Multi-Rate Multi-Carrier CDMA with Multiuser Detection for Wireless Multimedia Communications

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***Abstract*—We study two multi-rate access methods of multi-carrier CDMA systems, based on transforming the concepts of multi-code (MC) and variable-spreading-length (VSL) to frequency domain. A general and uniform system model accommodating both access methods is constructed, providing a framework of programmability for system integration. We thereof develop multiuser detections (MUD) applicable to both access methods. Chip-based filtering in frequency domain replaces the conventional match-filter bank and yield novel detection behaviors. The signal properties and detection performance in multi-rate traffic are analyzed and demonstrated with numerical simulations, which manifests the multi-rate approaches and the superiority of MC access over VSL access in uplink MC-CDMA. The compared study shows the domination of spreading length over the interference pattern in MUD performance.**

Keywords- MC-CDMA; multi-rate CDMA; OFDM; multiuser detection

I. INTRODUCTION

With the evolution to 3G systems, multimedia applications play increasingly important roles in wireless communications. Various demand in daily life introduces integrated services, including voice, image, and video etc., and each exhibits its own feature. One of the significances is the diverse data rates. In future wideband high-speed communication environment with highly hostile radio channels, realizing effective multi-rate physical-layer transmission is definitely a tough challenge. Recent years, some techniques based on the combining of Code-Division-Multiple-Access (CDMA) and Orthogonal-Frequency-Division-Multiplexing (OFDM) are proposed [1] [2], and one alternative, Multi-Carrier CDMA (MC-CDMA), creates a promising platform to overcome this challenge, which multiplexes source data stream and performs frequency-domain-spreading over orthogonal sub-carriers.

Issues about multi-rate transmission have drawn lots of studies in conventional single-carrier CDMA systems [3], such as the MC and VSL access methods; however, it is a new topic and rarely explored in multi-carrier cases. We adapt and transform the concepts of MC and VSL to realize multi-rate access in MC-CDMA systems. A general and systematic model of multi-rate MC-CDMA systems is built in this paper, which accommodates the two access schemes and thereof multiuser detections are easily to be applied, even though the observation vector is extracted form chip-based filtering in frequency domain. The optimum MUD of maximum-likelihood (ML) criterion is derived to analyze the fundamental properties and provide benchmark performance. Two detectors of linear complexity based on decorrelating and minimum-mean-squared error (MMSE) criterions are developed for practical utilizations. The previous researches showed the severe degradation of SUD in uplink channels due to the distortion of code orthogonality in frequency-selective fading [4]. These MUDs are demonstrated effective in uplink applications and applied in identical structure for both access methods with multi-rate traffics. Performance of MUDs in both access methods are analytically investigated and verified with simulations. The structural dimension resulted from chip-filtering yields novel properties out of the traditional cases, such as the asymptotic behaviors of the Decorrelating and LMMSE MUDs. In addition, compared studies of the two access methods show distinct results from DS-CDMA systems [5] and exhibit the advantages of MC access. Suggestions to system design based on the properties of two access methods with MUDs are summarized in the concluding section for various applications.

II. MULTI-RATE MC-CDMA

Assume there is a basic data rate $1/T_s$ in the system and all data rates fit an integer multiple of the basic rate. A user whose data rate is m times the basic is said a user with rate m in this paper.

A. Multi-Code access

The data stream of a user with rate m is multiplexed into m different streams of basic rate and each is treated as an effective user and assigned individual spreading codes. Each stream is then serial-to-parallel (S/P) converted into P parallel outputs. Symbols on each output is copied into F branches, where F is the constant spreading factor in MC access and the symbols on each branch are multiplied by the corresponding bit of the assigned spreading codes. There are PF parallel outputs of each effective user after such frequency-domain spreading. The parallel signals from all effective users are combined correspondingly and being transmitted by modulating PF orthogonal sub-carriers respectively. The baseband transmitted signal of the kth user with rate m is:

$$x_k^{(m)}(t) = \sum_{j=1}^{m}\sum_{p=1}^{P} A_j b_{jp} \sum_{f=1}^{F} c_{jf} e^{j2\pi((p-1)F+f-1)\Delta f t}, 0 \le t \le PT_s, \quad (1)$$

where $b_{jp} \in \{\pm 1\}$ is the pth symbol, A_k is the transmitted amplitude of the jth effective user, and $\Delta f = 1/(PT_s)$. $c_{jf} \in \{\pm 1\}$ denotes the fth bit of the spreading codes assigned to the jth effective user.

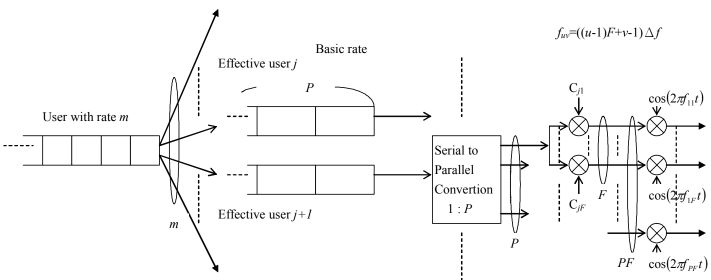

Fig. 1. Illustration of Multi-Code Accessed Multi-Rate MC-CDMA

B. Variable-Spreading-Length access

The data stream of a user with rate m is directly serial-to-parallel converted into Pm parallel outputs. Symbols at each output are copied into F/m branches and then respectively being multiplied by the corresponding bit of spreading codes whose length is F/m. F/m should be an integer in system design. Regardless any date rate, there are totally PF parallel signal outputs after such frequency domain spreading, and being transmitted simultaneously by modulating PF orthogonal sub-carriers. The bandwidth on each sub-carrier and the overall spectrum profile are the same as in MC access. The baseband transmitted signal of the kth user with rate m is

$$x_k^{(m)}(t) = \sum_{p=1}^{mP} A_k^{(m)} b_{kp}^{(m)} \sum_{f}^{F/m} c_{kf}^{(m)} e^{j2\pi[(p-1)\frac{F}{m}+f-1]\Delta f t}, \; 0 \le t \le PT_s, (2)$$

$b_{kp}^{(m)} \in \{\pm 1\}$ is the pth symbol, $c_{kf}^{(m)} \in \{\pm 1\}$ is the fth bit of the assigned spreading codes, and $A_k^{(m)}$ is the transmitted amplitude of the kth user with rate m respectively. Also the sub-carrier spacing is $\Delta f = 1/(PT_s)$.

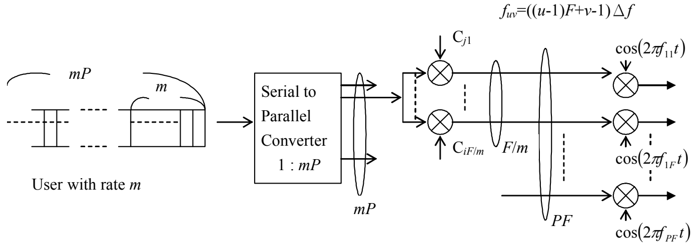

Fig. 2 Illustration of Variable-Spreading-Length Accessed Multi-Rate MC-CDMA

III. A General System Model

We consider the wireless channels frequency-selective fading with respect to the entire bandwidth. However, we can select enough number of sub-carriers such that the bandwidth of each sub-channel is smaller than the coherence bandwidth to yield flat fading. Furthermore, we assume sub-channels are mutually uncorrelated and slow Rayleigh fading [6].

In VSL systems, the signal formulation is intractable due to various symbol durations of diverse date rate. To make it compact and systematically converged with MC access, we regard a user with rate m in VSL access as m effective users, where the decomposition in frequency domain, corresponding spreading codes, and spectrum allocation are illustrated in Fig. 3. Assuming K_m denotes the number of users with rate m, there are therefore $K = \sum_{m=1}^{M} mK_m$ effective users in a system with maximum rate M, the same as in MC access. Consider the 'one-shot' transmission duration, that is, the duration transmitting mP symbols of a user with rate m, the received signal in a multi-rate MC-CDMA system, regardless of MC or VSL access, is:

$$r(t) = \sum_{k=1}^{K}\sum_{p=1}^{P} A_k b_{kp} \sum_{f=1}^{F} \alpha_{kpf} c_{kf} e^{j2\pi[(p-1)F+f-1]\Delta f t} + n(t), \quad (3)$$

where α_{kpf} is the channel response of the (p,f)th sub-channel of the kth effective user with normalized expectation $E[|\alpha_{kpf}|^2] = 1$, and $n(t)$ is AWGN with 2-sided PSD $N_0/2$. One can view $\{c_{k1}, c_{k2}, \cdots, c_{kF}\}$ as the virtual spreading codes of the kth effective user. Note that the guard time insertion is omitted in our one-shot model without affecting the generality for developing multi-rate access. Cyclic-extended guard samples can be properly added in practice to enhance ISI immunity.

After down conversion, the signals extracted from the corresponding F sub-carriers for detecting the pth symbol can be collected as a vector:

$$\mathbf{r} = \begin{bmatrix} r_{p1} \\ r_{p2} \\ \vdots \\ r_{pF} \end{bmatrix}_{(F\times 1)} = \begin{bmatrix} \sum_{k=1}^{K} \alpha_{kp1} A_k b_{kp} c_{k1} \\ \sum_{k=1}^{K} \alpha_{kp2} A_k b_{kp} c_{k2} \\ \vdots \\ \sum_{k=1}^{K} \alpha_{kpF} A_k b_{kp} c_{kF} \end{bmatrix} + \begin{bmatrix} n_{p1} \\ n_{p2} \\ \vdots \\ n_{pF} \end{bmatrix} = \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b}_p + \mathbf{n}_p, \quad (4)$$

where $\bar{\mathbf{I}} = [\mathbf{I} \cdots \mathbf{I}]^T$, $\mathbf{I}$ denotes a $K \times K$ identity matrix ,and

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_{1p1} & \cdots & \alpha_{Kp1} & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \alpha_{1p2} & \cdots \alpha_{Kp2} & 0 & \cdots & 0 \\ \vdots & & & & & \ddots & & \vdots \\ 0 & & \cdots & 0 & \cdots & 0 & \alpha_{1pF} \cdots & \alpha_{KpF} \end{bmatrix}_{(F\times KF)}.$$

The matrix $\mathbf{C} = diag(A_1 c_{11} \cdots A_K c_{K1} \cdots\cdots A_1 c_{1F} \cdots A_K c_{KF})$ is a $KF \times KF$ diagonal signature matrix of effective users. Form (4), it exhibits an equivalent multi-input-multi-output (MIMO) system structure, and systems of the two access methods are modeled in the same formulation, while the pattern of $\mathbf{C}$ distinguishes them. In VSL access, $\mathbf{C}$ contains zeros which implies symbols from users of higher rate involve less interference.

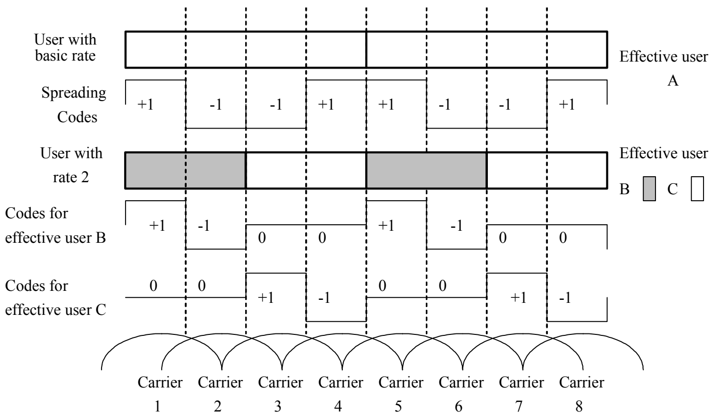

Fig. 3 Illustration of the concept of effective users, corresponding spreading codes, and spectrum distribution in VSL access. This is an example of one basic rate user with spreading codes (+1-1-1+1) and two effective users with spreading codes (+1-1 0 0) and (0 0 +1-1) respectively from a user of rate 2.

IV. MULTIUSER DETECTIONS

Due to the orthogonality among sub-carriers, $\{r_{p1}, r_{p2}, \cdots r_{pF}\}$ are sufficient to detect the pth symbols of all effective users. Although partial observations may be sufficient in VSL access for some users of shorter effective spreading length, we use the whole vector for joint MUD for all users. This will get better performance for VSL access without additional processing delay because of parallel transceiving, unlike in multi-rate DS-CDMA [3], and it unifies the MUD procedures for both access methods. To simplify notations, the sub-script p is omitted. The following developed detections are applicable to both MC access and VSL access in the bases of effective users.

A. Maximum-Likelihood Detections

Given channel coefficients, the ML decision of the transmitted symbols **b** of effective users is made by

$$\hat{\mathbf{b}} = \arg\max_{\mathbf{b}} \{\Pr(\mathbf{r}|\mathbf{b})\} = \arg\max_{\mathbf{b}} \left\{ \prod_{j=1}^{F} \Pr(r_j|\mathbf{b}) \right\}$$
$$= \arg\max_{\mathbf{b}} \left\{ \sum_{j=1}^{F} \log \Pr(r_j|\mathbf{b}) \right\} = \arg\min_{\mathbf{b}} \left\{ \sum_{j=1}^{F} \left| r_j - \sum_{k=1}^{K} \alpha_{kj} A_k b_k c_{kj} \right|^2 \right\} \quad (5)$$

By matrix notations,

$$\hat{\mathbf{b}} = \arg\min_{\mathbf{b}} \left\{ (\mathbf{r} - \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b})^H (\mathbf{r} - \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b}) \right\}$$
$$= \arg\max_{\mathbf{b}} \left\{ \mathrm{Re}\{\mathbf{b}^H \bar{\mathbf{I}}^H \mathbf{C}^H \boldsymbol{\alpha}^H \mathbf{r}\} - \tfrac{1}{2} |\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b}|^2 \right\} \quad (6)$$

The terms $\{r_j A_k \alpha_j c_{kj} \,|\, j = 1 \sim F,\ k = 1 \sim K\}$ compose the sufficient statistics to detect the pth symbols of all effective users. Unlike DS-CDMA whose spreading is operated in time domain, channel-selective fading makes the conventional matched-filter bank based on signature waveforms no longer produce sufficient statistics, and chip-matched filtering is necessary instead. Thus, chip-based instead of signature-sequence-based processing on the vector **r** is performed. Although the ML detector achieves minimum error probability, the computational complexity increases exponentially with the number of total effective users. However, it is regarded as a benchmark for other MUDs.

B. Decorrelating Detection

The symbol of the desired user can be extracted by eliminating the components from other users. However, the conventional orthogonality-restoring-combining (ORC) for MC-CDMA [4] is not useful in uplink channels. Instead, given the channel coefficients and effective spreading codes of all effective users, let $\mathbf{M} = \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}$, a $F \times K$ matrix, denote the equivalent channel for **b**. Assume **M** is of full rank,

1. If $F \geq K$: a system of over-determined linear equations, and the decision variable of **b** is
$$\tilde{\mathbf{b}} = \mathbf{M}^{+}\mathbf{r} = \left(\mathbf{M}^H \mathbf{M}\right)^{-1} \mathbf{M}^H \mathbf{r} \quad (7)$$
where $^{+}$ denotes the pseudo-inverse operator.
2. If $F < K$: a system of under-determined linear equations,
$$\tilde{\mathbf{b}} = \mathbf{M}^{+}\mathbf{r} = \mathbf{M}^H \left(\mathbf{M}\mathbf{M}^H\right)^{-1} \mathbf{r} \quad (8)$$
which yields the minimum-norm solution, i.e. a least-square estimate [7] of **b**.

In either case, the decision on **b** is made by $\hat{\mathbf{b}} = sign(\mathrm{Re}\{\tilde{\mathbf{b}}\})$.

The case $F \geq K$, which is usually satisfied in practices, is preferable since it provides correct recovery of **b** without background noise. Whatever, there may be noise enhancement problem if $|\mathbf{M}^{+}|$ is relatively large.

C. LMMSE Detection

Denote the linear filter for b_k as a vector $\mathbf{W}_k$. To minimize the mean-squared-error $E[|\mathbf{W}_k^H \mathbf{r} - b_k|^2]$, the principle of orthogonailty [7] yields the MMSE solution:

$$\mathbf{W}_k = \left(E\left[\mathbf{r}\mathbf{r}^H\right]\right)^{-1} E\left[\mathbf{r}b_k^*\right] = \left(\boldsymbol{\alpha}\mathbf{C}\boldsymbol{\Gamma}\mathbf{C}^H\boldsymbol{\alpha}^H + N_0\mathbf{I}\right)^{-1} \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{1}_k, \quad (9)$$

where $\boldsymbol{\Gamma} \equiv \begin{bmatrix} \mathbf{I} & \cdots & \mathbf{I} \\ \vdots & \ddots & \vdots \\ \mathbf{I} & \cdots & \mathbf{I} \end{bmatrix}$ and $\mathbf{1}_k$ is a $K \times 1$ vector whose components are all zeros except that the kth is one. Symbol decision is made by $\hat{b}_k = sign\{\mathrm{Re}[\mathbf{W}_k^H \mathbf{r}]\}$ for the kth effective user. The linear MMSE detector jointly deal with multiple-access interference (MAI), multiple channel effects, and background noise. Note that at high signal-to-noise power ratio (SNR),

$$\mathbf{W}_k \approx \left(\boldsymbol{\alpha}\mathbf{C}\boldsymbol{\Gamma}\mathbf{C}^H\boldsymbol{\alpha}^H\right)^{-1} \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{1}_k = \left(\mathbf{M}\mathbf{M}^H\right)^{-1} \mathbf{M}\mathbf{1}_k \quad (10)$$

which is identical to the decorrelating detector only if $K \geq F$. That is, whether the Decorrelating detection converges to LMMSE detection asymptotically is case-dependent in MC-CDMA systems, which is attributed to the chip-based processing. However, $K > F$ is not practical in cases of using orthogonal spreading codes or some codes with good cross-correlation for multiple access, and an error floor exits even when noise diminishes.

V. PERFORMANCE ANALYSIS OF MUDS

Since the system model is similar to the conventional DS-CDMA, the performance of MUD could be analyzed basically by applying the well-developed methodologies. For ML MUD, we adopt an alternative expression of Q function [8] to reveal the system property in multi-rate traffic. The error probabilities derived below are in terms of effective users for both access methods.

A. Maximum-likelihood Detection

The closed form of error probability in ML detection is generally difficult to derive and the bounds were calculated instead. Define the error vector as [9]:

$$\boldsymbol{\varepsilon} = [\varepsilon_1 \varepsilon_2 \cdots \varepsilon_K]^T \equiv \tfrac{1}{2}(\mathbf{b} - \hat{\mathbf{b}}) \text{ where } \varepsilon_i \in \{0, \pm 1\}. \quad (11)$$

It represents any possible error decision. According to (6), it follows the metric function conditioned on the channel coefficients $\boldsymbol{\alpha}$:

$$\Omega(\mathbf{b}|\boldsymbol{\alpha}) \equiv \mathrm{Re}\{\mathbf{b}^H \bar{\mathbf{I}}^H \mathbf{C}^H \boldsymbol{\alpha}^H \mathbf{r}\} - \tfrac{1}{2} |\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b}|^2 \quad (12)$$

Assume $\mathbf{b}$ is transmitted, and the conditional error probability of the kth effective user is upper bounded by

$$P_k(\boldsymbol{\alpha}) \le \sum_{\boldsymbol{\varepsilon} \in E_k} \Pr\{\Omega(\mathbf{b} - 2\boldsymbol{\varepsilon}|\boldsymbol{\alpha}) \textit{ is } \max, \boldsymbol{\varepsilon} \textit{ is admissable}\}^1$$

$$\le \sum_{\boldsymbol{\varepsilon} \in E_k} \Pr\{\boldsymbol{\varepsilon} \textit{ is admissable}\} \cdot \Pr\{\Omega(\mathbf{b} - 2\boldsymbol{\varepsilon}) \ge \Omega(\mathbf{b})\} \quad (13)$$

where $E_k \equiv \{\boldsymbol{\varepsilon} | \varepsilon_k \neq 0\}$. It results from the union bound of all possible joint error decisions and the fact that whether an error vector is admissible depends only on the transmitted symbols [9] with probability $2^{-w(\boldsymbol{\varepsilon})}$ for equal-likely transmitted bits, where $w(\boldsymbol{\varepsilon})$ denotes the Hamming weight of $\boldsymbol{\varepsilon}$. Define $\|M(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|^2 = \mathbf{F}^H \mathbf{F}$, where $\mathbf{F} \equiv \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\boldsymbol{\varepsilon}$. It can be shown that the expected error probability over the channel realizations,

$$P_k \le \sum_{\boldsymbol{\varepsilon} \in E_k} 2^{-w(\boldsymbol{\varepsilon})} \int_{\boldsymbol{\alpha}} Q\left(\frac{2\|M(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|}{\sqrt{N_0}}\right) p(\boldsymbol{\alpha}) d\boldsymbol{\alpha}, \quad (14)$$

where $p(\boldsymbol{\alpha})$ denotes the probability density function (pdf) of $\boldsymbol{\alpha}$. The integral in (14) is often formulated as a function of the eigenvalues of $E[\mathbf{F}\mathbf{F}^H]$ in Rayleigh fading channels [10]. Unfortunately, these eigenvalues are usually duplicated such that this commonly used result is invalid. We apply the alternative expression of $Q(.)$ verified in [8],

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta \quad (15)$$

Therefore, $\int_{\boldsymbol{\alpha}} Q\left(\frac{2\|M(\boldsymbol{\varepsilon}|\boldsymbol{\alpha})\|}{\sqrt{N_0}}\right) p(\boldsymbol{\alpha}) d\boldsymbol{\alpha} = \int_{\boldsymbol{\alpha}} Q\left(\frac{2\|\mathbf{F}\|}{\sqrt{N_0}}\right) p(\mathbf{F}) d\mathbf{F}$

$$= \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \left| \frac{4\Sigma}{N_0 \sin^2\theta} + \mathbf{I} \right|^{-1} d\theta \quad (16)$$

where $p(\mathbf{F})$ is the pdf the zero-mean Gaussian vector $\mathbf{F}$ with variance Σ and $\mathbf{I}$ is a $F \times F$ identity matrix . Although it is still not a closed-form, this formula could be more easily calculated by numerical methods. In addition, it depicts that the error probability depends only on Σ.

$$\Sigma = diag\left(\sum_{i \in \Im} A_i^2 c_{i1}^2, \sum_{i \in \Im} A_i^2 c_{i2}^2, \cdots, \sum_{i \in \Im} A_i^2 c_{iF}^2\right), \quad (17)$$

where $\Im$ is the set of $\{n | \varepsilon_n \neq 0, n = 1, 2, \cdots, K\}$. The error probability is therefore bounded by

$$P_k \le \sum_{\boldsymbol{\varepsilon} \in E_k} 2^{-w(\boldsymbol{\varepsilon})} \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{f=1}^{F} \left(\frac{4\sum_{i \in \Im} A_i^2 c_{if}^2}{N_0 \sin^2\theta} + 1 \right)^{-1} d\theta \quad (18)$$

It implies that error probability decreases with the increase of the spreading factor F and the transmitted power A_k^2 . Thus, there is a tradeoff in choosing MC access, which has longer spreading codes, or VSL access, which has higher transmitting power, for multi-rate transmission.

This upper bound could be tightened by indecomposable error vectors [9] [10], where we define it for multi-rate MC-CDMA systems:

Definition: For any error vector decomposed as $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_a + \boldsymbol{\varepsilon}_b$, *where* $\boldsymbol{\varepsilon}_b \in E_k$, *if* $\mathrm{Re}(\boldsymbol{\varepsilon}_a^H \bar{\mathbf{I}}^H \mathbf{C}^H \boldsymbol{\alpha}^H \boldsymbol{\alpha}^H \boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\boldsymbol{\varepsilon}_b) \ge 0$, $\boldsymbol{\varepsilon}$ *is said decomposable.*

All decomposable error vectors can be eliminated from the summation in (18) to make the bound tighter.

To derive a lower bound, we assume all the symbols of other effective users are known. The error vector set of the kth effective user is reduced to $E_k = \{[00\cdots0\pm10\cdots0]^T\} = \{\pm\boldsymbol{\varepsilon}_L\}$. Following the same principles as deriving the upper bound,

$$P_k \ge E\left[Q\left(\frac{2\|M(\boldsymbol{\varepsilon}_L|\boldsymbol{\alpha})\|}{\sqrt{N_0}}\right)\right] = E\left[Q\left(\sqrt{\frac{4}{N_0}\sum_{j=1}^{F}|\alpha_{kj}|^2 c_{kj}^2 A_k^2}\right)\right]. \quad (19)$$

This bound also suggests larger spreading factor and higher transmitted power support lower error probability. The numerical examples in section VI show the dominance of the two trade-off factors.

B. Decorrelating Detection

We derive the case of $K \le F$. The decision vector is

$$\tilde{\mathbf{b}} = \mathbf{b} + (\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H \mathbf{n} = \mathbf{b} + \tilde{\mathbf{n}} \quad (20)$$

where $\tilde{\mathbf{n}} \equiv \mathbf{M}^+ \mathbf{n}$. For the kth effective user

$$\hat{b}_k = sign(\mathrm{Re}\{\tilde{b}_k\}) = sign(\mathrm{Re}\{b_k + \mathbf{1}_k^H \tilde{\mathbf{n}}\}) \quad (21)$$

and $\tilde{b}_k$ is Gaussian distributed with variance $\tfrac{N_0}{2}[(\mathbf{M}^H \mathbf{M})^{-1}]_{kk}$, where the sub-script kk denotes the (k,k) element of matrix. Thus, the error probability of kth effective user under channels of p.d.f. $p(\boldsymbol{\alpha})$:

$$P_k = E[P_k(\boldsymbol{\alpha})] = \int_{\boldsymbol{\alpha}} Q\left(\frac{1}{\sqrt{\tfrac{N_0}{2}[(\mathbf{M}^H \mathbf{M})^{-1}]_{kk}}}\right) p(\boldsymbol{\alpha}) d\boldsymbol{\alpha}, \quad (22)$$

where the distribution of $[(\mathbf{M}^H \mathbf{M})^{-1}]_{kk}^{-1}$ is standard chi-square with $2(F\text{-}K\text{+}1)$ degrees of freedom. Higher $(F\text{-}K)$ implies higher robustness to noise and longer spreading codes with

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[1] $\boldsymbol{\varepsilon}$ is said admissible with a given $\mathbf{b} \in \{\pm 1\}^K$ if $\varepsilon_i = b_i$ *or* $0, \forall i$. [9]

higher transmitted power contribute more at each degree of freedom in decreasing the error probability.

C. LMMSE Detections

The decision for kth effective user is made as

$$\hat{b}_k = sign\{\mathrm{Re}[\tilde{b}_k]\} = sign\{\mathrm{Re}[\boldsymbol{I}_k^H \mathbf{H}(\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\mathbf{b}+\mathbf{n})]\}, \tag{23}$$

where $\mathbf{H}=\bar{\mathbf{I}}^H\mathbf{C}^H\boldsymbol{\alpha}^H\left(\boldsymbol{\alpha}\mathbf{C}\boldsymbol{\Gamma}\mathbf{C}^H\boldsymbol{\alpha}^H+N_0\mathbf{I}\right)^{-1}$. Decompose $\tilde{b}_k$ as

$$\tilde{b}_k=\sum_{i=1}^{K} X_i b_i + \mathbf{1}_k^H\mathbf{H}\mathbf{n} = X_k\left(b_k+\sum_{i\neq k}^{K} x_i b_i\right)+\tilde{n}_k, \tag{24}$$

where $X_i=\left(\mathbf{H}\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\right)_{ki}$, $x_i = X_i / X_k$, and $\tilde{n}_k$ is Gaussian distributed with variance $\frac{No}{2}\left(\mathbf{H}\mathbf{H}^H\right)_{kk}$. Conditioned on the symbols of other effective users and $\boldsymbol{\alpha}$, the error probability of the kth effective user is:

$$P_k\left(\boldsymbol{\alpha},\{b_j, j\neq k\}\right)=Q\left(\frac{\left(\mathbf{H}\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\right)_{kk}}{\sqrt{\frac{No}{2}\left(\mathbf{H}\mathbf{H}^H\right)_{kk}}}\left(1+\sum_{i\neq k}^{K} x_i b_i\right)\right) \tag{25}$$

For uniformly binary-distributed source symbols,

$$P_k(\boldsymbol{\alpha})=2^{1-K}\sum_{\{b_j, j\neq k\}\in\{\pm1\}^{K-1}} Q\left(\frac{\left(\mathbf{H}\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\right)_{kk}}{\sqrt{\frac{No}{2}\left(\mathbf{H}\mathbf{H}^H\right)_{kk}}}\left(1+\sum_{i\neq k}^{K} x_i b_i\right)\right) \tag{26}$$

The Gaussian approximation for the accumulated interference terms [11] can be applied appropriately, especially with lots of effective users, to alleviate the exponentially increased computation load of (26). Hence, the error probability of the kth effective user is approximated as

$$P_k=\int_{\boldsymbol{\alpha}} Q\left(\frac{\left(\mathbf{H}\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\right)_{kk}}{\sqrt{\sum_{i\neq k}^{K}\left(\left(\mathbf{H}\boldsymbol{\alpha}\mathbf{C}\bar{\mathbf{I}}\right)_{ki}\right)^2+\frac{No}{2}\left(\mathbf{H}\mathbf{H}^H\right)_{kk}}}\right)p(\boldsymbol{\alpha})d\boldsymbol{\alpha}, \tag{27}$$

which is easier to be evaluated. In fact, it is also the approximated error probability of Decorrelating detection in cases of $K>F$ when noise diminishes.

VI. NUMERICAL EXAMPLES

We present several numerical examples to summarize the performance and properties of the two multi-rate access methods and the proposed MUDs in multi-rate MC-CDMA. Without loss of generality, we assume a two-rate system and the higher rate is twice the lower. There are two users of each rate respectively, which results in six effective users. The transmitted symbols are uniformly binary-distributed of ±1 and the signal of each user passes individual slow Rayleigh fading sub-channels with $E[|\alpha_{kf}|^2]=1$ for all k and f.

A. BER of MUDs in both access methods

Assume the consumed energy per symbol, E_b, of each original user is the same, and Hadamard Walsh codes are used for spreading, where the code length F=16 for basic rate users. Fig. 4 (a), (b), and (c) show the bit error rate (BER) of ML, decorrelating, and LMMSE MUDs respectively in both access methods. The result of ML detection shows that the BER of low rate users in VSL systems is roughly the same as users in MC systems, but the BER of high-rate users in VSL systems is higher. The results of Decorrelating and LMMSE detections exhibit similar relative performance among users in the two access methods as in ML detection, except for smaller performance gap. Thus, corresponding to the analysis in section V, the length of spreading codes dominates the MUD performance over the transmit power, and the better correlation among virtual spreading codes provides minor advantage for VSL access, unlike in single-carrier multi-rate DS-CDMA systems [5]. Fig. 4 (d) summarizes the BERs of MUDs in MC access, and the BER of a SUD with Minimum-Mean-Squared-Error Combining (MMSEC), the most promising SUD scheme of MC-CDMA in downlink [1], is presented as comparison. ML detection performs the best and the LMMSE detection rivals the Decorrelating detector. Decorrelating suffers from noise enhancement problem but these results show that although the two MUDs are not asymptotically the same when $F>K$, the performance of Decorrelating detection still approaches LMMSE detection as noise diminishes.

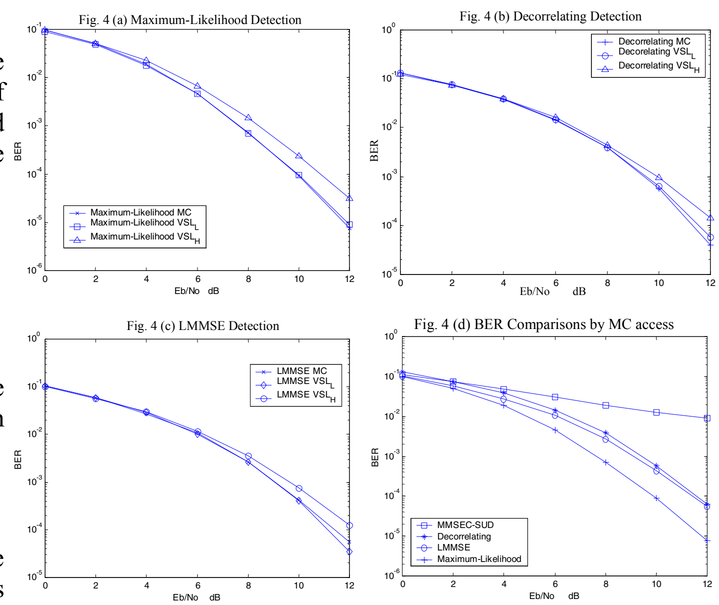

Fig. 4 Error probability of MUDs in multi-rate MC-CDMA systems.
L:Low-rate users H: High-rate users

B. Decorrelating v.s. LMMSE MUD

The unequal dimension of the spreading length and number of effective users results in novel relation between decorrelating and LMMSE MUDS. We take MC access for instance, Fig. 5 (a) shows the BERs at F = 32, 16, and 8, where the number of total effective users, K, is kept fixed as 6. It demonstrates that sub-carrier diversity improves the performance and the performance gap between LMMSE and Decorrelating detections is reduced with the increase of spreading length F. Fig. 5 (b) illustrates the compared simulation at F=8 with various numbers of effective users, where more users makes the performance gap larger. These results show that lower K/F merges the two detections at lower

E_b/N_0 and LMMSE MUD generally rivals Decorrelaing MUD, especially at higher loading (*K/F*) cases. The chip-filtering in MC-CDMA makes the superiority of LMMSE MUD over the noise-enhanced Decorrelating MUD more significant.

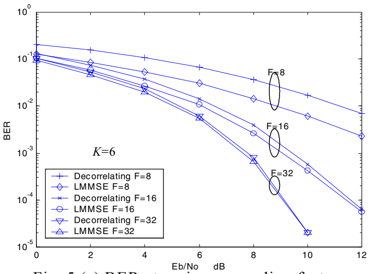

Fig. 5 (a) BER at various spreading factors.

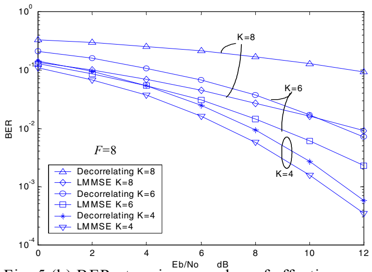

Fig. 5 (b) BER at various number of effective users

C. *BER at different rate combinations*

Fig.6 shows the results in a comparative scenario of two basic-rate users and one high-rate user with rate 4. There are still six effective users but the rate ratio of the high-rate users over the low-rate users becomes 4. The performance of basic-rate users in both access methods are not significantly affected, but high-rate users in VSL access systems degrades with the increase of *m* (decrease of code length). It exhibits again the domination of spreading length in MUD performance, and the error rate of a user is not significantly affected by interference pattern.

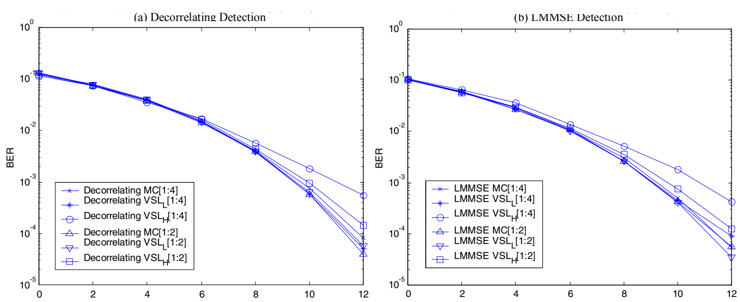

Fig. 6 BER of MUDs in multi-rate MC-CDMA systems. [1:2]: the case that high rate is twice the basic rate; [1:4]: the case that high rate is 4-times the basic rate. Totally six effective users.

VII. IMPLEMENTATION ISSUES

High peak-to-average power (PAP) ratio is the inherent drawback [4] of OFDM systems, and may be severer in MC-CDMA of Multi-Code access due to the parallel multiplexing for transmitting high-rate data streams. The PAP problem will degrade the efficiency of power amplifier, and thus some techniques have been proposed to alleviate the PAP ratio [12] in OFDM systems. It can be proved that the PAP ratio in MC access is *m*-times larger than in VSL access for transmitting a user with rate *m* in multi-rate MC-CDMA systems. On the other hand, processing delay is another critical issue of multi-carrier systems for real-time multimedia services. VSL access, direct spreading without codes-multiplexing, stands on the advantageous position in this aspect, although it lacks some module regularity. Therefore, despite of the worse BER than MC access, VSL access is still an attractive candidate for real-time applications. To realize multi-rate multi-carrier transmission, robust power amplifiers with larger linear region or more efficient PAP-ratio reduction mechanisms are another topic that needs further research.

VIII. CONCLUSIONS

In this paper, multi-rate transmission of the MC-CDMA systems has been studied and a general system model accommodating both MC and VSL access is built. We proposed three effective multiuser detections whose structures are identical in both access methods and analyzed their behaviors and detection performances. The chip-based LMMSE detection is suggested and practical to be used and it replaces the single-user detector effectively in uplink applications. Compared studies show that the BER of multiuser detections in MC-CDMA systems is dominated by the length of spreading codes, instead of the interference pattern. MC access is suggested for multi-rate uplink applications especially for high-rate services with critical BER requirement, and the costs are the effort to deal with higher PAP ratio of the transmitted signals and extra de/multiplexing processing. VSL access in contrast is suggested for real-time services of low requirement on BER, such as the voice communications.

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