/N$. Suppose that h denotes the effluent fee per unit of waste emissions. The amount of effluent fees paid by firm i, $F_i(k_i(t))$, is equal to h multiplied by $x_i(t)$ in (4), thus yielding

$$F_i(k_i(t)) = hk_i(t)^{-\theta}. \tag{7}$$

In (Cournot-Nash) short-run equilibrium, firm i will take the other firms' production strategies as given, while choosing an amount of output, denoted by $q_i(t)^*$, to maximize its operating flow profit, $\pi_i(\cdot)$ given by (5). Consequently, $q_i(t)^*$ is derived by setting the derivative of $\pi_i(\cdot)$ with respect to $q_i(t)$ equal to zero. This yields the equality of the marginal revenue, $1 - (q_i(t)/(\varepsilon Q(t)))$ multiplied by $P(t)$ defined in (1), with the short-run marginal costs $(\partial C_i(\cdot)/\partial q_i)$ given by $wq_i(t)^{g-1}x_a(t)^{\lambda g}Z(t)^{-g}/\alpha$. This equality, together with the equilibrium condition $q_j(t) = q_i(t)^*(j = 1, \ldots, N)$, yields $q_i(t)^*$ and its corresponding $P(t)^*$ as respectively given by

$$q_i(t)^* = [\frac{w}{\gamma}N^e(1 - \frac{e}{N})^{-1}]^{\frac{-1}{(e+g-1)}}[x_a(t)^{-\lambda g}Z(t)^g]^{\frac{1}{(e+g-1)}}, \tag{8}$$

and

$$P(t)^* = [\frac{w}{\gamma}(1 - \frac{e}{N})^{-1}]^{\frac{e}{(e+g-1)}}N^{\frac{-e(g-1)}{(e+g-1)}}[x_a(t)^{-\lambda g}Z(t)^g]^{\frac{-e}{(e+g-1)}}, \tag{9}$$

where $e = 1/\varepsilon$.

Evaluating $\pi_i(\cdot)$ in (5) at $q_j(t) = q_i(t)^*(j = 1, \ldots, N)$ yields the optimized value of firm i's flow profit, denoted by $\pi_i{}^1(k(t),Z(t))$, as shown by (10) below. Define $f = eg/(e + g - 1)$, then

$$\pi_i^1(k(t), Z(t)) = d_0x_a(t)^{\frac{-(1-e)f\lambda}{e}}Z(t)^{\frac{(1-e)f}{e}} - hk_i(t)^{-\theta}, \tag{10}$$

where $d_0 = (1 - (1 - \frac{e}{N})\gamma)N^{-f}(1 - \frac{e}{N})^{\frac{(1-e)}{(e+g-1)}}(\frac{w}{\gamma})^{\frac{(e-1)}{(e+g-1)}}$.

In contrast, a social planner will internalize the externality from capital before investing. The social planner knows that firm i's capital stock, $k_i(t)$, is equal to the industry's average level of capital stock, $k_a(t)$, because all firms are identical. Substituting this equality and imposing $h = 0$ into (10) yields the optimized value of the social flow profit, denoted by $\pi_i^2(k_i(t), Z(t))$, as given by

$$\pi_i^2(k_i(t), Z(t)) = d_0k_i(t)^{\frac{(1-e)f\theta\lambda}{e}}Z(t)^{\frac{(1-e)f}{e}}. \tag{11}$$

3. Optimum Effluent Fees

For ease of exposition, I will both abstract from depreciation of capital and assume that the purchase price of capital is constant over time, denoted by P_K. I will then respectively examine the following two cases: (i) investment is costlessly reversible where the resale price of capital is equal to P_K; and (ii) investment is completely irreversible where the resale price of capital is equal to zero.

3.1. COSTLESSLY REVERSIBLE INVESTMENT

Suppose that investment is costlessly reversible, then $k_i(t)$ will be a choice variable rather than a state variable. Let ρ denote a given (risk-adjusted) discount rate. At each instant firm i will choose an amount of capital stock that equates the private marginal return to capital with the user cost of capital (Jorgenson 1963), taking the other firms' choice of capital stock as given, i.e.,

$$\frac{\partial \pi_i^1(k(t), Z(t))}{\partial k_i(t)} = \rho P_K, \tag{12}$$

where $\pi_i{}^1(\cdot)$ is given by (10). In (Cournot-Nash) industry equilibrium, each firm will choose an equal amount of capital stock, denoted by $k_{f1}(\cdot, h, t)$. Substituting this condition into (12) Yields[5]

$$\frac{d_0(1-e)f\theta\lambda}{eN} k_{f1}(\cdot, h, t)^{\frac{(1-e)f\theta\lambda}{e}-1} Z(t)^{\frac{(1-e)f}{e}} + h\theta k_{f1}(\cdot, h, t)^{-(1+\theta)} = \rho P_K. \tag{13}$$

Imposing $h = 0$ into (13), and then rearranging yields firm i's choice of capital stock in the absence of any regulations as given by

$$k_{f1}(\cdot, 0, t) = \left[\frac{\frac{d_0(1-e)f\theta\lambda}{eN} Z(t)^{\frac{(1-e)f}{e}}}{\rho P_K}\right]^{\frac{1}{1-\frac{(1-e)f\theta\lambda}{e}}}. \tag{14}$$

Similarly, the capital stock chosen by a social planner at each instant, denoted by $k_{f2}(\cdot, t)$, is found by equating the social marginal return to capital, with the user cost of capital, i.e.,[6]

$$\frac{\partial \pi_i^2(k_i(t), Z(t))}{\partial k_i(t)} = \rho P_K, \tag{15}$$

where $\pi_i{}^2(\cdot)$ is given by (11). Evaluating (15) at $k_i(t) = k_{f2}(\cdot, t)$, and then rearranging yields

$$k_{f2}(\cdot, t) = \left[\frac{\frac{d_0(1-e)f\theta\lambda}{e} Z(t)^{\frac{(1-e)f}{e}}}{\rho P_K}\right]^{\frac{1}{1-\frac{(1-e)f\theta\lambda}{e}}}. \tag{16}$$

Comparing $k_{f2}(\cdot, t)$ given by (16) with $k_{f1}(\cdot, 0, t)$ given by (14) suggests that in the absence of any effluent fees ($h = 0$), the social planner's choice of capital stock given by the right-hand side of (16) will outweigh firm i's choice of capital stock given by the right-hand side of (14). Given the abatement technology shown by (4), this implies that firm i will pollute more than is socially desirable. Suppose that the regulator imposes an effluent fee per unit of emissions, denoted by h_f, to correct that wedge. Accordingly, h_f is the h that satisfies the equality $k_{f1}(\cdot, h, t) = k_{f2}(\cdot, t)$. Solving this equality yields

$$h_f = \frac{d_0(1-e)f\lambda}{e}\left(1 - \frac{1}{N}\right) k_{f2}(\cdot, t)^{\theta\left(1+\frac{f(1-e)\lambda}{e}\right)} Z(t)^{\frac{(1-e)f}{e}}. \tag{17}$$

3.2. IRREVERSIBLE INVESTMENT

In the long-run, through Cournot-Nash competition, firm i will maximize its expected discounted flow profit, net of the investment costs, taking the strategies of the other firms as given (Baldursson 1998). Consequently, the Bellman value function for firm i is given by

$$V_1(k(t), Z(t)) = \max_{\{k_i(\tau)\}} E_t \left\{ \int_t^\infty e^{-\rho(\tau-t)} \left[\pi_i^1(k(\tau), Z(\tau)) d\tau - P_K dk_i(\tau) \right] \right\}, \tag{18}$$

where $E_t\{\cdot\}$ denotes conditional expectation taken at time t. The maximization problem faced by firm i amounts to choosing the optimal path for $k_i(t)$. There are $N + 1$ state variables in this maximization problem, $Z(t)$ and an n-tuple of strategies; $k(t)$; $Z(t)$, and $k_{-i}(t) = (k_1(t), \ldots, k_{i-1}(t), k_{i+1}(t), \ldots k_N(t))$ are exogenously given, while $k_i(t)$ is subject to control. However, all elements of $k_{-i}(t)$ should be equal to $k_i(t)$ in Cournot-Nash industry equilibrium.

As is well known in real options literature (see, e.g., Dixit and Pindyck 1994), the interaction of the stochastic evolution of $Z(t)$ and irreversibility indicates that firm i solves a problem of instantaneous control of Brownian motion; the optimal policy is to regulate the state variable $k_i(t)$ at a lower barrier, denoted by $k_{i^*}(t)$ (Harrison and Taksar 1985), which is also called the "desired" capital stock by Bertola and Caballero (1994). Given the abatement technology in (4), firm i should also regulate its amount of emissions, $x_i(t)$ in (4), at an upper barrier, denoted by $x_i{}^*(t)$, which is derived by evaluating $x_i(t)$ at $k_i(t) = k_{i^*}(t)$. Consequently, $x_i{}^*(t)$ is the maximum amount of waste emissions tolerable by firm i.

When $k_i(t) > k_{i^*}(t)$, the private marginal gain from increasing the capital stock, $v_I(\cdot) = \partial V_I(\cdot)/\partial k_i(t)$, is given by (see Appendix A)

$$v_1(\cdot) = A_1 B(\cdot)^\beta + \frac{d_0}{\phi(1)} B(\cdot) + \frac{h\theta k_i(t)^{-(1+\theta)}}{\rho}, \tag{19}$$

where

$$B(\cdot) = \frac{(1-e) f\theta\lambda}{eN} k_i(t)^{-\theta-1} x_a(t)^{\frac{-(1-e)f\lambda}{e}-1} Z(t)^{\frac{(1-e)f}{e}}, \tag{20}$$

A_1 is a constant to be determined, β is the larger root of τ in the quadratic equation given by (A4), and $\phi(1)$ is obtained by substituting $\tau = 1$ into (A5). On the right-hand side of (19), the first term is the private value of the option to install one more incremental unit of capital, while the sum of the last two terms is the private value of the last incremental of installed capital stock. Suppose that $k_{s1}(\cdot, h, t)$ denotes the $k_{i^*}(t)$ chosen by firm i when investment is completely irreversible. Two optimal conditions, the value-matching and smooth-pasting conditions, must be satisfied at $k_i(t) = k_{s1}(\cdot, h, t)$ (Bertola and Caballero 1994). They are respectively given by $v_1(\cdot) = P_K$ and $\partial v_1(\cdot)/\partial Z(t) = 0$. Substituting the equilibrium condition

$k_j(t) = k_{s1}(\cdot, t)(j = 1, \ldots, i, \ldots N$, into these two conditions, and then solving them simultaneously yields

$$\frac{d_0\alpha}{\rho(\alpha+1)}\left(\frac{(1-e)f\theta\lambda}{eN}\right)k_{s1}(\cdot, h, t)^{\frac{(1-e)f\theta\lambda}{e}-1}Z(t)^{\frac{(1-e)f}{e}} + \frac{h\theta k_{s1}(\cdot,h,t)^{-(1+\theta)}}{\rho} = P_K, \quad (21)$$

where $-\alpha$ is the smaller root in the quadratic equation given by (A4). Imposing h = 0 into (21) and then rearranging yields firm i's choice of "desired" capital stock in the absence of any regulations as given by

$$k_{s1}(\cdot, 0, t) = m_1 k_{f1}(\cdot, 0, t), \quad (22)$$

where $m_1 = \left[\frac{\alpha}{1+\alpha}\right]^{\frac{1}{1-\frac{(1-e)f\theta\lambda}{e}}}$ and $k_{f1}(\cdot, 0, t)$ is given by (14).

The factor m_1 is smaller than or equal to one. It will be equal to one as the size of uncertainty σ approaches zero because α will then approach infinity, and therefore, $\alpha/(1+\alpha)$ will approach one; otherwise, m_1 will be smaller than 1.

Now consider the case for a command economy. In the long-run a social planner will internalize the external effect when choosing an optimal path of $k_i(t)$ to maximize the expected discounted social flow profit, net of the investment costs. The Bellman value function for the social planner is thus given by

$$\begin{aligned} &V_2(k_i(t), Z(t)) = \\ &\max_{\{k_i(\tau)\}} E_t\left\{\int_t^\infty e^{-\rho(\tau-t)}\left[\pi_i^2(k_i(\tau), Z(\tau))d\tau - P_K dk_i(\tau)\right]\right\}. \end{aligned} \quad (23)$$

The maximization problem faced by the social planner has two state variables, $Z(t)$ and $k_i(t)$; $Z(t)$ is exogenously given while $k_i(t)$ is subject to control. Suppose that $k_{i^*}(t)$ still denotes the "desired" capital stock. When $k_i(t) > k_{i^*}(t)$, the social marginal gain from increasing the capital stock, $v_2(\cdot) = \partial V_2(\cdot)/\partial k_i(t)$, is given by (see Appendix A)

$$\begin{aligned} &v_2(\cdot) = \\ &A_2[k_i(t)^{\frac{(1-e)f\theta\lambda}{e}-1}Z(t)^{\frac{(1-e)f}{e}}]^\beta + \frac{d_0(1-e)f\theta\lambda}{\phi(1)e}k_i(t)^{\frac{(1-e)f\theta\lambda}{e}-1}Z(t)^{\frac{(1-e)f}{e}}, \end{aligned} \quad (24)$$

where A_2 is a constant to be determined.

Suppose that $k_{s2}(\cdot, t)$ denotes the $k_{i^*}(t)$ chosen by a social planner when investment is completely irreversible. The value-matching and smooth-pasting conditions applied to $v_2(\cdot)$ are respectively given by $v_2(\cdot) = P_K$ and $\partial v_2(\cdot)/\partial Z(t) = 0$. These two equations must be satisfied at $k_i = k_{s2}(\cdot, t)$. Solving these two conditions simultaneously yields

$$k_{s2}(\cdot, t) = m_1 k_{f2}(\cdot, t), \quad (25)$$

where m_1 is given by (22), and $k_{f2}(\cdot, t)$ is given by (16).

Comparing $k_{s2}(\cdot, t)$ given by (25) with $k_{s1}(\cdot, 0, t)$ given by (22) suggests that in the absence of any effluent fees (h = 0), a social planner's choice of "desired" capital stock given by the right-hand side of (25) will outweigh firm i's choice of "desired" capital stock given by the right-hand side of (22). Accordingly, the social planner will ask for a cleaner environment than will the firm. An effluent fee will be optimal if it causes both the firm and the social planner to have identical "desired" capital stocks because both will then have identical investments (Xepapadeas 1999). Suppose that this optimal effluent fee per unit of emissions is denoted by h_s, which is the h that satisfies the equality $k_{s1}(\cdot, h, t) = k_{s2}(\cdot, t)$. Solving this equality yields

$$h_s = \frac{d_0(1-e)f\lambda\alpha}{e(\alpha+1)}(1-\frac{1}{N})k_{s2}(\cdot, t)^{\theta(1+\frac{f(1-e)\lambda}{e})}Z(t)^{\frac{(1-e)f}{e}}. \tag{26}$$

Dividing h_s in (26) by h_f in (17) yields

$$h_s = m_2 h_f, \quad \text{where } m_2 = {m_1}^{1+\theta}. \tag{27}$$

The factor m_2 will be equal to one as the size of uncertainty σ approaches zero; otherwise, m_2 will be smaller than 1. Proposition 1 will thus follow.

Proposition 1: *If uncertainly arises, then the regulator should impose lower effluent fees on irreversible investments than on costlessly reversible ones. Otherwise, the regulator should impose equal effluent fees on both types of investments*.[7]

The intuition behind Proposition 1 is as follows. Suppose that firms make irreversible investments in the presence of uncertainty. Accordingly, firms may experience a low emissions shock in the future. If they have already employed a large stock of abatement capital, then they can't take it back. However, if firms experience a high emissions shock in the future, then it is always possible for firms to add more abatement capital. This asymmetry of the response to a high or low emissions shock induces firms more prudent in investing in abatement capital as compared to the case where either there is no uncertainty or there is complete reversibility (The factor m_1 shown by (22) which is smaller than one when uncertainty arises verifies this implication). Given the abatement technology shown by (4), this also implies that the optimal level of pollution will be higher for firms that make irreversible investments under uncertainty as compared to the case where either there is no uncertainty or there is complete reversibility.

The role of the externality is reduced to decide the optimal level of pollution. Given that abatement capital exhibits externality, the market solution will result in less investments, and therefore, more pollutants than socially desirable. Consequently, there is a reason for the regulator to impose effluent fees. In the presence of uncertainty ($\sigma > 0$), the wedge between the social and private optimal stocks of abatement capital when investment is irreversible, $k_{s2}(\cdot, t) - k_{s1}(\cdot, 0, t)$, will be lower than its counterpart when investment is costlessly reversible, $k_{f2}(\cdot, t) - k_{f1}(\cdot, 0, t)$; this is because subtracting (22) from (25) yields $k_{s2}(\cdot, t) - k_{s1}(\cdot, 0, t) = m_1(k_{f2}(\cdot, t) - k_{f1}(\cdot, 0, t))$, where $m_1 < 1$ when $\sigma > 0$. Given the abatement technology, this also implies that the wedge between the private and social optimal levels of pollution will be lower when investment is irreversible than when investment is costlessly reversible. Accordingly, the regulator should impose lower effluent fees on irreversible investments than on costlessly reversible ones when uncertainty arises.

Differentiating the factor m_2 given by (27) with respect to its underlying parameters yields Proposition 2.

Proposition 2: *When uncertainty arises, the regulator should discriminate more against costlessly reversible investments i.e., m_2 will be lower) in the following cases. (i) uncertain is greater (σ is higher); (ii) abatment capital is more efficient (θ is higher); and (iii) environmental externality is more significant (λ is higher).*

Proof: See Appendix B.

In Appendix B, I show that in the three cases stated in Proposition 2, the factor m_2 will be lower. Nevertheless, the intuition behind Proposition 2 is better understood from the fact that in all these cases, the factor m_1 given by (22) will be lower. Given the equality $k_{s2}(\cdot, t) - k_{s1}(\cdot, 0, t) = m_1(k_{f2}(\cdot, t) - k_{f1}(\cdot, 0, t))$, a lower m_1 indicates that the wedge between the social and private optimal stocks of abatement capital when investment is irreversible will be smaller that its counterpart when investment is costlessly reversible. Given the abatement technology, this also implies that the wedge between the private and social optimal levels of pollution when investment is irreversible will be smaller than its counterpart when investment is costlessly reversible. Accordingly, the regulator should adopt an effluent fee policy that discriminates more against costlessly reversible investments.

4. Conclusions

This article investigates the issue of optimal effluent fees in a framework where waste emissions are abated by investing in capital of which the pay-off is uncertain and the cost is fully sunk. The stock of waste emissions harms an individual firm's production, but the firm will underestimate this external effect upon investing. Consequently, the firm will invest less capital, and thereby, pollute more than is socially desirable. The regulator, who can use effluent fees to correct this, should impose lower effluent fees on irreversible investments than on costlessly reversible ones when uncertainty arises.

Future research may relax several assumptions in this article. First, some extensions that add technical difficulty but not conceptual ones include that (i) the costs on investment may be partially sunk; and (ii) the costs on purchasing capital may rise over time (Dixit and Pindyck 1998). One may then investigate how changes in these two asset characteristics affect the required effluent fees. Second, asymmetric information between firms and the regulator may arise because firms usually have better information about their own cost conditions than the regulator, as suggested by Weitzman (1974). One may incorporate the study by Gaudet, Lasserre and Long (1998), which examines real investment decisions with adjustment costs under asymmetric information, to address the issue of optimal effluent fees. Finally, one may extend the study by Dixit (1991), which examines real investment decisions with price ceilings, to examine the issue of optimal effluent standards.[8]

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Notes

1. Glazer (1997) considers the negative externality generated by the consumption of a good. He shows that a linear tax imposed on a monopolist may not attain the first-best social optimum. However, the type of externality and market structure considered by him differ from those of this article.
2. Equation (3) indicates that information about an individual firm's production technology arriving in time is independent of its decision to invest. This contrasts with that of Kolstad (1996) and Ulph and Ulph (1997) where uncertainty can be resolved through learning.
3. Some articles on the environmental economics allow a firm to employ both productive and abatement capital (see, e.g., Kort 1996). The empirical study by Gray and Shadbegian (1998) indicates that these two types of capital tend to crowd out each other. Several articles on real options literature also allow two types of capital investment (see, e.g., Dixit 1997; Eberly and Van Mieghem 1997). However, equation (2) indicates that productive capital is ignored because including it complicates analysis while adding little insight on the issue on which this article focuses.
4. This differs from positive externalities of capital on production through the "learning by doing" effect (Arrow 1962).
5. In what follows, I will assume that $k_{f1}(\cdot, h, t)$ is decreasing with P_K. This requires that, $e > (1 - e) f\theta\lambda$ which is more likely to hold if either ε (demand elasticity), θ (the efficiency of abatement capital), or λ (environmental externality) is small.
6. The term e is required to be smaller than one to assure that the private and social marginal returns to capital are both positive. I will assume this in what follows.
7. This can be formally proven by using the procedures in Kongsted (1996). I omit such a proof to save space.
8. One can also use the model in this article to investigate the issue of optimal investment tax credits. It is straightforward to show that the regulator should give an equal rate of investment tax credits to both costlessly reversible investments and irreversible ones.

Appendix

A. DERIVATION OF $v_1(\cdot)$ AND $v_2(\cdot)$

I will solve $V_1(\cdot)$ given by (18) first, and later for $V_2(\cdot)$ given by (23). Suppose that $k_i(t) > k_{i^*}(t)$. Treating $V_1(\cdot)$ as an asset value, according to Itô's lemma, and using (3) yields its expected capital gain as

$$E_t \frac{dV_1(\cdot)}{dt} = \frac{1}{2}\sigma^2 Z(t)^2 \frac{\partial^2 V_1(\cdot)}{\partial Z(t)^2}. \tag{A1}$$

ENVIRONMENT, ASSET CHARACTERISTICS, AND OPTIMUM EFFLUENT FEES

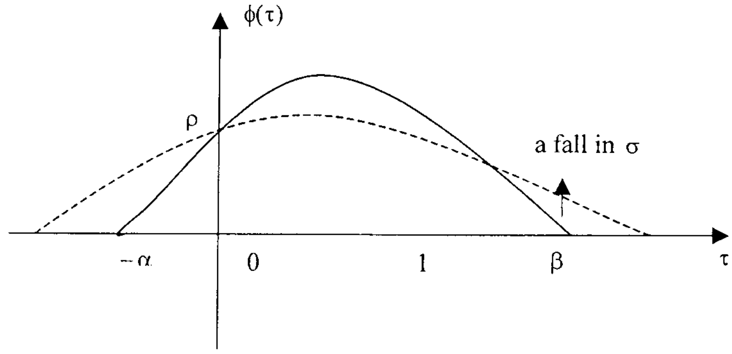

Figure 1. $\phi(\tau)$ vs. τ.

This expected capital gain plus the dividend, $\pi_i^1(\cdot)$ given by (10), i.e., $d_0 x_a(t)^{\frac{-(1-e)f\lambda}{e}} Z(t)^{\frac{(1-e)f}{e}} - hk_i(t)^{-\theta}$ should be equal to the normal return $\rho V_1(\cdot)$ to prevent any arbitrage profits from arising. This yields the differential equation

$$\frac{1}{2}\sigma^2 Z(t)^2 \frac{\partial^2 V_1(\cdot)}{\partial Z(t)^2} - \rho V_1(\cdot) + d_0 x_a(t)^{\frac{-(1-e)f\lambda}{e}} Z(t)^{\frac{(1-e)f}{e}} - hk_i(t)^{-\theta} = 0. \qquad \text{(A2)}$$

Let $\partial V_1(\cdot)/\partial k_i(t) = v_1(\cdot)$. Differentiating (A2) term by term with respect to $k_i(t)$ yields

$$\frac{1}{2}\sigma^2 Z(t)^2 \frac{\partial^2 v_1(\cdot)}{\partial Z(t)^2} - \rho v_1(\cdot) + d_0 B(k(t), Z(t)) + h\theta k_i(t)^{-(1+\theta)} = 0, \qquad \text{(A3)}$$

where $B(\cdot)$ is given by (20). As suggested by Bertola and Caballero (1994, Appendix) the term $[B(k(t), Z(t))]^\tau$ solves the homogeneous part of (A3). Substituting this into (A3) yields the quadratic equation

$$\phi(\tau) = -\frac{1}{2}\sigma^2 \frac{(1-e)f\tau}{e}(\frac{(1-e)f\tau}{e} - 1) + \rho = 0. \qquad \text{(A4)}$$

Denote β and $-\alpha$ as respectively the larger and smaller roots in the quadratic equation given by (A4). Equation (A4) can also be rewritten as

$$\phi(\tau) = \frac{1}{2}[\frac{(1-e)f}{e}]^2 \sigma^2 (\alpha + \tau)(\beta - \tau) = \frac{\rho(\alpha+\tau)(\beta-\tau)}{\alpha\beta}, \qquad \text{(A5)}$$

where $\phi(\tau) > 0$ if $-\alpha < \tau < \beta$. Figure 1 depicts $\phi(\tau)$ as a function of τ.

One particular solution from the non-homogeneous part of (A3) is given by

$$v_{1P}(\cdot) = \frac{d_0}{\phi(1)} B(k(t), Z(t)). \qquad \text{(A6)}$$

Since the value function $V_1(\cdot)$ must approach zero as $Z(t)$ approaches zero so that only the positive root in (A4) should be considered. The general solution of (A3), which is composed solutions from both the homogeneous and non-homogeneous parts of (A3), is shown by (19). Following similar procedures as above yields $v_2(\cdot) = \partial V_2(\cdot)/\partial k_i$ as that shown by (24).

B. PROOF OF PROPOSITION 2

Let $E = \alpha/(1+\alpha) < 1$ and $G = (1+\theta)/[1-(1-e)f\theta\lambda/e] > 0$, then $m_2 = E^G$. Differentiating m_2 with respect to σ, λ and θ yields the following results.

$$\frac{\partial m_2}{\partial \sigma} = \frac{m_2 G}{\alpha(1+\alpha)}\frac{\partial \alpha}{\partial \sigma} < 0, \tag{B1}$$

where $\frac{\partial \alpha}{\partial \sigma} = \frac{\partial \phi(-\alpha)/\partial \sigma}{\partial \phi(-\alpha)/\partial \tau} < 0$,

since $\frac{\partial \phi(-\alpha)}{\partial \sigma} = -\sigma[\frac{(1-e)f(-\alpha)}{e}][\frac{(1-e)f(-\alpha)}{e} - 1] < 0$,

and $\frac{\partial \phi(-\alpha)}{\partial \tau} > 0$, as suggested by Figure 1;

$$\frac{\partial m_2}{\partial \theta} = -m_2 \ \ln\frac{(\alpha+1)}{\alpha}\frac{\partial G}{\partial \theta} < 0; \tag{B2}$$

$$\frac{\partial m_2}{\partial \lambda} = -m_2 \ \ln\frac{(\alpha+1)}{\alpha}\frac{\partial G}{\partial \lambda} < 0. \tag{B3}$$

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