Passivity Based Control of the Double Inverted Pendulum Driven by a Linear Induction Motor

Chin-I Huang, and Li-Chen Fu

Abstract— In this paper, we will propose a nonlinear passivity based controller for a double inverted pendulum system, consisting of a two-link pendulum mounted on a pad which is moved by a linear induction motor so that the double pendulum points straight-up. The proposed controller not only can drive the double pendulum from its "natural stable hanging position" to its "unstable upright inverted position" while linear the induction motor's displacement is brought to zero but also can guarantee that the motor with very unique end-effect can achieve servo motion tracking with high performance. Also, computer simulations are provided to demonstrate the effectiveness of the hereby presented controller design.

Index Terms— Double Inverted Pendulum, Underactuated Mechanical System, Linear Induction Motor, Passivity Based Control, Adaptive Backstepping Control

I. INTRODUCTION

The general double inverted pendulum system is a two-link (two-degree of freedom) manipulator mounted on a cart (one degree of freedom), where both links are not actuated but the cart is actuated. The main goal is to bring up the two-link pendulum from its "natural stable hanging position" to its "unstable upright inverted position" while the cart is brought to the origin. Since if the cart needs to be driven by a rotary actuator and to move translationally, then a gear and a rotary-to-linear converter will be required to fulfill the purpose. But the most obvious advantage of the linear motor is that it has no power loss due to gears and is free of mechanical rotary-to-linear converters to produce linear motion. Hence, in this paper the cart of the system is in particular actuated by a linear induction motor. For the double inverted pendulum, it is a typical underactuated mechanical system. So far, there have been many similar systems proposed in the literature, such as Acrobot[24], Pendubot [18-20], etc.

Many studies have been conducted for controlling nonlinear underactuated mechanical systems [18-20, 23], which can be roughly classified into two groups respectively associated with swing up control and a balance control. For implementation of the first control strategy, researchers use the concept of partial feedback linearization and passivity to design the swing up controller. For implementation of the latter, people consider the linear quadratic regulator (LQR) design or the pole placement technique based on the plant model linearized around the desired equilibrium stabilizing the pendulum at the desired position.

On the other hand, linear induction motors (LIMs) widely used in many industrial applications including transportation, conveyor systems, actuators, material handling, pumping of liquid metal, and sliding door closers, etc. with satisfactory performance. Besides LIMs, linear electric motors also include linear D.C. motors, linear synchronous motors and linear stepping motors. Among these motors, the LIM has many advantages such as simple replacement about the motion device of the motor, reduction of mechanical power losses and of the size of the motion device, silence, high starting thrust force, and easy maintenance, repairing and replacement.

In the early researches, Yamamura has first discovered a particular phenomenon of the end-effect on LIM [1]. A control method, decoupling the control of thrust and the attractive force of a LIM using a space vector control inverter, was presented in [2], i.e. by selecting voltage vectors of PWM inverters appropriately.

Although the parameters of the simplified equivalent circuit model of an LIM can be measured by conventional methods (no-load and locked secondary tests), due to limited length of the machine the realization of the no-load test is almost impossible. Thus, the applicability of conventional methods for calculating the parameters of the equivalent model is limited. In order to measure the parameters, application of the finite element (FE) method for determining the parameters of a two-axis model of a three-phase linear induction motor has been proposed in [3]. Another method is proposed by removing the secondary [4].

To resolve the unique end-effect problem for an LIM, speed dependent scaling factors are introduced to the magnetizing inductance and series resistance in the d-axis equivalent circuit of the rotary induction motor (RIM) [5] to correct the deviation caused by the "end-effect". On the other hand, there is a thrust correction coefficient introduced by [6,7] to calculate an actual thrust to compensate for the end effect. A related method to deal with the problem is that an external force corresponding to the end effect is introduced into the RIM model to provide a more accurate modeling of an LIM under consideration of end effect as shown in [8].

The paper is organized as follows. In section 2, we introduce the system model and state the control objectives. Section3 will develop a passivity-based nonlinear control of the double inverted pendulum to solve the task of bringing up to pendulum from its "natural stable hanging position" to its "unstable upright inverted position". In Section 4, we propose an adaptive backstepping approach to fulfill the tracking linear induction motor. To

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demonstrate the effectiveness of the hereby developed controllers, Section 5 will provide extensive computer simulations with satisfactory performance. Finally, Section 6 gives some concluding remarks.

II. PROBLEM FORMULATION

The system of double inverted pendulum consists of a two-linked pendulum mounted on a cart actuated by a linear induction motor to move translationlly along a horizontal track so as to maintain two-link inverted pendulums on the cart. Each link is free to rotate on the vertical plane about an axis associated with an articulated joint, defining the two DOFs, $\theta_1(t)$ and $\theta_2(t)$. In our set-up, we assume that no torque can be around the axis of $\theta_1(t)$ and $\theta_2(t)$, which are two passive joints. The linear induction motor is an actuator, which exerts a force along the translational DOF x. Figure 1 illustrates the system of double inverted pendulum on a cart driven by a linear induction motor. The swing up problem consists in driving the double pendulum from its "natural stable hanging position" to its "unstable upright inverted position" while the linear induction motor displacement is brought to zero.

In this paper, we not only consider the pendulum dynamics but also consider the dynamics of the actuator, i.e. linear induction motor. The mathematical model of the double inverted pendulum is constructed using Euler-Lagrange method under the following assumptions to simplify the analysis:

(A.1) The links of the pendulum are rigid bodies;
(A.2)The inherent friction of the pendulum can be neglected;
(A.3) Three phases of the LIM are balanced;
(A.4) The magnetic circuit of the LIM is unsaturated;
(A.5) The LIM is without end effect (we will relax this assumption later in the stage of controller design),
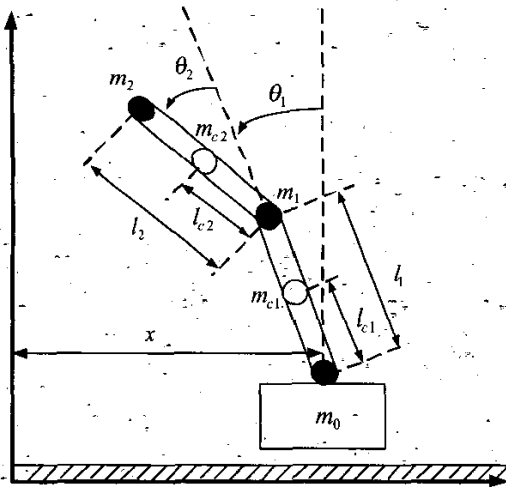

Fig 1: The DIP on a LIM configuration
the dynamics of the entire system can be rearranged into the following more compact form

$$\dot{i}_q = -a_1 i_q + a_2 \lambda_q - \beta p v_r \lambda_d + c V_{qs}$$

$$\dot{i}_d = -a_1 i_d + \beta p v_r \lambda_q + a_2 \lambda_d + c V_{ds}$$

$$\dot{\lambda}_q = a_3 i_q - a_4 \lambda_q + p v_r \lambda_d$$

$$\dot{\lambda}_d = a_3 i_d - a_4 \lambda_d - p v_r \lambda_q$$

$$F_e = K_f (\lambda_d i_q - \lambda_q i_d)$$

$$F = M(q)\ddot{q} + C(q,\dot{q}) + G(q) \quad (1)$$

where $q = [x\ \theta_1\ \theta_2]^T$ is the generalized coordinates and $F = [F_e\ 0\ 0]^T$ is vector of the external force. The notations of the inertia matrix M, the Coriolis C, and gravity G, and the parameters involved are shown in Table 1.

III. PASSIVE SYSTEM AND PASSIVITY BASED CONTROL

In this section, we try to design the controller which achieves the goal of moving the double pendulum from its "natural stable hanging position" to its "unstable upright inverted position" while the cart driven by the LIM is brought to origin. To facilitate the controller design, we briefly review some of the basic definitions and the theorem of passivity theory [25].

3.1 Basic concepts and definitions of Passive System

Consider an MIMO nonlinear system of the form

$$\dot{x} = f(x) + g(x)u$$
$$y = h(x) \quad (2)$$

where $x \in R^n$ is the state, $u \in R^m$ is the input, $y \in R^m$ is the output, f and g are C^∞ vector fields, and h is a smooth mapping (i.e. C^∞). It is assumed that the vector field f has at least one equilibrium and without loss of generality we assume that $f(0) = 0$ and $h(0) = 0$. Let U be the set of admissible inputs that consisting of all real-valued piecewise continuous function defined on R^m. It is also assumed that for any $u \in U$ and for any $x^0 = x(0) \in R^n$, the output $y(t) = h(\phi(t, x^0, u))$ of system (2), where $\phi(t, x^0, u)$ denotes the flow of $f(x) + g(x)u$ for any initial condition $x^0 \in R^n$ and for any $u \in U$, is such that $\int_0^t |y^T(s) u(s)| ds < \infty$, for all $t \geq 0$, i.e. the energy stored in system (2) is bounded above.

Definition 1[25]: _A system (2) is said to be C^r passive if there exists a C^r nonnegative function $V : R^n \to R$, called the storage function, which satisfies $V(0) = 0$, such that for all $u \in U$, $x^0 \in R^n$, $t \geq 0$_

$$V(x) - V(x^0) \leq \int_0^t y^T(s) u(s) ds \quad (3)$$

Definition 2[25]: _A passive system Σ with storage function V is said to be lossless if for all $u \in U$, $x^0 \in R^n$, $t \geq 0$_

$$V(x) - V(x^0) = \int_0^t y^T(s) u(s) ds \quad (4)$$

Definition 3[25]: _A passive system Σ with storage function V is said to be strictly passive if there exists a_

positive definite function $S : R^n \to R$ such that for all $u \in U$, $x^0 \in R^n$, $t \geq 0$

$$V(x) - V(x^0) = \int_0^t y^T(s)u(s)ds - \int_0^t S(x(s))ds \quad (5)$$

Theorem 1[25]: *Suppose Σ is passive with a storage function V which is positive definite with no critical point other than $x = 0$ and such that $L_f V(x) = 0$ (i.e. lossless system) and if $D = span\{ad_f^k g_i : 0 \leq k \leq n-1, 1 \leq i \leq m\}$ has dimension n at each $x \neq 0$, then the control law*

$$u = -(L_g V(x))^T = -h^T(x) \quad (6)$$

globally asymptotically stabilize the system.

3.2 Partial Feedback Linearization

It has been shown that the dynamics of the double inverted pendulum (1) are not feedback linearizable with static state feedback nor through some nonlinear coordinate transformation [20]. The double inverted pendulum system belongs to the class of underactuated mechanical systems, which have fewer control inputs than the degrees of freedom. We first consider a general underactuated mechanical system with n generalized coordinates and m actuators. By partitioning the vector q, we get $q = [q_\theta^T, q_x^T]$, with q_θ corresponding to the passive variables and q_x corresponding to the actuated variables. The dynamics of n degrees of freedom of the general underactuated mechanical system with q_θ passive coordinates and q_x actuated coordinates can be written in the following general form [20]:

$$M_{\theta\theta}\ddot{q}_\theta + M_{\theta x}\ddot{q}_x + C_\theta(q,\dot{q}) + g_\theta(q) = 0$$
$$M_{x\theta}\ddot{q}_\theta + M_{xx}\ddot{q}_x + C_x(q,\dot{q}) + g_x(q) = f_x \quad (7)$$

Here, $M(q)$ is the symmetric, positive definite inertia matrix, the $C(q,\dot{q})$ includes Coriolis and centrifugal terms, G contains the terms derived from the potential energy, such as gravitational and elastic generalized forces, and f_x represents the input of the generalized forces produced by the m actuators at q_x. In the underactuated mechanical system (7), we first assume that all the parameters are known. Thus we may achieve a linear response from either link, but not both, by applying suitable nonlinear partial feedback linearization. Considering the first formula in (7), one gets

$$\ddot{q}_\theta = -M_{\theta\theta}^{-1}(M_{\theta x}\ddot{q}_x + C_\theta + g_\theta) \quad (8)$$

By substituting (8) into the second formula in (7), one obtains

$$\bar{M}_{xx}\ddot{q}_x + \bar{C}_x + \bar{g}_x = f_x \quad (9)$$

where

$$\bar{M}_{xx} = M_{xx} - M_{x\theta}M_{\theta\theta}^{-1}M_{\theta x}$$
$$\bar{C}_x = C_x - M_{x\theta}M_{\theta\theta}^{-1}C_\theta \quad (10)$$
$$\bar{g}_x = g_x - M_{x\theta}M_{\theta\theta}^{-1}g_\theta$$

Hence, a partial feedback linearizing controller can be defined according to equations (9) and (10) yielding

$$f_x = \bar{M}_{xx}u + \bar{C}_x + \bar{g}_x \quad (11)$$

where u is regarded as a new control input. As a result, the complete system can be rewritten as

$$M_{\theta\theta}\ddot{q}_\theta + C_\theta + g_\theta = -M_{\theta x}u$$
$$\ddot{q}_x = u \quad (12)$$

Let the control input u be designed into two parts $u = u_x + u_\theta$. Then, u_x and u_θ are designed such that q_x and q_θ will converge to zero as $t \to \infty$, respectively.

3.3 Passivity Based Controller Design

The partial feedback linearization approach transfers the original system (7) into a simpler one (12) both in concept and in structure. For the reason of convenience, we rewrite system (12) in a more general form

$$\dot{\theta} = f(\theta) + g(\theta)u$$
$$\dot{x} = Ax + Bu \quad (13)$$

According to nonlinear θ-subsystem in (13), there exists the total energy $V_\theta(\theta)$. The time derivative of the total energy $V(\theta)$ can be derived as

$$\dot{V}_\theta(\theta) = \frac{\partial V_\theta(\theta)}{\partial\theta}\dot{\theta} = \frac{\partial V_\theta(\theta)}{\partial\theta}(f(\theta) + g(\theta)u)$$
$$= L_f V_\theta + L_g V_\theta u \quad (14)$$

By the assumption A.2, the double inverted pendulum system is lossless system. Thus, we have

$$L_f V_\theta = 0 \quad (15)$$

According to Theorem 1, if the control input is chosen as

$$u = -(L_g V_\theta)^T \quad (16)$$

then the θ subsystem is globally asymptotically stable.

On the other hand, the x-subsystem is globally asymptotically stable if the control input u is selected as $u = -K_x x$ such that the transfer function $K_x(sI - A)^{-1}B$ is strictly positive real. From the Passivity Theorem [22, 27], we know that the whole system can be rendered passive with the combined control input

$$u = -(L_g V_\theta)^T - K_x x \quad (17)$$

Remark: *In order to achieve the former control objective, i.e. to stabilize the double inverted pendulum in the "unstable upright inverted position", the following two conditions should be satisfied.*
A) $x = 0$, $\dot{x} = 0$;

B) $V_\theta = V_{up} = g(2m_{c2}l_{c2} + 2m_2l_2 + m_1l_1 + m_2l_1 + m_{c2}l_1 + m_{c1}l_{c1})$;
where the second condition refers to the double inverted pendulum at the desired unstable upright inverted position with zero linear displacement and zero linear velocity. The explicit energy expression of the double inverted pendulum, when under the above conditions, is:

$$V_\theta = \frac{1}{2}M_{22}\dot{\theta}_1^2 + \frac{1}{2}M_{33}(\dot{\theta}_1 + \dot{\theta}_2)^2 + M_{23}\dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2$$
$$+ g(2(m_{c2}l_{c2} + m_2l_2)\cos(\theta_1 + \theta_2) + (m_1l_1 + m_2l_1 + m_{c2}l_1 + m_1l_{c1})\cos\theta_1)$$
$$= V_{up} \quad (18)$$

The control input which can successfully swing up the double pendulum from its "natural stable hanging position" to its "unstable upright inverted position" is shown to be

$$u = -k_p x - k_v \dot{x}$$
$$+ k_5(V_\theta - V_{up})((m_1l_1 + m_2l_1 + m_1l_{c1} + m_2l_1)\dot{\theta}_1\cos\theta_1 + 2(m_2l_2 + m_{c2}l_{c2})(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2))$$

However, the real control input is not "force", but is "voltage" according to the full dynamics of double inverted pendulum system driven by LIM. Since the above proposed passivity based controller can make the system of pendulum globally asymptotically stable, in the following context we will assume that the desired motion trajectory x_d of LIM can be sought. Hence, the problem to find the

799

force/torque control to swing up the double inverted pendulum problem can be changed as motion tracking problem for an LIM. In the next section, we will propose a position controller to achieve the objective of motion tracking of the LIM. An adaptive position controller is proposed to deal with uncertainty of the mutual inductance.

IV. ADAPTIVE BACKSTEPPING CONTROLLER DESIGN

The fundamental difference between a rotary induction motor and a LIM is the finite length of the magnetic and electric circuit of the LIM along the direction of the travelling field. The open magnetic circuit causes an initiation of the so-called longitudinal end-effects [5].

For a LIM, the end effect with the load force can be represented as a function of the speed $\dot{x}$, which can be normally simplified into the form

$$F_E = \sum_{n=0}^{2} b_n \dot{x}^n + M_e \ddot{x} \triangleq M_e \ddot{x} + b_0 + b_1 \dot{x} + b_2 \dot{x}^2$$

In this paper, the end-effect is assumed in the aforementioned form as $F_E = \Theta V_r^T$ with the unknown constant parameters $\Theta = \begin{bmatrix} M_e & b_0 & b_1 & b_2 \end{bmatrix}$, and a known function vector $V_r^T = \begin{bmatrix} \ddot{x} & \dot{x}^0 & \dot{x}^1 & \dot{x}^2 \end{bmatrix}$. The joint mass $M = m_0 + M_e$ is therefore also unknown, which leads to the total mechanical load with motor itself as $F_e = \Theta^T V_r$, where $\Theta^T = [M \; b_0 \; b_1 \; b_2]$. To proceed further, we introduce some more assumption as shown below:

(A.6) $x_2 = \lambda_q^2 + \lambda_d^2 > 0$,

(A.7) The desired speed should be a bounded smooth function with known first and second order time derivatives, then further simplify the dynamics shown in (1) by introducing a nonlinear coordinate transformation given as follows:

$$x_1 = i_q^2 + i_d^2$$
$$x_2 = \lambda_q^2 + \lambda_d^2$$
$$x_3 = i_q \lambda_q + i_d \lambda_d$$
$$x_4 = i_q \lambda_d - i_d \lambda_q$$
$$x_5 = \dot{x}$$

Remarks: The transformation is trying to make the secondary flux norm, the electric force and the rotor speed as individual variables x_2, x_4 and x_5, respectively, and certainly the nonlinear transformation is not unique.

Intially, we adopt the stator voltage inputs as $cV_{ds} = \dfrac{-\lambda_q}{\sqrt{\lambda_d^2 + \lambda_q^2}} V, cV_{qs} = \dfrac{\lambda_d}{\sqrt{\lambda_d^2 + \lambda_q^2}} V$ [17,22], with such

transformation, then the dynamical equations shown in (1) can thus be transformed into the following dynamic model:

$$\dot{x}_1 = -2a_1 x_1 + 2a_2 x_3 + (2x_4 / \sqrt{x_2})V$$
$$\dot{x}_2 = -2a_4 x_2 + 2a_3 x_3$$
$$\dot{x}_3 = a_3 x_1 + a_2 x_2 - (q + a_4)x_3 + p x_5 x_4$$
$$\dot{x}_4 = -px_5 x_3 - \beta px_5 x_2 - (q + a_4)x_4 + \sqrt{x_2}V$$
$$M\dot{x}_5 = K_f x_4 - \sum_{n=0}^{2} b_n x_5^n \qquad (19)$$

To control the system (19) we develop the position controller to achieve the goal $x_r \to x_d$ as introduced in the following section.

Now, we introduce another state

$$x_6 = x_r \qquad (20)$$

to facilitate investigation of the development of a position controller. Then, define the tracking errors as follows:

$$e_p = x_r - x_d \triangleq e_6 \qquad (21)$$

Normally, while the position tracking error is driven to zero, the speed is also regulated to zero. Thus, we naturally define a joint error signal S as follows:

$$S = \dot{e}_p + ae_p = e_5 + ae_6, \qquad (22)$$

where a is a positive scalar gain, and note the case with $a=0$ will be degenerated back speed tracking problem.

Theorem 2. Consider an LIM whose dynamics are governed by system (19) under the assumptions (A.6). Given a third-time differentiable smooth desired position trajectory x_d with x_d, $\dot{x}_d$, $\ddot{x}_d$ and $\dddot{x}_d$ being all bounded, then the following control input can achieve the control objective $x_r \to x_d$ (i.e. $x_6 = x_r$ will follow x_d asymptotically) with the control input

$$V_{qs} = \frac{\lambda_d}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c}, V_{ds} = \frac{-\lambda_q}{\sqrt{\lambda_q^2 + \lambda_d^2}} \frac{V}{c},$$

and

$$V = \frac{1}{\sqrt{x_2}}[g_2(x) + \hat{\Theta}_2 W_2 - K_f z_1 - \rho_2 z_2],$$

with adaptation law

$$\dot{\hat{\Theta}} = \dot{\tilde{\Theta}} = -\Gamma_1 z_1 W, \; \dot{\hat{\Theta}}_2 = \dot{\tilde{\Theta}}_2 = -\Gamma_2 z_2 W_2,$$

where $\Gamma_1, \Gamma_2 > 0$, and $z_1 = S$, $z_2 = x_4 - \alpha_2$,

$$\alpha_2 = -\rho_1 MS + \frac{1}{K_f} \Theta^T W - \frac{a}{K_f} Me_5$$

for some $\rho_1, \rho_2 > 0$, and

$$g_2(x) = px_3 x_5 + \beta x_2 x_5 + (q + a_4 - a)x_4 - \rho_1(K_f x_4 + \kappa Je_6)$$

$$\Theta_2^T W_2 = (\rho_1 + \frac{a}{K_f})\Theta^T W + \frac{1}{K_f}\Theta^T \dot{W} = (\rho_1 + \frac{a}{K_f})\Theta^T W + \frac{1}{K_f}\Theta' W'$$

with the parameter vector Θ' as well as the known function vector W' satisfying $\Theta^T \dot{W} = \Theta'^T W'$.

Proof:
Step 1. Choose a different stabilizing function α_2 as follows

$$\alpha_2 = -\rho_1 MS + \frac{1}{K_f} \hat{\Theta}^T W - \frac{a}{K_f} Me_5 \qquad (23)$$

where $\hat{\Theta}$ denotes the on-line parameter estimate. And, redefine the new error variables $z_1 = S$, $z_2 = x_4 - \alpha_2$.

Evaluate the time derivative of the Lyapunov function candidate

$$V_1 = \frac{1}{2} Mz_1^2 + \frac{1}{2\Gamma_1} \tilde{\Theta}^T \tilde{\Theta}, \qquad (24)$$

along the solution trajectories to obtain

$$\dot{V}_1 = -\rho_1 K_f Mz_1^2 + K_f z_1 z_2 + \tilde{\Theta}^T(\frac{1}{\Gamma_1}\dot{\tilde{\Theta}} + z_1 W) \qquad (25)$$

Devise the adaptation law as

$$\dot{\hat{\Theta}} = \dot{\tilde{\Theta}} = -\Gamma_1 z_1 W \qquad (26)$$

for some proper positive adaptation gain Γ_1, then (25) can be slightly simplified as:

$$\dot{V}_1 = -\rho_1 K_f M z_1^2 + K_f z_1 z_2 \qquad (27)$$

Step 2. The time derivative of z_2 is now expressed as

$$\dot{z}_2 = \dot{x}_4 - \dot{\alpha}_2 = -g_1(x) - \Theta_1^T W_1 + \sqrt{x_2} V \qquad (28)$$

where the function are as previously defined. Thus, we need to select a Lyapunov function candidate and design V to render its time derivative nonpositive. We want to apply the augmented Lyapunov function candidate as:

$$V_2 = V_1 + \frac{1}{2} z_2^2, \qquad (29)$$

whose time derivative is found to be

$$\dot{V}_2 = -\rho_1 K_f M z_2^2 + K_f z_1 z_2 + z_2[-g_1(x) - \Theta_1^T W_1 + \sqrt{x_2} V], \qquad (30)$$

The control law V should be able to cancel the indefinite term in (17). On the other hand, to deal with the unknown parameters Θ_2, we will try to employ the current estimates $\hat{\Theta}_1$, i.e.,

$$V = \frac{1}{\sqrt{x_2}}[g_1(x) + \hat{\Theta}_1 W_1 - K_f z_1 - \rho_2 z_2], \qquad (31)$$

From this resulting derivative

$$\dot{V}_2 = -\rho_1 K_f M z_1^2 + z_2 \hat{\Theta}_1 W_1 - \rho_2 z_2^2 \qquad (32)$$

in order to cancel the last term in (32), we modify the Lyapunov function as below:

$$V_3 = V_2 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\Theta}_1^T \tilde{\Theta}_1, \qquad (33)$$

and the time derivative of V_3 hence is

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 + \hat{\Theta}_1^T (z_2 W_2 + \frac{1}{\Gamma_2} \dot{\hat{\Theta}}_1) \qquad (34)$$

Now, the term with $\tilde{\Theta}_3$ can be eliminated completely with the update law

$$\dot{\hat{\Theta}}_2 = \dot{\tilde{\Theta}}_2 = -\Gamma_2 z_2 W_2 \qquad (35)$$

for some positive adaptation gain Γ_2, which thus yields

$$\dot{V}_3 = -\rho_1 K_f M z_1^2 - \rho_2 z_2^2 \qquad (36)$$

which guarantees boundedness of all parameter estimates $\tilde{\theta}$, $\hat{\theta}_1$ and z_1, z_2, and $z_1 \in L^2 \cap L^\infty$. To show boundedness of the rest of states, we can rearrange the dynamical equations from system (19) as shown below:

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2a_1 & 0 & 2a_2 \\ 0 & -2a_4 & 2a_3 \\ a_3 & a_2 & -(a_1 + a_4) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \dfrac{2x_4}{\sqrt{x_2}} V \\ 0 \\ p x_5 x_4 \end{bmatrix},$$

$$= AX + u$$

where A can be shown to be Hurwitz. After reviewing definitions of x_3 and V, repectively, we found that the first entry of u will be bounded because x_2 grows no slower than x_3 if x_3 does grow unbounded (due to the second equation of (19)). As a result, u is apparently bounded, and hence X will be bounded. This then proves the boundedness of all the states. We note that $\dot{z}_1$ is also bounded, and hence by Barbalat's lemma we can conclude $\dot{z}_1 \in L_\infty$ so that

$$\lim_{t \to \infty} z_1 \to 0, \text{ i.e., } x_r \to x_d \text{ as } t \to \infty. \quad \square$$

V. SIMULATION RESULTS

In our simulation, the parameters of the linear induction motor we use are according to Table 2. All the results will be demonstrated in the following sections.

The two-link pendulum start form its "natural stable hanging position" ($\theta_1 = \pi$, $\theta_2 = 0$) to its "unstable upright inverted position" ($\theta_1 = 0$, $\theta_2 = 0$) while the LIM is brought to the origin.
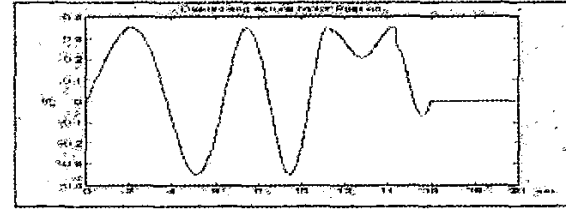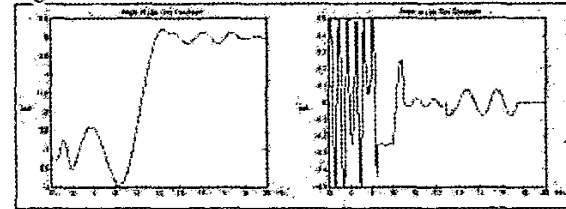

Fig. 2 Desired and Actural Motor Position x

Fig. 3 Angle of Link One θ_1 Fig 4. Angle of link Two θ_2

VI. CONCLUSION

In this paper, we have proposed a nonlinear passivity controller and an adaptive backstepping controller for double inverted pendulum mounted on a linear induction motor. The controller which not only swing up but also balance the two linked inverted pendulums on the motor. Stability analysis based on Lyapunov theory is performed to guarantee the controller design is stable. Finally, the simulation results confirm the effectiveness of our control design.

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Table 2: Specification and parameter of the motor

Specification		Parameters normal value
3 Phase (Y-connected)		$R_s = 13.2\,\Omega$
Rated Power	1HP	$R_r = 11.78\,\Omega$
Rated Air gap	0.125in	
Rated Current	5A	$L_s = 0.42H$
Rated Voltage	240V	$L_r = 0.42H$
Rated Poles	4	$L_m = 0.4H$
Pitch	46.5mm	$M = 4.775\,Kg$
Secondary length	82cm	$B = 53Kg/sec$
$m_0 = 4.775kg$		$l_1 = 0.39m$
$m_1 = 0.12kg$		

$m_2 = 0.02kg$	$l_2 = 0.395m$
$m_{c1} = 0.08kg$	$l_{c1} = 0.195$
$m_{c2} = 0.08kg$	$l_{c2} = 0.1975m$
	$I_i = m_c l_i^2 / 12$

Table 1: Nomenclature

m_o, m_1, m_2	Mass of LIM and joints
m_{c1}, m_{c2}	Mass of each pendulum
l_1, l_2	Length of each pendulum
I_1, I_2	Moment of inertia of each pendulum
l_{c1}, l_{c2}	Length of each pendulum from axis to centre of mass
$V_{qs}(V_{ds})$	q-(d-) Axis input stator voltage
$i_q(i_d)$	q-(d-) Axis input stator current
$R_s(R_r)$	Primary (secondary) resistance
$L_s(L_r)$	Primary (secondary) inductance
$\lambda_{qr}(\lambda_{dr})$	q-(d-) Axis rotor flux
v_r	Linear speed of the primary
p_r	Position of the primary
M_m	Primary mass
B	Viscous friction coefficient
F_e	Electromagnetic force
F_L'	Mechanical load force
L_m	Mutual inductance
P	Number of pole pairs
τ	Pole pitch
K_f	Force constant $(= 3PL_m \pi / 2\tau L_r)$

$$a_2 = \beta R_r / L_r, \ a_3 = L_m R_r / L_r, \ a_4 = R_r / L_r, D = L_s L_r - L_m^2,$$
$$p = P\pi / \tau, \ \beta = L_m / D, \ a_1 = (R_s L_r) / D + \beta L_m R_r / L_r$$

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \ C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix},$$

$$G(q) = \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix}, q = \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix}, F = \begin{bmatrix} F_e \\ 0 \\ 0 \end{bmatrix}$$

$M_{11} = m_0 + m_{c1} + m_1 + m_2 + m_{c2}$

$M_{22} = m_{c1}l_{c1}^2 + I_1 + (m_1 + m_2 + m_{c2})l_1^2 + 2(m_{c2}l_{c2} + m_2l_2)l_1\cos\theta_2 + m_{c2}l_{c2}^2 + I_2 + m_2l_2^2$

$M_{33} = m_{c2}l_{c2}^2 + I_2 + m_2l_2^2$

$M_{12} = M_{21} = -(m_{c1}l_{c1} + m_1l_1 + m_{c2}l_1 + m_2l_1)\cos\theta_1 - (m_{c2}l_{c2} + m_2l_2)\cos(\theta_1 + \theta_2)$

$M_{13} = M_{31} = -(m_{c2}l_{c2} + m_2l_2)\cos(\theta_1 + \theta_2)$

$M_{23} = M_{32} = (m_{c2}l_{c2} + m_2l_2)l_1\cos\theta_2 + m_{c2}l_{c2}^2 + I_2 + m_2l_2^2$

$C_{11} = C_{21} = C_{31} = C_{33} = 0$

$C_{12} = (m_1l_1 + m_2l_1 + m_{c2}l_1 + m_{c1}l_{c1})\dot{\theta_1}\sin\theta_1 + (m_2l_2 + m_{c2}l_{c2})(\dot{\theta_1} + \dot{\theta_2})\sin(\theta_1 + \theta_2)$

$C_{13} = (m_2l_2 + m_{c2}l_{c2})(\dot{\theta_1} + \dot{\theta_2})\sin(\theta_1 + \theta_2)$

$C_{22} = (m_2l_2 + m_{c2}l_{c2})l_1(\dot{\theta_1} - \dot{\theta_2})\sin\theta_2$

$C_{23} = -(m_2l_2 + m_{c2}l_{c2})l_1(\dot{\theta_1} + \dot{\theta_2})\sin\theta_2$

$C_{32} = (m_2l_2 + m_{c2}l_{c2})l_1\dot{\theta_1}\sin\theta_2$

$C_1 = 0$

$G_2 = g((m_{c2}l_{c2} + m_2l_2)\cos(\theta_1 + \theta_2) + (m_1l_1 + m_2l_1 + m_{c2}l_1 + m_{c1}l_{c1})\cos\theta_1)$

$G_3 = g(m_2l_{c2} + m_2l_2)\cos(\theta_1 + \theta_2)$