Transactions Briefs_____

Design of a Class of IIR Eigenfilters With Time- and Frequency-Domain Constraints

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Abstract—**An effective eigenfilter approach is presented to design special classes of infinite-impulse response (IIR) filters with time- and frequency-domain constraints is presented. By minimizing a quadratic measure of the error in the passband and stopband, an eigenvector of an appropriate real symmetric and positive-definite matrix is computed to get the filter coefficients. Several IIR filters such as notch filters, Nyquist filters and partial response filters can be easily designed by this approach. Some numerical design examples are illustrated to show the effectiveness of this approach.**

Index Terms—**Infinite-impulse response (IIR) eigenfilter, notch filter, Nyquist filter, partial response filter.**

I. INTRODUCTION

THE eigenfilter approach has been recently used to effectively design linear phase finite-impulse response (FIR) digital filters [1], FIR Hilbert transformers [2], and digital differentiators [3]. This approach has also been applied to design complex FIR filters with arbitrary complex frequency response [4] and [5]. The design of 1-D and 2-D infinite-impulse response (IIR) eigenfilters in time domain has been studied in [6]. In the present communication, we compute the filter coefficients by approximating an ideal desired impulse response. Recently, the eigenfilter approach is extended to design stable causal IIR filters in frequency domain with an arbitrary number of zeros and poles [7] and [8]. These methods work out in the frequency domain and allow to design filters with arbitrary prescribed magnitude frequency response. In this paper, the eigenfilter approach is used to design IIR notch filters, Nyquist filters, and partial response filters which have special frequency domain or time domain constraints.

In many signal processing applications, it is necessary to eliminate narrowband or sinusoidal disturbance while leaving the broardband signal unchanged. Usually this work can be done by the notch filters characterized by a unit gain over the whole frequency domain except at some certain frequencies in which their gain are zero. So far, several methods to effectively design IIR or FIR notch filters have been developed [9]. Adaptive notch filter design has been studied too, [10] and [11]. When the frequencies of narrowband interferences are known in advance, fixed notch filters can be used. In Section II, we study the properties of notch filters and formulate the design algorithm by adding the frequency-domain constraints to the eigenfilter approach. In Section III, we present the effectiveness of this method by showing some examples with arbitrarily chosen frequencies of notches.

Nyquist filters play an important role in digital data transmission for its intersymbol interference (ISI)-free property. Also they can be

adopted in decimation or interpolation multirate systems. To achieve zero ISI, Nyquist filters must satisfy some criteria in time domain that they should have zeros equally spaced in the impulse response coefficients except one specified. There are two conventional methods, which are IIR and FIR filter forms, to implement Nyquist filters. So far the design procedures on FIR Nyquist filters have been widely studied [12] and [13]. However, FIR Nyquist filters usually require higher filter order to meet the magnitude specification. In contrast to FIR filters, IIR filters have lower orders, but their impulse responses are more difficult to keep the zero-crossing time constraint property and the problem of filter stability should also be carefully examined.

Recently, Nakayama and Mizukami have proposed a novel expression of transfer functions for IIR Nyquist filters that can keep exact zero intersymbol interference [14]. They have used the iterative Chebyshev approximation procedure but without considering its filter stability [15]. After adopting the proposed transfer function, the frequency response of the filters can be optimized without taking into account its time domain constraints (zero-crossing property). In this paper, we study this transfer function of Nyquist filters and present a new process based on eigenfilter method to design a Nyquist filter satisfying the arbitrarily selected time and frequency criteria.

The eigenfilter approach can also be applied to design the partial response filters which have similiar zero-crossings as Nyquist filters but sinusoidal shape magnitude response over the passband [16]. Partial response filters have received considerable attention since they can be employed at an increased bit rate under a prescribed available bandwidth for data transmission [16] and [17]. They play an important role in binary data communication for that the intersymbol interference can be reduced by introducing rolloff around the Nyquist frequency. The designing procedures of class 1–5 partial response filters are mainly based on Nyquist filter design for we can take one partial response filter as the multiplication of one Nyquist filter and a polynomial which shapes the filter. In Section III experimental results are presented.

II. PROBLEM FORMULATION AND DESIGN PROCEDURES

A. Notch Filters

Assume the transfer function of a notch filter is defined by

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}. \quad (1)$$

If we need this filter to have a notch at frequency ω_i, i.e., the ideal desired frequency response is unity everywhere in the frequency domain except at $\omega = \omega_i$. The transfer function (1) should have a pair of zeroes at $z = e^{\pm j\omega_i}$, that is

$$\left(1 - 2 \cdot \cos(\omega_i) \cdot z^{-1} + z^{-2}\right) | N(z). \quad (2)$$

Equation (2) means that if we need K different notch positions in the frequency domain, the order of numerator polynomial should be at least $2K$.

When the positions of the notches of the desired filter are given by (1) and (2), the transfer function of this filter can be written as

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 \cdot C(z)}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_{2K} z^{-2K}} \quad (3)$$

TABLE I
PROPERTIES OF CLASS 1–5 PARTIAL RESPONSE FILTERS [16]

Class	Impulse response $h(n)$					$H(f)$ over the passband $[0 < f < F]$
	k_1	k_2	k_3	k_4	k_5	
Binary	1					1
1	1	1				$2\cos(\pi f / 2F)$
2	1	2	1			$4\cos^2(\pi f / 2F)$
3	2	1	-1			$2 + \cos(\pi f / F) - \cos(2\pi f / F) + j[\sin(\pi f / F) - \sin(2\pi f / F)]$
4	1	0	-1			$2\sin(\pi f / F)$
5	-1	0	2	0	-1	$4\sin^2(\pi f / F)$

where

$$C(z) = \prod_{i=1}^{K}\left(1 - 2 \cdot \cos(\omega_i) \cdot z^{-1} + z^{-2}\right). \qquad (4)$$

Clearly, $C(z)$ is known when we have chosen the position of the notches. Then we will use the eigenfilter approach to find the optimal b_0 and denominator coefficients.

Let $H^d(\omega)$ denote the desired target function representing the notch filter with some specified notch bandwidth $\Delta\omega$ at each chosen notch position, and we have set $\Delta\omega = 0.001\pi$ in our examples. With K notches, we can divide the frequency domain into $3K + 1$ intervals to express different desired frequency response in each interval. The response inside the notch consists of two transition bands: one decreasing function from 1–0 and the other increasing function from 0–1, respectively, and is unity elsewhere.

To proceed with the formulation of notch filters, consider a cost function related to the error of the difference between the calculated response $H(\omega)$ and target response $H^d(\omega)$

$$E(\omega) = (H^d(\omega) - H(\omega))D(\omega). \qquad (5)$$

Let $E_k(\omega)$ be the cost function in the frequency interval I_k, so

$$E_k(\omega) = D(\omega)H_k^d(\omega) - b_0\prod_{i=1}^{K}\left(1 - 2 \cdot \cos(\omega_i) \cdot e^{-j\omega} + e^{-2j\omega}\right). \qquad (6)$$

To minimize $E(\omega)$ somehow means a weighted minimization of the error, in which $D(\omega)$ acts as the weighting function. Consider the global square cost function given by $\Phi = \sum_k \beta_k \phi_k$, where β_k is a positive constant that weights the kth band cost function. And ϕ_k is given by

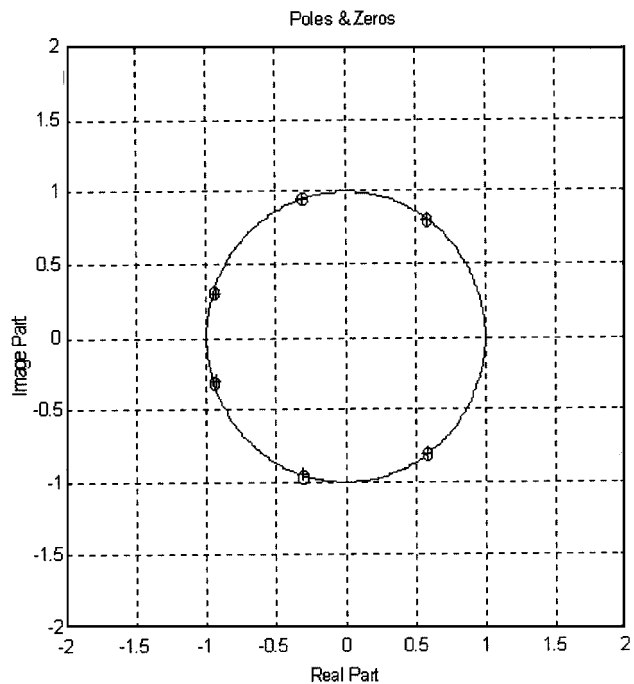$$\phi_k = \int_{I_k} |E_k(\omega)|^2 W_k(\omega)d\omega \qquad (7)$$

where $W_k(\omega)$ denotes the positive weighting function. Consider the vectors (8) and (9) shown at the bottom of the next page. The cost function ϕ_k can be expressed as

$$\phi_k = \int_{I_k} A^T C_k^*(\omega)C_k^T(\omega)A W_k(\omega)d\omega \qquad (10)$$
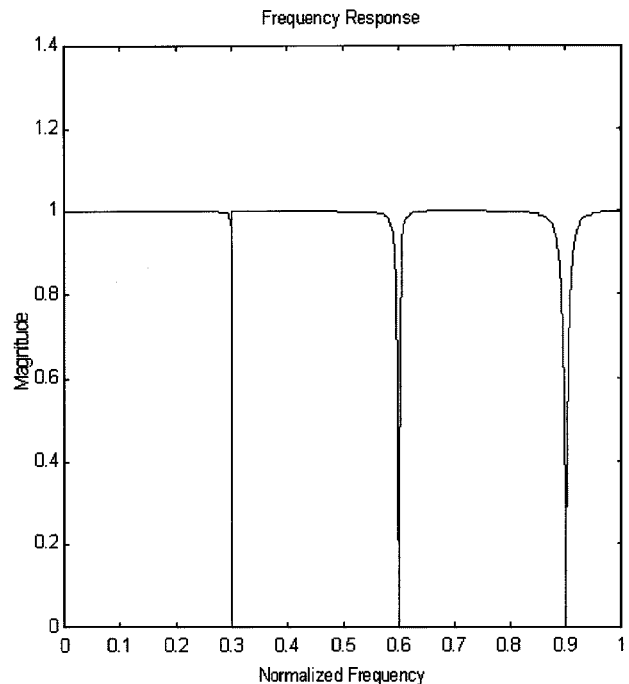
Fig. 1. (a) Magnitude response of example 1. (b) Pole-zero plotting of example 1.

where superscript T and $*$ denote the transposition and conjugation operations, respectively, ($A^T = A^*$ since the filter coefficients are assumed real). Therefore, ϕ_k is given by

$$\phi_k = A^T P_k A \qquad (11)$$

and

$$P_k = \int_{I_k} C_k^*(\omega)C_k^T(\omega)W_k(\omega)d\omega \qquad (12)$$

TABLE II
RESULTING COEFFICIENTS OF EXAMPLE 1

b_0	0.29801430260418	a_0	0.30692859574000
b_1	0.40070303293878	a_1	0.39958611590026
b_2	0.36147942821254	a_2	0.35985811773165
b_3	0.38956042881218	a_3	0.38939406634299
b_4	0.36147942821254	a_4	0.36292519208517
b_5	0.40070303293878	a_5	0.40128160491205
b_6	0.29801430260418	a_6	0.28887025331247

is a real, symmetric, positive-definite $(2K + 2) \times (2K + 2)$ matrix. The global square cost function now can be expressed as

$$\Phi = \sum_k \beta_k \phi_k = A^T \left(\sum_k P_k \right) A = A^T P A. \qquad (13)$$

Applying the eigenfilter approach, the optimum filter coefficients, which minimize the cost function, are the elements of the eigenvector of the matrix P corresponding to the minimum eigenvalue. After some calculation iterations and computing the corresponding eigenvector, we obtain the solution of $A = [a_0 \quad a_1 \quad \cdots \quad a_{2K} \quad b_0]^T$. Then we can get the transfer function coefficients of numerator and denominator by (3) and (4).

Experimentally, some remarks need to be mentioned.

1) **Updating of the target function phase:** Reference to [1], here we only approximate the magnitude response of $H^d(\omega)$. Yet the cost function $E(\omega)$ also depends on the phase of $H^d(\omega)$. In absence of any information, we can initially assume the desired phase response linear, i.e., $\varphi(\omega) = M\omega$, where M is a given constant. Reference to [1] and [8], to have a well-behaved response and fast convergence, an iterative phase updating can be adopted. Here we describe this method in brief. At the nth iteration, let $A^{(n)}$ be the coefficient vector obtained and $H^{(n)}(\omega)$ the corresponding frequency response. Assume that $\varphi^{(n+1)}(\omega)$ be the phase of $H^d(\omega)$ at the $(n+1)$th iteration. Assign $\varphi^{(n+1)}(\omega) = \angle H^{(n)}(\omega)$, and redo the design procedure until some criterion is met.

2) **Choice of weighting function $W_k(\omega)$:** As discussed previously, to compute the quadratic matrix P in (12) and (13), we need to decide the weighting function. Let $W^{(n)}(\omega)$ be the weighting function in the nth iteration. We adopt the recommendation of [8] in our experiments, let

$$W^{(n)}(\omega) = \frac{1}{|D^{(n-1)}(\omega)|} \qquad (14)$$

where $D^{(n)}(\omega)$ is the denominator of $H^{(n)}(\omega)$. So at each iteration, we can give higher weights to the regions where $D^{(n-1)}(\omega)$ is small and the error will be smaller in the regions. Another recursive weighting function $W^{(n)}(\omega) = 1/[D^{(n-1)}(\omega)]^2$ can be used here and suggested in [18].

3) **Stability check:** The stability of the filters we designed should be carefully concerned. To ensure the stability of the IIR filters, we need to calculate the roots of denominator polynomial to find unstable poles and substitute them with their inverse conjugate. So that we can make this filter always stable without changing its magnitude response.

To summarize the method to design IIR multiple notch filters, we can take the following steps.

1) Decide the positions of notches and calculate polynomial $C(z)$ according to (4). The initial state can be set $\varphi^{(0)}(\omega) = K\omega$ (where K denotes the number of notches we need) and $W_k^{(0)}(\omega) = 1 \, \forall \, k$.

2) Compute the matrix P by using (12)–(14), where P is a function of $\varphi^{(n)}(\omega)$ and $W_k^{(n)}(\omega)$. Let $A^{(n)}$ be the eigenvector corresponding to the minimum eigenvalue of P, and let $H^{(n)}(z)$ be the relative transfer function; find unstable poles and substitute them with their inverse conjugate.

3) Update the phase of the target function by using $\varphi^{(n+1)}(\omega) = \angle H^{(n)}(\omega)$; update the weighting function $W_k^{(n+1)}(\omega)$ according to (14).

4) Compute the poles outside the unit circle and substitute them with their inverse conjugate.

5) Repeat steps (2)–(4) until some stop criterion is met. In our experiments, we take fifty iterations in each result for that the error converges within twenty iterations.

6) Using (1) and (2) to get the desired transfer function coefficients.

B. Nyquist Filters and Partial Response Filters

1) Nyquist Filters: As mentioned in the introduction, to obtain the zero intersymbol interference, Nyquist filter $H(z)$ has the impulse response $h(n)$ with the time domain constraints, i.e.

$$h(K + kN) = \begin{cases} c \neq 0, & \text{if } k = 0 \\ 0, & \text{otherwise} \end{cases} \qquad (15)$$

where K and N are integers. Then the impulse response crosses the time axis every N samples. According to [15], the conditions in (15) can be rearranged and written as

$$h(K + kN) = \begin{cases} \frac{1}{N} \neq 0, & \text{if } k = 0 \\ 0, & \text{otherwise} \end{cases} \qquad (16)$$

where K and N are integers. It is known that the transfer function of an IIR Nyquist filter can be expressed in the form

$$H(z) = b_K z^{-K} + \frac{\sum\limits_{i=0, \, i \neq kN+K}^{N_n} b_i z^{-i}}{\sum\limits_{i=0}^{N_d/N} a_{iN} z^{-iN}}, \qquad b_K = \frac{1}{N} \qquad (17)$$

where N_n, N_d are integers, and all the filter coefficients a_i, b_i are real, $a_0 = 1$. N_d is a multiple of N. Hence, only frequency-response optimization should be considered when the above transfer function is employed. The specification of Nyquist filters in frequency domain

$$C_k(\omega) = [H_k^d(\omega) \quad H_k^d(\omega)e^{-j\omega} \quad \cdots \quad H_k^d(\omega)e^{-j2K\omega} \quad -\prod_{i=1}^k \left(1 - 2 \cdot \cos(\omega_i) \cdot e^{-j\omega} + e^{-j2\omega}\right)]^T \qquad (8)$$

and

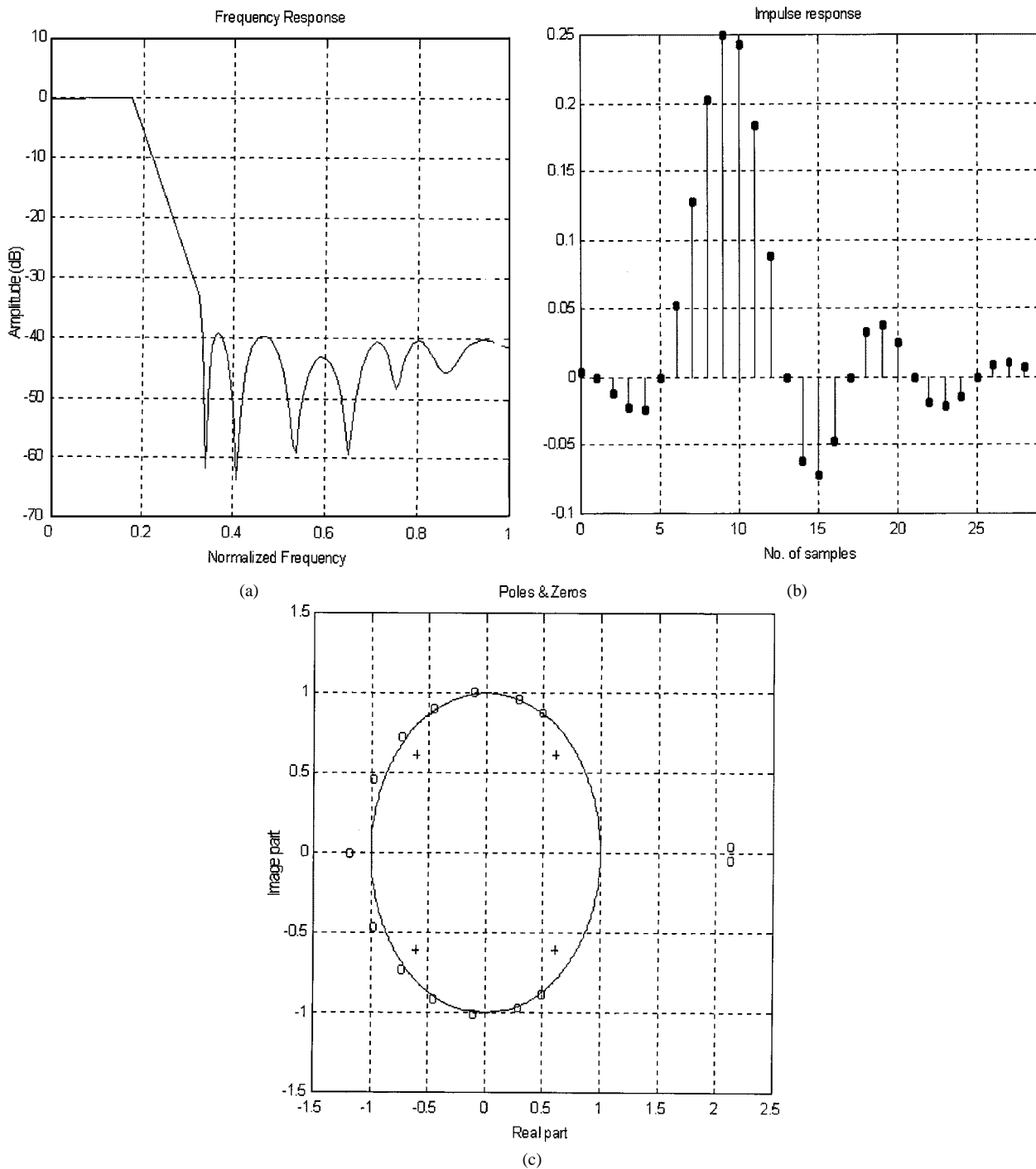$$A = [a_0 \quad a_1 \quad \cdots \quad a_{2K} \quad b_0]^T. \qquad (9)$$

Fig. 2.   (a) Magnitude response of example 2. (b) Impulse response of example 2. (c) Pole-zero plotting of example 2.

should be a lowpass filter with passband and stopband cutoff frequency ω_p and ω_s expressed as

$$\omega_p = \frac{1-\rho}{N}\pi \tag{18a}$$

$$\omega_s = \frac{1+\rho}{N}\pi \tag{18b}$$

where N is the interpolation ratio and ρ is the rolloff rate.

2) Partial Response Filters: The partial response filters can be thought as a modification of Nyquist filters but have some specified magnitude and impulse response. These filters can be classified to five classes and summarized in Table I [16].

In Table I, each binary symbol C_n is chosen to be a prescribed superposition of n successive transmitted impulses $b_1, b_2, \ldots, b_n$ as $C_n = k_1 b_n + k_2 b_{n-1} + \cdots + k_n b_1$ [16]. Partial response filters can achieve high data rates with better error rate performance. And the class of binary in Table I is the original Nyquist filter.

The transfer function of partial response filters is assumed as $H(z) = \mathrm{Num}(z)/\mathrm{Den}(z)$, to obtain the impulse and magnitude response prescribed, the numerator polynomial of the transfer function must be divided by some specified polynomial. That is

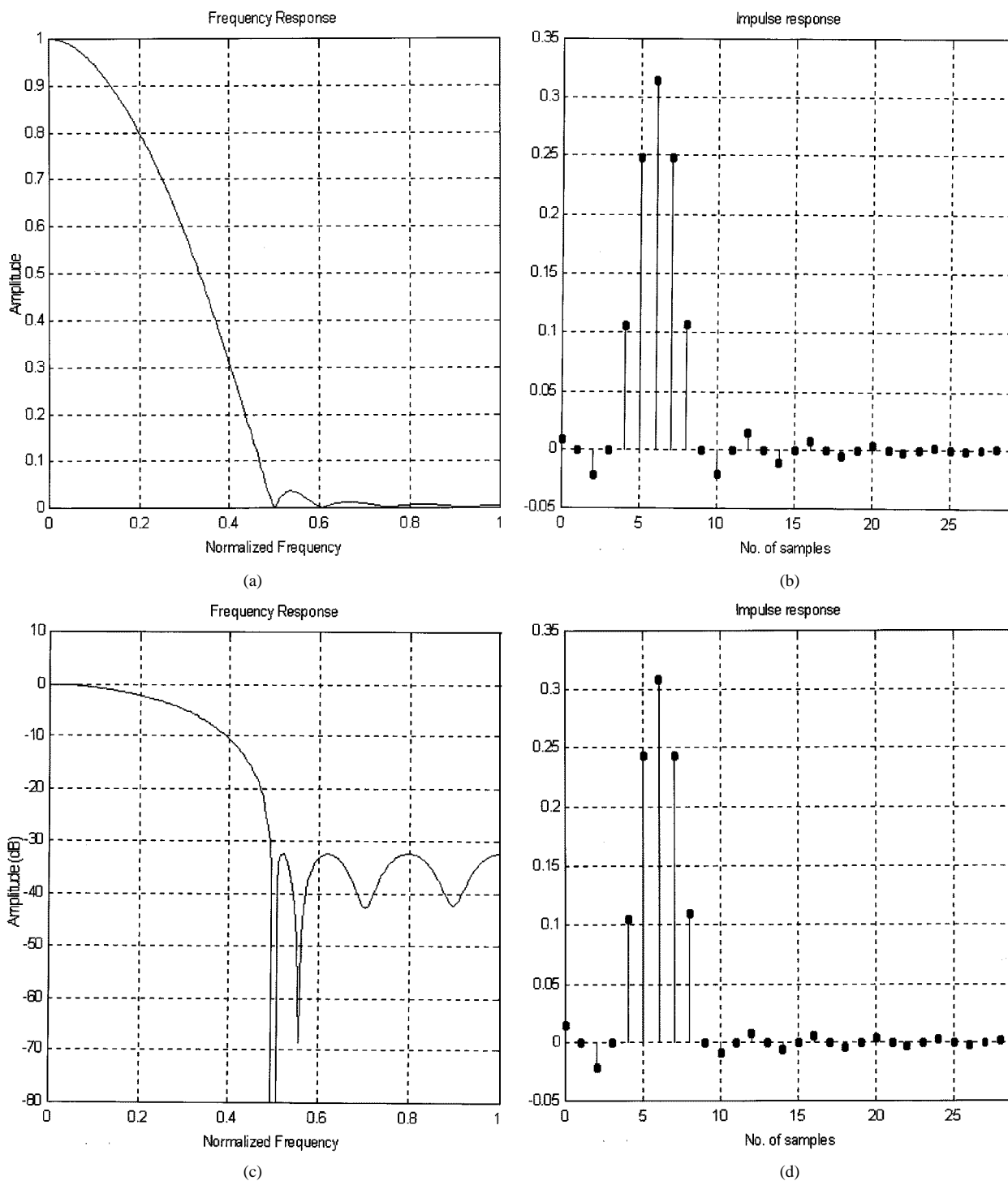$$R(z) \mid \mathrm{Num}(z) \tag{19}$$

Fig. 3. (a) Linear magnitude response of example 3: class 1 partial response filter. (b) Impulse response of example 3. (c) Magnitude response of equiripple case of example 3. (d) Impulse response of equiripple case of example 3.

where

$$R(z) = \begin{cases} 1 & \text{Nyquist filter} \\ 1 + z^{-N} & \text{class (1)} \\ 1 + 2z^{-N} + z^{-2N} & \text{class (2)} \\ 2 + z^{-N} - z^{-2N} & \text{class (3)} \\ 1 - z^{-2N} & \text{class (4)} \\ -1 + 2z^{-2N} - z^{-4N} & \text{class (5).} \end{cases} \quad (20)$$

So the transfer functions of the classified partial response filters can be thought as Nyquist filters cascade with some specified $R(z)$. Nyquist filter is a special case of partial response filter with $R(z) = 1$.

We can assume the calculated frequency response of a partial response filter as

$$H(e^{j\omega}) = \left[\frac{1}{N} e^{-jK\omega} + \frac{\displaystyle\sum_{i=0,\ i \neq kN+K}^{N_n} b_i e^{-ij\omega}}{\displaystyle\sum_{i=0}^{N_d/N} a_{iN}(e^{-j\omega})^{iN}} \right] R(e^{j\omega}) \quad (21)$$

and the desired frequency response $H^d(e^{j\omega})$ is given in Table I, then the current error is

$$\varepsilon(\omega) = H^d(e^{j\omega}) - H(e^{j\omega}). \quad (22)$$

Let the cost function be

$$
\begin{aligned}
E(\omega) &= \varepsilon(\omega) \sum_{i=0}^{N_d/N} a_{iN}(e^{-j\omega})^{iN} \\
&= \left(H^d(e^{j\omega}) - \frac{1}{N}e^{-jK\omega}D(e^{j\omega}) \right) \sum_{i=0}^{N_d/N} a_{iN}(e^{-j\omega})^{iN} \\
&\quad - R(e^{j\omega}) \sum_{i=0,\ i\neq kN+K}^{N_n} b_i e^{-ij\omega}
\end{aligned}
\tag{23}
$$

and the total error function of the whole frequency range will be

$$
\Phi(\omega) = \int_\omega |E(\omega)|^2 W(\omega)\,d\omega.
\tag{24}
$$

If

$$
C(\omega) = \begin{bmatrix} A' & B' \end{bmatrix}
\tag{25a}
$$

where (25b), (25c), and (26) are shown at the bottom of the page. Then we can rewrite the cost function as

$$
\Phi(\omega) = \int_\omega A^T C^*(\omega) C(\omega) A W(\omega)\,d\omega = A^T P A
\tag{27}
$$

and

$$
P = \int_\omega C^*(\omega) C(\omega) W(\omega)\,d\omega.
\tag{28}
$$

In (27) and (28), $W(\omega)$ is the weighting function of each frequency grid.

Since (27) results an eigenform, we can solve the minimization problem by eigenfilter approach to obtain the optimal solution. At last, according to (21), we should multiply the calculated numerator by $R(z)$ specified by (20) and obtain the desired transfer function coefficients. The resulting numerator order will be $N_n + \deg[R(z)]$.

As the notch filter design discussed in Section II-A, we need a weighting function $W(\omega)$ when computing the matrix P. Here we can use the weighting function prescribed in (14) to get a nonequiripple response. If the equiripple response is desired, another kind of weighting function should be employed. In the previous works proposed [8], a recursive updating of the weighting function was introduced to obtain an almost equiripple solution. Also we adopted the recursive procedure to get a convergent solution satisfying the given specification. An adequate choice of the recursively updating weighting function is given below.

Let $A^{(n)}$ be the solution vector at the nth iteration and $H^{(n)}(\omega)$ the corresponding frequency response. The magnitude error is

$$
e^{(n)}(\omega) = \left| |H^d(\omega)| - |H^{(n)}(\omega)| \right|
\tag{29}
$$

and the weighting function used in the $(n+1)$ iteration will be

$$
W^{(n+1)}(\omega) = W^{(n)}(\omega)\operatorname{env}(e^{(n)}(\omega))
\tag{30}
$$

where $\operatorname{env}(e^{(n)}(\omega))$ is the envelope of the positive function $e^{(n)}(\omega)$. Then the resulting solution will be almost equiripple.

The Nyquist filters and partial response filters are lowpass filters and have no constraints of the phase response. Usually we use a linear phase as the initial target phase response. So that we can initially set $\angle H(\omega) = -M\omega$, in which M is a constant, as the desired phase re-

TABLE III
RESULTING COEFFICIENTS OF EXAMPLE 2

b_0	0.00373701676675	b_{11}	0.25294719373311
b_1	0	b_{12}	0.19736549464621
b_2	-0.01124615922963	b_{13}	0.13402787767668
b_3	-0.02174419712868	b_{14}	0.06879712348272
b_4	-0.02126344099409	b_{15}	0.02727697661389
b_5	0	a_0	1.0
b_6	0.04660624892311	a_1	0
b_7	0.11623394636996	a_2	0
b_8	0.19049647956317	a_3	0
b_9	0.25	a_4	0.53611151070671
b_{10}	0.27100557266568		

sponse when we calculated $H_d(\omega)$. Experimentally we use the given integer K to be M, then

$$
H_d(\omega) = |H_d(\omega)|e^{-jK\omega}.
\tag{31}
$$

III. EXPERIMENTAL RESULTS

A. Notch Filter Design

Here we present some design examples using our proposed method. First for the notch filters, we show the effectiveness of the proposed approach with arbitrarily prescribed notch frequencies and bandwidth.

Example 1: A notch filter with three different notches at $\omega = 0.3\pi$, 0.6π, and 0.9π. Besides that, we can arbitrarily specify the notch bandwidth. Here the widths of three notches are 0.002π, 0.02π, and 0.04π, respectively. The responses are shown in Fig. 1, and Table II gives the resultant coefficients. It is interesting that the mirror-image symmetry relation exists between the numerator and denominator polynomials of allpass filter, and the resultant IIR notch filter $H(z)$ can be expressed as $H(z) = (1 + A(z))/2$, where $A(z)$ is an allpass filter [19]. The main advantage of this form is that it can be realized by a computationally efficient lattice structure with low sensitivity [19].

Designing the notch filter using the eigenfilter approach with constraints can obtain exact zeroes at the desired locations and very flat passband elsewhere, and the solution converges after several iterations.

B. Nyquist Filter Design

Example 2: A Nyquist filter with $N_n = 15$, $N_d = 4$, $\rho = 0.3$, $N = 4$, $K = 9$ and the weighting of ripple ratio $\delta_p/\delta_s = 1000$. To express the properties of Nyquist filters completely, the equiripple magnitude response, impulse response, and pole-zero plotting are given in Fig. 2. The resultant coefficients are given in Table III. The specification of this example is the same as illustrated in [14]. Compared with the attenuation 38 dB in [14], an improved minimum stopband attenuation of almost 40 dB is obtained but the ripples are not quite equal.

C. Partial Response Filter Design

At last we design the class 1 partial response filter in example 3. Both equiripple and nonequiripple cases are displayed.

and

$$
A' = \left(H^d(e^{j\omega}) - \frac{1}{N}e^{-jK\omega}R(e^{j\omega}) \right) \begin{bmatrix} 1 & e^{-j\omega} & e^{-jN\omega} & e^{-j2N\omega} & \dots & e^{-jN_d\omega} \end{bmatrix}
\tag{25b}
$$

$$
B' = R(e^{j\omega}) \begin{bmatrix} -1 & -e^{-j\omega} & -e^{-j2\omega} & \dots & -e^{-ji\omega} & \dots & -e^{-jN_n\omega} \end{bmatrix}, \qquad i \neq kN+K
\tag{25c}
$$

$$
A = \begin{bmatrix} a_0 & a_N & a_{2N} & \dots & a_{N_d} & b_0 & b_1 & b_2 & \dots & b_i & \dots & b_{N_n} \end{bmatrix}^T, \qquad i \neq kN+K
\tag{26}
$$

TABLE IV
RESULTING COEFFICIENTS OF EQUIRIPPLE CASE OF EXAMPLE 3

b_0	0.01446798102295	b_7	0.45110472197426
b_1	0	b_8	0.37240643153797
b_2	-0.00961818862706	b_9	0.20708477391566
b_3	0	b_{10}	0.08488537433446
b_4	0.08634022042579	a_0	1.0
b_5	0.24401994805860	a_1	0
b_6	0.39794744727930	a_2	0.84863870992193

Example 3: Class 1 partial response filter with the specifications: $N_n = 10$, $N_d = 2$, $N = 2$, $K = 6$. The magnitude responses and impulse responses of both equiripple and nonequiripple cases are given in Fig. 3. To express the cosine shape of the magnitude response of the class 1 partial response filter, the magnitude response of the nonequal ripple is given in linear scale in Fig. 3(a). Other magnitude responses are given in log scale to show the attenuation of the stopband. The coefficients of equiripple case of class 1 filter are given in Table IV. Compared with the design example of 30 dB stopband attenuation using the complicated three step optimization in [20], the designed filter with better 32-dB stopband rejection can be obtained by our effective one-step eigen approach.

IV. CONCLUSION

In this paper, we have presented a new method to design digital IIR notch filters, Nyquist filters, and partial response filters. To design these filters, many methods have been proposed before. However, designing these filters with time-domain or frequency-domain constraints, IIR eigenfilter approach is a better choice and easier to handle. Here we first formulated the properties of these IIR filters and extended the existing IIR eigenfilter approach to design these filters. The effectiveness of eigenfilter approach has been revealed for adding the time-domain or frequency-domain constraints. We have employed an iteration process to obtain an equiripple, stable solution of the desired IIR filters.

REFERENCES

[1] P. P. Vaidyanathan and T. Q. Nguyen, "Eigenfilters: A new approach to least-squares FIR filter design and applications including Nyquist filters," *IEEE Trans. Circuits Syst.*, vol. CAS-34, pp. 11–23, Jan. 1987.
[2] S. C. Pei and J. J. Shyu, "Design of FIR Hilbert transformers and differentiators by eigenfilter," *IEEE Trans. Circuits Syst.*, vol. 35, pp. 1457–1461, Nov. 1993.
[3] ——, "Eigenfilter design of higher-order digital differentiators," *IEEE Trans. Signal Processing*, vol. 37, pp. 505–511, Apr. 1989.
[4] T. Q. Nguyen, "The design of arbitrary FIR digital filters using the eigenfilter method," *IEEE Trans. Signal Processing*, vol. 41, pp. 1128–1139, Nov. 1993.
[5] S. C. Pei and J. J. Shyu, "Complex eigenfilter design of arbitrary complex coefficient FIR digital filters," *IEEE Trans. Circuits Syst. II*, vol. 40, pp. 32–40, June 1993.
[6] ——, "Design of 1-D and 2-D IIR eigenfilters," *IEEE Trans. Signal Processing*, vol. 42, pp. 962–966, Apr. 1994.
[7] X. Zhang and H. Iwakura, "Design of IIR digital filters based on eigenvaule problem," *IEEE Trans. Signal Processing*, vol. 44, pp. 1325–1333, June 1996.
[8] F. Argenti and E. Del Re, "Design of IIR eigenfilters in the frequency domain," *IEEE Trans. Signal Processing*, vol. 46, pp. 1694–1698, June 1998.
[9] T.-H. Yu, S. K. Mitra, and H. Babic, "Design of linear phase FIR notch filters," *Sadhana*, vol. 15, no. 3, pp. 133–155, Nov. 1990.
[10] A. Nehorai, "A minimal parameter adaptive notch filter with constrained poles and zeros," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-33, pp. 983–966, Aug. 1985.
[11] T. S. Ng, "Some aspects of an adaptive digital notch filter with constrained poles and zeros," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. ASSP-35, pp. 158–161, Feb. 1987.
[12] J. K. Liang, R. J. Pefigueriredo, and F. C. Lu, "Design of optimal Nyquist, partial response, Nth band and nonuniform tap spacing FIR digital filters using linear programming techniques," *IEEE Trans. Circuits Syst.*, vol. CAS-32, pp. 386–392, Apr. 1985.
[13] X. Zhang and H. Iwakura, "Design of FIR Nyquist filters using Remez exchange algorithm," *IEICE Trans.*, vol. J79-A, no. 8, pp. 1378–1384, Aug. 1996.
[14] K. Nakayama and T. Mizukami, "A new IIR Nyquist filter with zero intersymbol interference and its frequency response approximation," *IEEE Trans. Circuits Syst.*, vol. CAS-29, pp. 23–34, Jan. 1982.
[15] S. J. Maeng and B. G. Lee, "A design of linear-phased IIR Nyquist filters," *IEEE Trans. Select. Areas Comm.*, vol. 13, pp. 167–175, Jan. 1995.
[16] E. R. Kretzmer, "Generalization of a technique for binary data communication," *IEEE Trans. Commun. Technol.*, vol. COM-14, pp. 1966–1969, Feb. 1966.
[17] R. W. Lucky, J. Salz, and E. J. Weldon Jr., *Principles of Data Communication*. New York: McGraw-Hill, 1968.
[18] W. S. Lu, S. C. Pei, and C. C. Tseng, "A weighted least-squares method for the design of stable 1-D and 2-D IIR digital filters," *IEEE Trans. Signal Processing*, vol. 46, pp. 1–10, Jan. 1998.
[19] S. C. Pei and C. C. Tseng, "IIR multiple notch filter design based on allpass filter," *IEEE Trans. Circuits Syst. II*, vol. 44, pp. 133–136, Feb. 1997.
[20] D. Y. Yu, S. Q. Wu, and L. M. Li, "Design of partial response data transmission filters with specified stopband attenuation," *Proc. Inst. Elect. Eng.*, pt. G, vol. 138, no. 3, pp. 347–350, June 1991.

The Design of Peak-Constrained Least Squares FIR Filters With Low-Complexity Finite-Precision Coefficients

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Abstract—A method for the design of peak-constrained least squares (PCLS) finite-impulse response (FIR) digital filters with low-complexity finite-precision coefficients (FPC) based on Adams' optimality criterion and an efficient local search is presented. Simple quantization of the infinite precision coefficients typically leads to filter designs which fail to meet the frequency response, passband to stopband energy ratio (PSR) and coefficient complexity (number of coefficient adders and subtractors) specifications. It is shown that it is possible to design a filter with an acceptable PSR that meets the frequency response specification while using a reduced number of adders and subtractors.

Index Terms—Canonic signed digit (CSD), finite-impulse response (FIR), local search, peak-constrained least squares (PCLS).

I. INTRODUCTION

Peak-constrained least squares (PCLS) filters are well suited for applications where both the minimum stopband attenuation (DBs) and the passband to stopband energy ratio (PSR) of the filter are important