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An adaptive variable structure control for a class of nonlinear systems*

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Abstract: Motivated by the recent advance in adaptive output-feedback control for a class of nonlinear systems which can be transformed into the so-called output-feedback form [7, 10, 11], an adaptive variable structure controller is proposed in this paper to solve the nonlinear model reference adaptive control problem. It is shown that an asymptotic output tracking performance can be achieved for this class of nonlinear system even if some nonlinearity is not available or some unknown parameters are fast time-varying.

Keywords: Adaptive variable structure; output-feedback control; nonlinear system.

1. Introduction

In the researches of adaptive control for affine nonlinear systems, it is usually assumed that the unknown parameters enter linearly into some known nonlinear vector fields. Based on the differential geometric approach [4, 13], an adaptive version of state-feedback linearizing control for such types of nonlinear systems has been developed [1, 8, 9, 15] and an adaptive version of state-feedback input–output linearizing control is given in [14]. However, besides a number of constraints on the nonlinear systems, the above researches require the common assumption that the state measurement is available. Recently, in order to relax such assumptions, a more challenging problem has been proposed in the field of adaptive output-feedback control. Kanellakopollos et al. first derived observer-based [5] and indirect [6] adaptive output-feedback control for a class of nonlinear systems which can be globally transformed into a linear system with input nonlinearity under the assumption of output matching and sector-type conditions. Later, based on the adaptive observer–integrator backstepping approach, a more general class of nonlinear system which can be transformed into the so-called output-feedback canonical form has been studied in [7], where no output matching or sector-type condition is needed. Using the filter transformation technique and the backstepping concept, [10, 11] also propose output-feedback controls for the same class of nonlinear systems.

In this paper we develop a new approach, which is different from those in [7, 10, 11], to solve the adaptive output-feedback control problem for a class of nonlinear systems using a variable structure method. The proposed adaptive variable structure scheme is also different from those in traditional linear model reference adaptive control (MRAC) systems [2, 3] where variable structure design is applied to the adaptation of some traditional control parameters. Using this new adaptation scheme, we release the standard requirement of the upper bounds on some unknown parameters which are frequently observed in the field of robust linear

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MRAC. Appropriate update laws are given to compensate for the unknown upper bounds on some unknown parameters. For this class of nonlinear system with relative degree one, very mild assumptions are needed for the controller design and the asymptotic output tracking performance can be achieved even when the nonlinearity is not available or some unknown parameters are fast time-varying. Under suitable conditions for control parameters, the tracking performance of the output error will in general be better than conventional adaptive controllers for this class of nonlinear system.

The paper is organized as follows: in Section 2, a detailed problem formulation for this class of nonlinear systems is given. An MRAC-based error model is derived in Section 3 and then the adaptive variable structure controller is proposed. Section 4 gives some numerical simulations to demonstrate the control performance of this adaptive scheme. Finally, a conclusion is drawn in Section 5.

2. Problem formulation

We consider an affine nonlinear system of the following form

$$\dot{x} = f(x) + \sum_{i=1}^{m} \psi_i^* f_i(x) + g(x)u, \qquad y = h(x), \tag{1}$$

where $\psi_1^*, \ldots, \psi_m^*$ are some unknown parameters which may be time-varying, $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the control output; $f, g, f_i, i = 1, \ldots, m$ are smooth vector fields with $f(0) = 0$, $g(0) \neq 0$ and h is a smooth function with $h(0) = 0, \forall x \in \mathbb{R}^n$. The following is the most relevant condition on the nonlinear system described above throughout this paper.

- **Assumption I.** The nonlinear system described in (1) can be transformed into the following output-feedback form:

$$\dot{z} = Az + p_0(y) + \sum_{i=1}^{m} \psi_i^* p_i(y) + b\sigma(y)u \triangleq Az + P(y)\Psi^* + b\sigma(y)u, \qquad y = c^{\mathrm{T}} z, \tag{2}$$

where

$$A = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}, \quad c^{\mathrm{T}} = [1 \quad 0 \ldots \quad 0],$$

$$P(y) = [p_0(y), p_1(y), \ldots, p_m(y)] \in \mathbb{R}^{n \times (m+1)}, \qquad \Psi^* = [1, \psi_1^*, \ldots, \psi_m^*]^{\mathrm{T}} \in \mathbb{R}^{m+1}.$$

In particular, (c, A) is an observable pair, $a_j, b_j, j = 1, \ldots, n$ are unknown constants, $\sigma(y) \neq 0$, $\forall y \in \mathbb{R}$ is a known nonlinear function and $p_i(y) \in \mathbb{R}^n$, $i = 0, \ldots, m$ are some nonlinear vectors. Here, we assume that $P(y)$ is not available but bounded by some known function $Q(y)$ such that $|P(y)| \leq \kappa |Q(y)| + \kappa$ for some unknown positive constant κ. Furthermore, $\sigma(y)$ and $P(y)$ are uniformly bounded if y is uniformly bounded and $|\Psi^*| \leq \kappa$ for some $\kappa > 0$ if ψ_i^* is time-varying.

The following proposition gives a sufficient and necessary condition to ensure the existence of the transformation stated in Assumption I.

Proposition 2.1 (Kanellakopoulos et al. [7], Marino and Tomei [10, 11]). *The system* (1) *can be transformed into* (2) *via a global state space transformation* $z = T(x)$, $T(0) = 0$ *if and only if the following conditions are satisfied for all* $x \in \mathbb{R}^n$,

(1) $\operatorname{rank}\{\mathrm{d}h(x), \mathrm{d}L_f h(x), \ldots, \mathrm{d}L_f^{n-1} h(x)\} = n$,
(2) $[\mathrm{ad}_f^i r, \mathrm{ad}_f^j r] = 0,\ i, j = 0, 1, \ldots, n-1$,

(3) $(-1)^n \mathrm{ad}_f^n r = \sum_{j=1}^{n} [(-a_j) + p'_{0_j}(y)](-1)^{n-j} \mathrm{ad}_f^{n-j} r$, *where* $p_0(y) = [p_{01}(y), \ldots, p_{0_n}(y)]^T$, $p_{0_j}(y) = \int_0^y p'_{0_j}(s)\,\mathrm{d}s$.

(4) $[f_i, \mathrm{ad}_f^j r] = 0$, $i = 1, \ldots, m$, $j = 0, 1, \ldots, n-2$,

(5) $[g, \mathrm{ad}_f^j r] = 0$, $j = 0, 1, \ldots, n-2$,

(6) $g = \sigma(y) \sum_{j=1}^{n} b_j (-1)^{n-j} \mathrm{ad}_f^{n-j} r$,

(7) *the vector fields* $\mathrm{ad}_f^j r$, $j = 0, \ldots, n-1$ *are complete, where* r *is the vector field satisfying*

$$L_r L_f^j h = \begin{cases} 0 & \text{if } j = 0, 1, \ldots, n-2, \\ 1 & \text{if } j = n-1. \end{cases} \tag{3}$$

In order to suit the controller design purpose here, another condition on the system in (1) is needed to guarantee the feasibility of applying output feedback and variable structure control. The following are the investigations of the condition and the resulting effect.

- **Assumption II.** The polynomial $b_1 s^{n-1} + \cdots + b_{n-1}s + b_n$ is Hurwitz and $b_1 \neq 0$.

Proposition 2.2 (Marino and Tomei [11]). *If the nonlinear system* (1) *satisfies Assumptions* I and II, *then, the zero dynamics of* (1) *is exponentially stable and can be expressed, in suitable global coordinates, by a linear asymptotically stable system as*

$$\dot{\bar{z}} = \begin{bmatrix} -b_2/b_1 & 1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & \vdots \\ -b_{n-1}/b_1 & 0 & 0 & \cdots & 1 \\ -b_n/b_1 & 0 & 0 & \cdots & 0 \end{bmatrix} \bar{z}$$

Finally, to make the problem posed above tractable, one more condition is necessary.

- **Assumption III.** The sign of b_1 is assumed to be known and without loss of generality, we assume $b_1 > 0$.

Remark 2.3. When Assumptions I–III are satisfied, the nonlinear system (1) with relative degree one can be transformed, via $z = T(x)$, into (2) and the transfer function

$$G(s) = c^T(sI - A)^{-1}b = \frac{b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n} \tag{4}$$

is a minimum phase system with known relative degree and sign of high-frequency gain. However, the parameters a_j, b_j in $G(s)$ are assumed to be unknown before controller design. Furthermore, we allow the unknown parameters Ψ^* to be time-varying and the nonlinearity $P(y)$ to be not available, which is more general than in [7, 10, 11].

3. The adaptive variable structure controller

For the nonlinear system described in Section 2, the control objective is to design the input $u(t)$ through an adaptive controller such that the output $y(t)$ of the nonlinear system tracks the output $y_m(t)$ of a linear time-invariant reference model described by $y_m(t) = M(s)[r_m](t)$, where $M(s)$ is a stable system with relative degree one and $r_m(t)$ is a uniformly bounded reference input.

The basic strategy used to construct the error model between y and y_m is the traditional model reference adaptive control concept [12]. However, instead of the conventional model reference adaptive control, a new adaptive variable structure control will be given in order to get better robustness and tracking performance. First, from the point of view of frequency-domain representation, (2) can be described as

$$y = G(s)[\sigma(y)u] + \sum_{j=1}^{n} G_j(s)[(P(y)\Psi^*)_j],$$

where $G(s)$ is defined as in (4),

$$G_j(s) = \frac{s^{n-j}}{s^n + a_1 s^{n-1} + \dots + a_n}$$

and $(P(y)\Psi^*)_j$ denotes the jth element of $P(y)\Psi^*$, which can be rewritten as

$$y = G(s)\left[\sigma(y)u + \sum_{j=1}^{n} \frac{s^{n-j}}{b_1 s^{n-1} + \dots + b_n}(P(y)\Psi^*)_j\right]. \tag{5}$$

Now define

$$v = \sigma(y)u + \sum_{j=1}^{n} \frac{s^{n-j}}{b_1 s^{n-1} + \dots + b_n}(P(y)\Psi^*)_j \triangleq \sigma(y)u + \bar{u}$$

then, from the traditional model reference adaptive control strategy [12], it can easily be shown that there exists $\Theta^* = [\theta_1^*, \dots, \theta_{2n}^*]^{\mathrm{T}} \in \mathbb{R}^{2n}$ and $\theta_{2n}^* > 0$ such that if

$$v = [\theta_1^*, \dots, \theta_{2n}^*]\begin{bmatrix} \dfrac{a(s)}{n(s)}[v] \\ \dfrac{a(s)}{n(s)}[y] \\ y \\ r_m \end{bmatrix}, \tag{6}$$

where $a(s) = [1, s, \dots, s^{n-2}]^{\mathrm{T}}$, $n(s)$ is a monic Hurwitz polynomial of degree $n-1$, then the closed-loop transfer function from r_m to y equals $M(s)$. From (6), one can readily find that u in fact satisfies

$$\sigma(y)u = [\theta_1^*, \dots, \theta_{2n}^*]\begin{bmatrix} \dfrac{a(s)}{n(s)}[\sigma(y)u] \\ \dfrac{a(s)}{n(s)}[y] \\ y \\ r_m \end{bmatrix} + [\theta_1^*, \dots, \theta_{n-1}^*]\frac{a(s)}{n(s)}[\bar{u}] - \bar{u}$$

$$\triangleq \Theta^{*\mathrm{T}} w + \sum_{j=1}^{n} \Delta_j(s)(P(y)\Psi^*)_j, \tag{7}$$

where

$$\Delta_j(s) = \left(\frac{\theta_1^* + \dots + \theta_{n-1}^* s^{n-2}}{n(s)} - 1\right)\frac{s^{n-j}}{b_1 s^{n-1} + \dots + b_n}$$

is proper stable or strictly proper stable. Note that (7) is usually referred to as the matching condition for the traditional model reference adaptive control scheme. But since a_j, b_j, Ψ^* are not known, Θ^* and $\Delta_j(s)$ will not be available in advance, and hence (7) cannot be used to design the controller. However, (7) can actually be used to describe the output function y from the input–output operator point of view as

$$y = M(s)\left[\theta_{2n}^{*-1}\left(\sigma(y)u - \Theta^{*\mathrm{T}}w - \sum_{j=1}^{n} \Delta_j(s)(P(y)\Psi^*)_j\right) + r_m\right].$$

If we define the tracking error e_0 as $y - y_m$, then the error model due to the unknown parameters can readily be found as follows:

$$e_0 = M(s)\theta_{2n}^{*-1}\left[\sigma(y)u - \Theta^{*\mathrm{T}}w - \sum_{j=1}^{n}\Delta_j(s)(P(y)\Psi^*)_j\right]. \tag{8}$$

By Assumption II, the reference model $M(s)$ can be chosen to be strictly positive real (SPR), and for the following state-space realization of (8),

$$\dot{e}_0 = A_m e + b_m\theta_{2n}^{*-1}\left(\sigma(y)u - \Theta^{*\mathrm{T}}w - \sum_{j=1}^{n}\Delta_j(s)(P(y)\Psi^*)_j\right), \quad e_0 = c_m^{\mathrm{T}}e \tag{9}$$

the triplet (A_m, b_m, c_m) will satisfy

$$P_m A_m + A_m^{\mathrm{T}}P_m = -2Q_m; \qquad P_m b_m = c_m; \qquad c_m^{\mathrm{T}}(sI - A_m)^{-1}b_m = M(s) \tag{10}$$

for some $P_m = P_m^{\mathrm{T}} > 0$, $Q_m > 0$. Let $|\cdot|$ denote the absolute value of any scalar or the norm of any vector or matrix. The adaptive variable structure controller is now designed as

$$u(t) = \frac{-\operatorname{sgn}(e_0)}{\sigma(y)}(\beta_1(t)|w(t)| + \beta_2(t)m(t) + \beta_3(t)),$$

$$\operatorname{sgn}(e_0) = \begin{cases} 1 & \text{if } e_0 > 0 \\ 0 & \text{if } e_0 = 0 \\ -1 & \text{if } e_0 < 0, \end{cases} \tag{11}$$

with $m(t)$ being defined as the bounding function

$$m(t) = \sup_{t \geq \tau}|Q(y(\tau))| \tag{12}$$

and $\beta_1(t)$, $\beta_2(t)$, $\beta_3(t)$ being the control parameters to be updated. Since $\Delta_j(s)$ is a stable proper or strictly proper transfer function, it is easy to show that there exist positive constants β_2^* and β_3^* (depending on $\Psi^*, \theta_1^*, \ldots, \theta_{n-1}^*, b_1, \ldots, b_n, \kappa$) such that

$$\left|\sum_{j=1}^{n}\Delta_j(s)(P(y)\Psi^*)_j\right| \leq \beta_2^* m(t) + \beta_3^*. \tag{13}$$

So, if we define $|\Theta^*| = \beta_1^*$ and choose a Lyapunov function

$$V = \frac{1}{2}e^{\mathrm{T}}P_m e + \sum_{i=1}^{3}\frac{1}{2\gamma_i\theta_{2n}^*}(\beta_i - \beta_i^*)^2,$$

with $P_m^{\mathrm{T}} = P_m > 0$ satisfying (10) and $\gamma_1, \gamma_2, \gamma_3 > 0$, then,

$$\begin{aligned}\dot{V} &= -e^{\mathrm{T}}Q_m e + \frac{e_0}{\theta_{2n}^*}\left(\sigma(y)u - \Theta^{*\mathrm{T}}w - \sum_{j=1}^{n}\Delta_j(s)(P(y)\Psi^*)_j\right) + \sum_{i=1}^{3}\frac{1}{\gamma_i\theta_{2n}^*}(\beta_i - \beta_i^*)\dot{\beta}_i \\ &\leq -e^{\mathrm{T}}Q_m e - \frac{|e_0|}{\theta_{2n}^*}\left((\beta_i - \beta_1^*)|w| + (\beta_2 - \beta_2^*)m + (\beta_3 - \beta_3^*)\right) + \sum_{i=1}^{3}\frac{1}{\gamma_i\theta_{2n}^*}(\beta_i - \beta_i^*)\dot{\beta}_i.\end{aligned}$$

Let the adaptive law be designed as

$$\dot{\beta}_1 = \gamma_1|e_0||w|, \qquad \dot{\beta}_2 = \gamma_2|e_0|m, \qquad \dot{\beta}_3 = \gamma_3|e_0|, \tag{14}$$

with $\beta_i(0) > 0$, then $\dot{V}$ will now satisfy $\dot{V} \leq -q_m|e|^2$ for some constant $q_m > 0$.

Now we are ready to state the main result about the proposed controller.

Theorem 3.1. *Consider the nonlinear system* (1) *satisfying Assumptions* I–III. *If the controller is designed as in* (11)–(12) *and the parameter update law is chosen as in* (14), *then the tracking error* e_0 *will converge to zero asymptotically and all signals inside the closed-loop system remain uniformly bounded.*

Proof. Since $\dot{V} \leq -q_m|e|^2$, we have $e \in L_2 \cap L_\infty$ and $\beta_1, \beta_2, \beta_3, y, P(y), m \in L_\infty$. Let $w_1 = \frac{a(s)}{n(s)}[\sigma(y)u]$, from (5) w_1 in fact satisfies

$$w_1 = \frac{a(s)}{n(s)}\left[G^{-1}(s)[y] - \sum_{j=1}^{n} \frac{s^{n-j}}{b_1 s^{n-1} + \cdots + b_n}(P(y)\Psi^*)_j\right].$$

Since y is uniformly bounded, this implies w_1 is uniformly bounded and hence u and all signals inside the closed-loop system are uniformly bounded. Finally, we conclude that $\dot{e} \in L_\infty$ by (9) and e will converge to zero asymptotically by Barbalat's lemma [12]. This completes our proof. □

Corollary 3.2. *Consider the system setup in Theorem* 3.1. *If* $\beta_i(0) \geq \beta_i^*$, $i = 1, 2, 3$, *then the output error will converge to zero in finite time with all signals inside the closed-loop system remaining uniformly bounded.*

Proof. Consider the Lyapunov function $V(e) = \frac{1}{2}e^{\mathrm{T}}P_m e$, where P_m satisfies (10). Then under the adaptive variable structure controller (11), (12) and (14), we have

$$\dot{V} = -e^{\mathrm{T}}Q_m e - \frac{|e_0|}{\theta_{2n}^*}((\beta_1 - \beta_1^*)|w| + (\beta_2 - \beta_2^*)m + (\beta_3 - \beta_3^*)) \leq -e^{\mathrm{T}}Q_m e,$$

since $\beta_i(t) \geq \beta_i(0)$, $\forall t \geq 0$. This implies e is uniformly bounded and e approaches zero at least exponentially fast. Furthermore, by the fact that

$$e_0\dot{e}_0 = e_0 c_m^{\mathrm{T}}\dot{e} \leq |e_0|\{|c_m^{\mathrm{T}}A_m||e| - ((\beta_1 - \beta_1^*)|w| + (\beta_2 - \beta_2^*)m + (\beta_3 - \beta_3^*))\}$$

there exists a finite time T_f such that $e_0\dot{e}_0 \leq -\kappa|e_0| < 0$ for all $t \geq T_f$ and $|e_0| \neq 0$, and hence, the sliding surface $e_0 = 0$ is guaranteed to be reached in finite time. Consequently, β_i and all signals inside the closed-loop system will now be uniformly bounded. □

Remark 3.3. In this paper, the bounds on β_1^*, β_2^* and β_3^* are not assumed to be available and suitable integral update laws on β_1, β_2 and β_3 are given so that only asymptotical tracking performance is obtained. However, in order to get a good tracking performance, it is reasonable that the initial conditions $\beta_1(0)$, $\beta_2(0)$ and $\beta_3(0)$ are set large enough in the beginning according to Corollary 3.2. Simulation results really show that the choice of $\beta_i(0)$ affects the transient behavior of the tracking performance.

Remark 3.4. Theoretically, the adaptive variable structure scheme will stabilize the closed-loop system no matter what $\beta_i(0)$'s are. However, as discussed in Remark 3.3, the low magnitude of the control parameters will to some extent affect the tracking performance. The adaptation gains γ_1, γ_2 and γ_3 in (14) will now play an important role in improving the transient behavior of the tracking performance. This is because large adaptation gains will provide high adaptation speed and, hence, increase the control parameters to a suitable level of magnitude so as to achieve the aforementioned feature as quickly as possible.

Remark 3.5. It is well known that chattering behavior will be observed in variable structure design. The remedy for this kind of behavior is the introduction of the boundary layer into the controller design. In our adaptive variable structure scheme, it is easy to modify $u(t)$ in (11) as

$$u(t) = \frac{-\pi(e_0)}{\sigma(y)}(\beta_1(t)|w(t)| + \beta_2(t)m(t) + \beta_3(t))$$

$$\pi(e_0) = \begin{cases} \mathrm{sgn}(e_0) & \text{if } |e_0| > \varepsilon \\ e_0/\varepsilon & \text{if } |e_0| \leq \varepsilon \end{cases}$$

for some small $\varepsilon > 0$. Note that $\pi(e_0)$ is now a continuous function.

4. Simulations

We consider the same nonlinear system in [11]

$$\dot{x}_1 = x_2 + \psi^*(e^y - 1), \qquad \dot{x}_2 = x_3, \qquad \dot{x}_3 = u, \qquad y = x_1 + 2x_2 + x_3, \tag{15}$$

which is not state-feedback linearizable and does not satisfy the global Lipschitz condition. However, this example is a relative degree one system which satisfies Assumptions I and II. Indeed, the linear transformation $z_1 = x_1 + 2x_2 + x_3$, $z_2 = x_2 + 2x_3$, $z_3 = x_3$ brings the system (15) into the following output-feedback form

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \psi^*\begin{bmatrix}e^y - 1\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}1\\ 2\\ 1\end{bmatrix}u, \qquad y = [1 \;\; 0 \;\; 0]\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix}.$$

Let the initial condition of (15) be $y(0) = 1$. The adaptive controller is designed as follows:

- **Controller 1** (adaptive variable structure controller)

$$u = -\operatorname{sgn}(e_0)(\beta_1|w| + \beta_2 m + \beta_3)$$

- **Controller 2** (adaptive variable structure controller with boundary layer $\varepsilon = 0.01$)

$$u = \pi(e_0)(\beta_1|w| + \beta_2 m + \beta_3)$$

where

$$m = \sup_{t \geq \tau} e^{y(\tau)}, \qquad w = \left[\frac{1}{s+10}[u] \quad \frac{1}{s+10}[y] \quad y \quad r_m\right]^{\mathrm{T}} \qquad \text{and} \qquad \beta_i,\ i = 1, 2, 3$$

are the control parameters updated by (14) with $\gamma_1 = \gamma_2 = \gamma_3 = 1$. The reference model and reference input are chosen as

$$M(s) = \frac{5}{s+5}, \qquad r_m(t) = \begin{cases} 1 & \text{if } t < 2\\ -1 & \text{if } 2 \leq t < 4\\ 1 & \text{if } 4 \leq t < 6\\ -1 & \text{if } 6 \leq t < 8.\end{cases}$$

Four simulation cases are now described.

(i) Figure 1(a)–(d) are the simulations for $\psi^* = 5$ (time-invariant case) with Controller 1. The initial conditions for update law (14) are set to be $\beta_1(0) = 3$, $\beta_2(0) = 2$, $\beta_3(0) = 0.1$. Nice tracking performance between y and y_m is achieved in Fig. 1(a). However, chattering behavior in control input is also observed in Fig. 1(c).

(ii) Figure 2(a)–(d) are the repeated simulations as in case (i) except that Controller 1 is replaced by Controller 2. The control performances are hardly affected by the redesigned scheme but obviously the chattering is drastically improved.

(iii) Figure 3(a)–(d) are repeated simulations as in case (ii) except that $\psi^* = 5\sin(10t)$ (time-varying case).

(iv) Finally, in order to study the effect of initial choice of $\beta_i(0)$, we repeat the simulation case (iii) but reduce $\beta_i(0)$ to be $\beta_1(0) = 1.5$, $\beta_2(0) = 1$, $\beta_3(0) = 0.1$. As shown in Fig. 4(a), the tracking performances between y and y_m are not as good as those in simulation case (iii). However, the large output error results in rapid increase in the magnitudes of control parameters β_i, and hence, after a transient period, satisfactory tracking performance is again ensured.

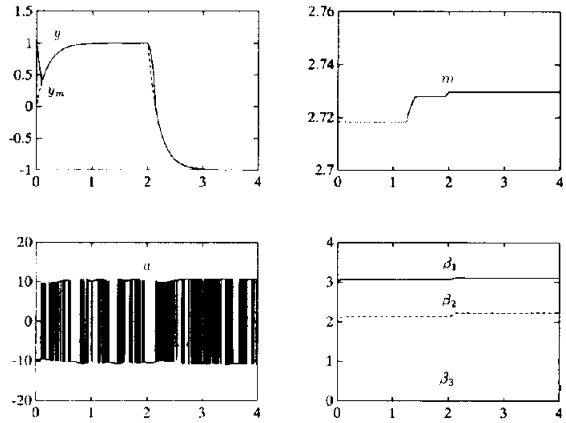

Fig. 1. Simulations for $\psi^* = 5$ with Controller 1: (a) plant output versus model output; (b) bounding function; (c) control input; (d) control parameters.

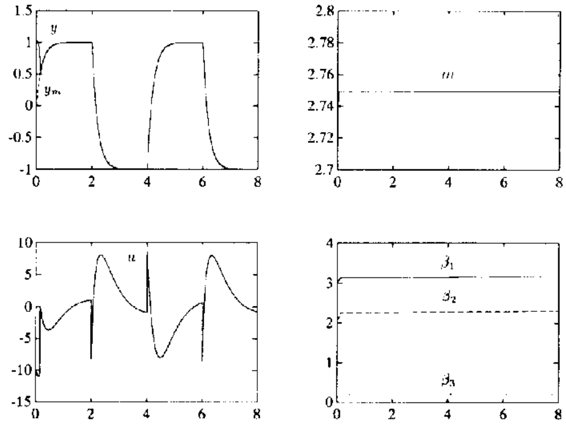

Fig. 2. Simulations for $\psi^* = 5$ with Controller 2: (a) plant output versus model output; (b) bounding function; (c) control input; (d) control parameters.

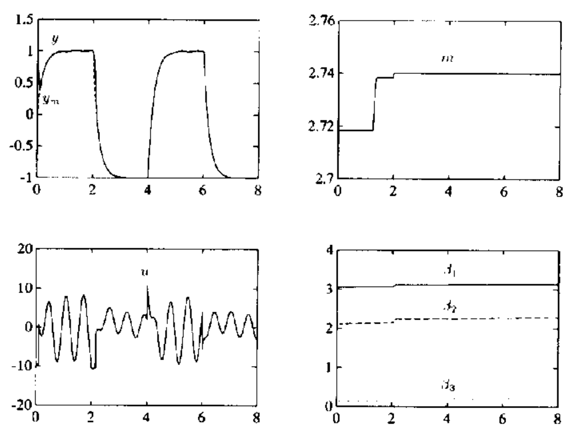

Fig. 3. Simulations for $\psi^* = 5\sin(10t)$ with Controller 2: (a) plant output versus model output; (b) bounding function; (c) control input; (d) control parameters.

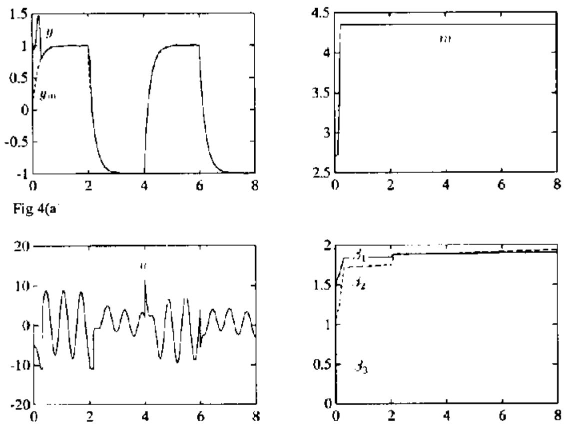

Fig. 4. Simulations for the case as in Fig. 3 but with reduced β_i: (a) plant output versus model output; (b) bounding function; (c) control input; (d) control parameters.

5. Conclusion

Under some suitable coordinate free geometric condition, a affine nonlinear system can be transformed, via a state transformation, into a so-called output-feedback form which is a linear system driven by some nonlinear functions. An adaptive variable structure controller is then proposed in this paper to solve the nonlinear model reference adaptive control problem. It is shown that the asymptotical output tracking performance can be achieved for this class of nonlinear systems with relative degree one even when some nonlinearity is not available or some unknown parameters are fast time-varying. Under suitable conditions for control parameters, the tracking performance of the output error will in general be better than conventional adaptive controllers for this class of nonlinear system.

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