Data-Aided Maximum Likelihood Frequency Synchronization for OFDM Systems

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***Abstract*— Distinct from conventional approaches, we apply a generalized data-aided signal model which can jointly consider the cyclic prefix with the preamble/pilot to propose three data-aided frequency offset estimation algorithms based on maximum likelihood criterion. The proposed algorithms differ from each other in the processing domain and the observation space. They can not only systematically estimate both the integer part and the fractional part of the frequency offset, but also possess the maximum acquisition range of a discrete time system. The effectiveness of the proposed schemes is verified by mathematical analysis and computer simulation.**

I. Introduction

In recent years, multicarrier transmission, in particular, orthogonal frequency division multiplexing (OFDM) has attracted much more attention for its possible application to the design of broadband wireless communication systems. Multicarrier transmission schemes are resistant to wireless impairments such as multipath fading and impulsive noise. Therefore, its most important application for future wireless multimedia is high speed data transmission over fading channels.

In OFDM systems, the sensitivity to carrier frequency offset (CFO) is one of the major challenges for practical implementation. CFO is mainly caused by Doppler shift, Doppler spread in fading channels and transmitter-receiver oscillator instabilities. Such an offset can be dozens of the subcarrier spacing and is usually classified as the *integer part* and the *fractional part*. The integer part is a multiple of the subcarrier spacing while the fractional part is confined to half the subcarrier spacing. Without properly compensation, the former results in a shift of the subcarrier indices and the latter produces inter-carrier interference [2].

Data-aided systems using pilot or preamble symbols are more suitable for packet oriented applications which require fast and reliable synchronization. Several data-aided schemes are proposed in the literature. The method proposed in [3] gives the maximum likelihood (ML) frequency estimator based on the observation of two consecutive and identical OFDM symbols. The estimation range is limited to half of the subcarrier spacing. In other words, an ambiguity of multiple subcarrier spacing exists even though this limit can be extended by shortening the training symbol duration at the cost of reduced estimation accuracy. A joint timing and frequency estimation is proposed in [4], where two training OFDM symbols are employed for both timing and frequency offset estimation. In this scheme, the first symbol has two identical halves and serves to measure the fractional part. The second symbol serves to resolve the remaining ambiguity, however the estimation accuracy is still not satisfied.

On account of the significance of data-aided systems, it is substantial to investigate the data-aided estimation of CFO in OFDM systems. Data-aided frequency synchronization algorithms can be essentially classified by the processing domain (time or frequency), the observation space (considering cyclic prefix or not) and the data-assistance (preamble-aided or pilot-aided). In this paper, different from conventional approaches, we apply ML criterion to propose a class of estimators based on a generalized data-aided signal model which can completely describe the foregoing affecting factors and only assume one general training symbol or pilot signal. The proposed schemes can effectively estimate both parts of CFO and have a wide acquisition range up to the whole OFDM transmission bandwidth.

In Section II, the generalized data-aided OFDM signal model is described. The class of data-aided estimators for the integer part are presented in Section III. And the estimation for the fractional part is given in Section IV. Performance evaluation via mathematical analysis and computet simulation are addressed in Section V. Finally, Section IV discusses and concludes this paper.

II. Generalized Data-Aided OFDM Signal Model

We consider a generalized data-aided OFDM system using N-point inverse fast Fourier transform (IFFT) for modulation. Each OFDM symbol is composed of N_u complex symbols $X_{l,m}$ where l denotes the OFDM symbol index and m denotes the subcarrier index. Let $\mathcal{D}$ denote the set of indices for N_d data-conveying subcarriers and $\mathcal{P}$ is the set of indices for N_p pilot subcarriers. Then, the set of indices of the N_u useful subcarriers $\mathcal{U}$ can be defined as

$$\mathcal{U} = \mathcal{D} \cup \mathcal{P} \tag{1}$$

and

$$N_u = (N_d + N_p) \leqslant N. \tag{2}$$

The output of the IFFT has a duration of T seconds which is equivalent to N samples. A cyclic prefix of duration T_g seconds or N_g samples longer than the channel impulse response is preceded to eliminate the inter-symbol interference

(ISI). The resulting training signal is of duration $T_s = T + T_g$ seconds or equivalently, $N_s = N + N_g$ samples. And the transmitted baseband complex signal can be represented by

$$s(t) = \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m\in\mathcal{U}} X_{l,m} e^{j2\pi(m/T)(t-T_g-lT_s)} g(t-lT_s) \quad (3)$$

where $g(t)$ is the rectangular pulse given by

$$g(t) = \begin{cases} 1, & t \in [0, T_s) \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The transmitted signal can be separated into two parts and modelled as

$$s(t) = d(t) + p(t), \quad (5)$$

where

$$d(t) = \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m\in\mathcal{D}} X_{l,m} e^{j2\pi(\frac{m}{T})(t-T_g-lT_s)} g(t-lT_s), \quad (6)$$

$$p(t) = \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m\in\mathcal{P}} X_{l,m} e^{j2\pi(\frac{m}{T})(t-T_g-lT_s)} g(t-lT_s). \quad (7)$$

Here, we assume that $\{X_{l,m}\}$ belong to some constellation with zero mean and two kinds of average power $\sigma_X^2 = \{E\{|X_{l,m}|^2\} | m \in \mathcal{D}\}$, and $\sigma_P^2 = \{E\{|X_{l,m}|^2\} | m \in \mathcal{P}\}$. When $\mathcal{D}$ is a null set, this model can also characterize the preamble signal.

In the following, we assume a zero mean additive white Gaussian noise $n(t)$ and a frequency offset Δf. At the receiver, timing recovery is assumed to be accomplished and the received signal sampled at $t_k = kT/N$ is

$$r(\frac{kT}{N}) = d(\frac{kT}{N})e^{j\frac{2\pi\Delta f kT}{N}} + p(\frac{kT}{N})e^{j\frac{2\pi\Delta f kT}{N}} + n(\frac{kT}{N}). \quad (8)$$

In order to simplify the interpretation and terminology, we use $r(k)$ to represent $r(\frac{kT}{N})$ hereafter. In addition, ϵ is used to represent the frequency offset normalized to the subcarrier spacing. Therefore, the received signal can be rearranged as

$$r(k) = d(k)e^{j\frac{2\pi\epsilon k}{N}} + p(k)e^{j\frac{2\pi\epsilon k}{N}} + n(k) \quad (9)$$

The signal-to-noise ratio is defined as $\eta \triangleq (\sigma_d^2 + \sigma_p^2)/\sigma_n^2$ with $\sigma_d^2 \triangleq E\{|d(k)|^2\}$, $\sigma_p^2 \triangleq E\{|p(k)|^2\}$ and $\sigma_n^2 \triangleq E\{|n(k)|^2\}$. Let $r_{l,k}$ denotes the kth sample of the lth received OFDM symbol. Then, the lth received OFDM symbol can be represented by

$$\tilde{\boldsymbol{r}}_l \triangleq [r_{l,0}, r_{l,1}, \cdots, r_{l,N_s-1}]^T, \quad (10)$$

after removing the guard interval, the lth received OFDM symbol is represented by

$$\boldsymbol{r}_l \triangleq \left[r_{l,N_g}, r_{l,N_g+1}, \cdots, r_{l,N_s-1}\right]^T. \quad (11)$$

Taking FFT to $\boldsymbol{r}_l$, the lth frequency domain decision vector

$$\boldsymbol{R}_l \triangleq [R_{l,0}, R_{l,1}, \cdots, R_{l,N-1}]^T \quad (12)$$

is obtained for further inner-receiver processing like equalization and detection. With these vector variables defined, we are ready to develop the maximum-likelihood estimator.

III. DA-ML Frequency Estimation: Integer Part

Here, we consider the data-aided maximum likelihood estimation for the integer part of the CFO in OFDM systems. Taking the processing domain and the observation space into account, there are three possible schemes to approach this problem, they can be categorized as

- ❖ Time domain approach with cyclic prefix.
- ❖ Time domain approach without cyclic prefix.
- ❖ Frequency domain approach.

The first two approaches differ from each other in the observation space while the third one diverges from others in the processing domain. We shall systematically derive the synchronization algorithms based on ML criterion with the assist of preamble or pilot signal.

A. Stochastic Characteristics of $r(k)$

As in [5], we can simplify the statistical characteristics of $r(k)$ to derive a tractable estimator. First, with the assumption of sufficiently large number of data-conveying subcarriers on which the modulating symbols are uncorrelated, the central limit theorem can be applied to model $d(k)$ as a zero mean complex Gaussian random variable with variance $\sigma_d^2 = N_d\sigma_X^2/N$. It is evident that $\{d(k)\}$ are independent with each other provided that they are not cyclic prefix pair. Since $d(k)$ and $n(k)$ are both zero-mean complex Gaussian distributed and $p(k)$ is a deterministic signal which is known at the receiver, we can claim that $r(k)$ is also complex Gaussian distributed with time-varying mean $p(k)e^{j\frac{2\pi\epsilon k}{N}}$ and variance $\sigma_d^2 + \sigma_n^2$. Besides, $\{r(k)\}$ are independent with each other provided that they are not cyclic prefix pair.

Second, in OFDM systems employing cyclic prefix, $r(k)$ and $r(k+N)$ ($k \in [lN_s, lN_s + N_g - 1]$ for some l) are correlated since $d(k) = d(k+N)$. This correlation must be taken into account in the time domain approach with cyclic prefix. To simplify the derivation of the likelihood function, we shall find out the conditional complex Gaussian PDF of $r(k)$ and the conditional joint complex Gaussian PDF of $r(k)$ and $r(k+N)$ given ϵ.

1) Conditional Complex Gaussian PDF of $r(k)$: From the preceding discussion, it is straightforward to obtain the conditional complex Gaussian PDF of $r(k)$ as

$$f(r(k)|\epsilon) = \frac{1}{\pi(\sigma_d^2 + \sigma_n^2)} e^{-\frac{\left|r(k)-p(k)e^{j\frac{2\pi\epsilon k}{N}}\right|^2}{\sigma_d^2+\sigma_n^2}}. \quad (13)$$

2) Conditional Joint Complex Gaussian PDF of the Cyclic Prefix Pair: $r(k)$ and $r(k+N)$: Here, we assume that $r(k)$ and $r(k+N)$ belong to the same OFDM symbol and are cyclic prefix pair, this condition implies that $s(k) = s(k+N)$. Let $[r(k), r(k+N)]^T$ be denoted by $\boldsymbol{x}$, of which the mean vector and the covariance matrix are

$$\boldsymbol{\mu} = \begin{bmatrix} p(k)e^{j2\pi\frac{\epsilon k}{N}} \\ p(k+N)e^{j2\pi\frac{\epsilon(k+N)}{N}} \end{bmatrix} \quad (14)$$

$$f(r(k), r(k+N)|\epsilon) = \frac{e^{-\frac{|r(k)-p(k)e^{j\frac{2\pi\epsilon k}{N}}|^2 - 2\rho\Re\{[r(k)-p(k)e^{j\frac{2\pi\epsilon k}{N}}][r(k+N)-p(k+N)e^{j\frac{2\pi\epsilon(k+N)}{N}}]^* e^{j2\pi\epsilon}\} + |r(k+N)-p(k+N)e^{j2\pi\frac{\epsilon(k+N)}{N}}|^2}{(\sigma_d^2+\sigma_n^2)(1-\rho^2)}}}{\pi^2(\sigma_d^2+\sigma_n^2)^2(1-\rho^2)} \quad (17)$$

$$\begin{aligned} \Lambda_{cp}^l(\epsilon) = &- \sum_{k=0}^{N_g-1} \left| r_{l,k} - \alpha^l p_{l,k} e^{j\frac{2\pi\epsilon k}{N}} \right|^2 - \sum_{k=0}^{N_g-1} \left| r_{l,k+N} - \alpha^l p_{l,k+N} e^{j\frac{2\pi\epsilon(k+N)}{N}} \right|^2 - (1-\rho^2) \sum_{k=N_g}^{N-1} \left| r_{l,k} - \alpha^l p_{l,k} e^{j\frac{2\pi\epsilon k}{N}} \right|^2 \\ &+ \sum_{k=0}^{N_g-1} 2\rho\Re\left\{ \left[r_{l,k} - \alpha^l p_{l,k} e^{j\frac{2\pi\epsilon k}{N}} \right] \left[r_{l,k+N} - \alpha^l p_{l,k+N} e^{j\frac{2\pi\epsilon(k+N)}{N}} \right]^* e^{j2\pi\epsilon} \right\} \end{aligned} \quad (19)$$

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and

$$\boldsymbol{C_x} = \begin{bmatrix} \sigma_d^2 + \sigma_n^2 & \sigma_d^2 e^{-j2\pi\epsilon} \\ \sigma_d^2 e^{j2\pi\epsilon} & \sigma_d^2 + \sigma_n^2 \end{bmatrix}. \quad (15)$$

Then, the conditional joint complex Gaussian PDF of $r(k)$ and $r(k+N)$ can be expressed as

$$f(r(k), r(k+N)|\epsilon) = \frac{e^{-(\boldsymbol{x}-\boldsymbol{\mu})^H \boldsymbol{C_x}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})}}{\pi^2 \det(\boldsymbol{C_x})}. \quad (16)$$

To simplify the derivation, we define the correlation coefficient between $r(k)$ and $r(k+N)$ to be ρ. It is easy to find that $\rho = \sigma_d^2/(\sigma_d^2+\sigma_n^2)$ and the signal-to-noise ratio is related to this correlation coefficient by $\rho = \sigma_d^2\eta/[(1+\sigma_d^2)\eta + \sigma_p^2]$. After some algebraic manipulations, the explicit form of (16) can be obtained in (17). To avoid misunderstanding, we emphasize here that (16) and (17) only hold true for some k pertaining to cyclic prefix. Based on the complex Gaussian PDF obtained, we can now derive the DA-ML estimators.

B. Time Domain Approach with Cyclic Prefix

With the help of cyclic prefix in time domain, the DA-ML estimation should be based on the observation of $\tilde{\boldsymbol{r}}_l$. Hence, the conditional PDF of $\tilde{\boldsymbol{r}}_l$ given ϵ can be written as

$$f(\tilde{\boldsymbol{r}}_l|\epsilon) = \prod_{k=0}^{N_g-1} f(r_{l,k}, r_{l,k+N}|\epsilon) \prod_{k=N_g}^{N-1} f(r_{l,k}|\epsilon). \quad (18)$$

With some algebraic manipulations and defining $p_{l,k} \triangleq p(lN_s+k)$, the log-likelihood function corresponding to $\tilde{\boldsymbol{r}}_l$ can be obtained in (19), where $\alpha \triangleq e^{j\frac{2\pi\epsilon N_s}{N}}$ stands for the phase shift produced by the normalized frequency difference ϵ and the time difference N_sT/N. Thus, the time domain cyclic-prefix-assisted DA-ML frequency estimation corresponding to the lth OFDM symbol shall be

$$\hat{\epsilon}_{cp}^l = \arg\max_{\epsilon} \Lambda_{cp}^l(\epsilon). \quad (20)$$

C. Time Domain Approach without Cyclic Prefix

Without the help of cyclic prefix in time domain, the DA-ML estimation should be based on the observation of $\boldsymbol{r}_l$. The conditional PDF of $\boldsymbol{r}_l$ given ϵ can be written as $f(\boldsymbol{r}_l|\epsilon) = \prod_{k=0}^{N-1} f(r_{l,k+N_g}|\epsilon)$. After some algebraic manipulations, the log-likelihood function corresponding to $\boldsymbol{r}_l$ can be derived as

$$\Lambda_t^l(\epsilon) = \Re\left\{ \sum_{k=0}^{N-1} r_{l,k+N_g} \alpha^{-l} \beta^* p_{l,k+N_g}^* e^{-j\frac{2\pi\epsilon k}{N}} \right\}, \quad (21)$$

where $\beta \triangleq e^{j2\pi\frac{\epsilon}{N}N_g}$ stands for the phase shift produced by the normalized frequency difference ϵ and the time difference N_gT/N. Thus, the time domain DA-ML frequency estimation corresponding to the lth OFDM symbol shall be

$$\hat{\epsilon}_t^l = \arg\max_{\epsilon} \Lambda_t^l(\epsilon). \quad (22)$$

D. Frequency Domain Approach

In frequency domain, the DA-ML estimation should be derived based on $\boldsymbol{R}_l$. From similar derivation, the likelihood function in frequency domain corresponds to $\boldsymbol{R}_l$ is

$$f(\boldsymbol{R}_l|\epsilon) = \prod_{n=0}^{N-1} \frac{1}{N\pi\sigma_n^2} e^{-\frac{|R_{l,n} - \alpha^l \beta P_{l,n}(\epsilon)|^2}{N\sigma_n^2}} \quad (23)$$

where $P_{l,n}(\epsilon) \triangleq \mathcal{DFT}\left\{p_{l,k+Ng} e^{j2\pi\frac{\epsilon k}{N}}\right\}$ with $k, n \in \{0, 1, \cdots, N-1\}$. Please notice that when ϵ happens to be an integer, $P_{l,n}(\epsilon)$ is exactly the pilot symbol on the $((n-\epsilon))_N$ th subcarrier of the lth OFDM symbol, where the notation $((n))_N$ denotes (n modulo N). After some algebraic manipulations, the log-likelihood function can be obtain as

$$\Lambda_f^l(\epsilon) = \sum_{k=0}^{N-1} \Re\left\{R_{l,k} \alpha^{-l} \beta^* P_{l,k}^*(\epsilon)\right\}. \quad (24)$$

And the frequency domain DA-ML estimator shall be

$$\hat{\epsilon}_f^l = \arg\max_{\epsilon} \Lambda_f^l(\epsilon). \quad (25)$$

Comparing (21) and (24), we can find that they are essentially equivalent to each other.

E. DA-ML Frequency Acquisition Using Preamble

When it comes to "one-shot" synchronization, a preamble signal is usually suggested. The proposed algorithms are explicitly of pilot-aided scenario, however, it is straightforward to degenerate them to obtain their preamble versions. Here, we discuss two kinds of preamble, one is the "*mixed preamble*" which contains both training and data symbols in frequency domain. The other is the "*pure preamble*" which is purely composed of training symbols.

1) Mixed Preamble: The signal model of the mixed preamble is exactly consistent with the generalized data-aided OFDM system model described in Section II. Assume only one OFDM symbol is used, we can set l equals to 0 to get the mixed-preamble-aided algorithms.

2) Pure Preamble: Since the $d(k)$ in (9) is absent, the stochastic property (specifically the variance) of $r(k)$ is changed. We also assume only one OFDM symbol is employed, then set l equals to 0. With similar derivation, it can be shown that both the time domain approach without cyclic prefix and the frequency approach have the same results with (21) and (24), wherein $p_{0,k}$ and $P_{0,k}(\epsilon)$ now stand for the pure preamble signal in time and frequency domain respectively.

As for the time domain approach with cyclic prefix, we need to make some crucial modifications. Compared with other two alternatives, this approach relies on the correlation between cyclic prefix pairs to provide extra information. However, this correlation results solely from $d(k)$. When $d(k)$ vanishes, σ_d^2 and ρ become zero and each sample in one OFDM symbol become independent with each other. Under this condition, the two time domain approaches have the same form of log-likelihood function, except that considering the cyclic prefix gives a larger observation space. Briefly, when a pure preamble is adopted, the log-likelihood function of the time domain approach with cyclic prefix becomes

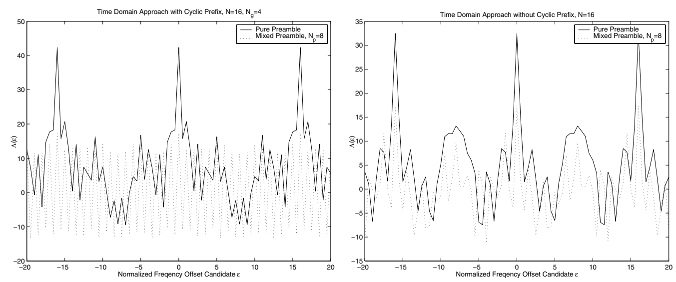
$$\Lambda_t^0(\epsilon) = \Re\left\{\sum_{k=0}^{N_s-1} r(k)p^*(k)e^{-j\frac{2\pi\epsilon k}{N}}\right\}. \tag{26}$$

F. Acquisition Range

The acquisition range is vital to a frequency estimator, and the conventional schemes generally suffer from small acquisition range within half to several subcarrier spacing. To examine the acquisition range of the three proposed approaches, the log-likelihood functions are depicted in Figure 1. The frequency domain approach is omitted due to its equivalence to the time domain approach without cyclic prefix. From these waveforms, we can find that the log-likelihood functions exhibit periodicity with the same period N (normalized frequency). Therefore, the acquisition range of the proposed schemes is $[-N/2, N/2)$. Please note that for a discrete-time OFDM system with Nyquist sampling rate of $1/T$, the recognizable frequency range is exactly $[-N/2, N/2)$, accordingly, the proposed frequency acquisition schemes possess the maximum acquisition range which is sufficient for any discrete-time OFDM system.

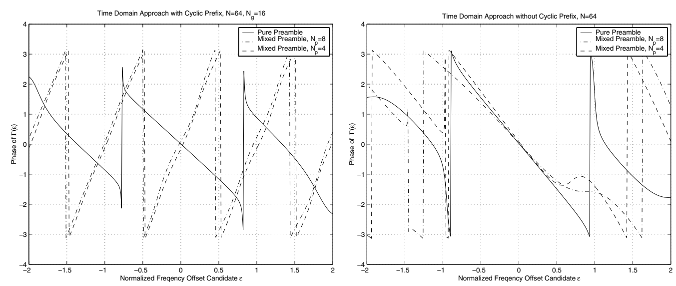
IV. DA-ML Frequency Estimation: Fractional Part

In [1], the phase of the log-likelihood function before taking real-part is used to estimate the frequency offset, this motivates us to examine the corresponding phases of the proposed



(a) Time domain approach with cyclic prefix. (b) Time domain approach without cyclic prefix.

Fig. 1. Waveforms of the log-likelihood functions of the proposed acquisition schemes. (the exact frequency offset=0, SNR=10 dB)



(a) Time domain approach with cyclic prefix. (b) Time domain approach without cyclic prefix.

Fig. 2. Phases of the log-likelihood functions before taking real part of the proposed acquisition schemes. (the exact frequency offset=0, SNR=10 dB)

algorithms. Prior to this, we define $\Re\{\Gamma_t^l(\epsilon)\} = \Lambda_t^l(\epsilon)$ and $\Re\{\Gamma_{cp}^l(\epsilon)\} = \Lambda_{cp}^l(\epsilon)$. Their phases are illustrated in Figure 2. We can observe that there exhibit a linear range if the frequency offset falls within the range of $[-1/2, 1/2]$. Provided that the linear range is acquired, we can use either a linear interpolation or a number control oscillator (NCO) to track the fractional frequency offset. Three kinds of scenario which can simultaneously compensate both the integer part and the fractional part of the frequency offset are thus suggested as follows:

- The proposed acquisition schemes followed by a linear interpolator to give an one-shot estimation or open-loop tracking.
- The proposed acquisition schemes followed by a NCO to give a close-loop tracking.
- The proposed acquisition schemes followed by a linear interpolator which feedback to a NCO to give a close-loop tracking.

The illustration of the third scenario is shown in Figure 3.

V. Performance Evaluation

In this section, we evaluate the performance of the proposed acquisition and tracking schemes via computer simulation and mathematical analysis. As for acquisition schemes, the channel is assumed to affect the signal by a CFO with discrete value and an additive white Gaussian noise. The acquisition resolution is set to 1 (normalized frequency). And the channel

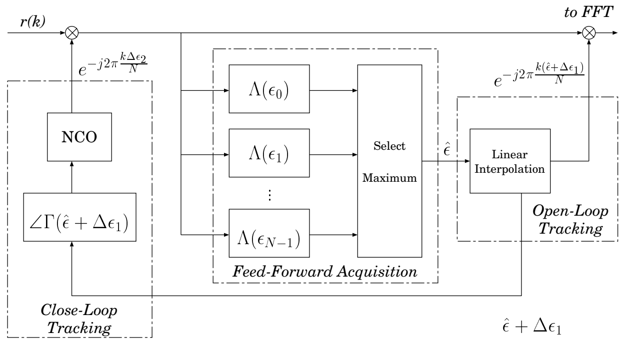
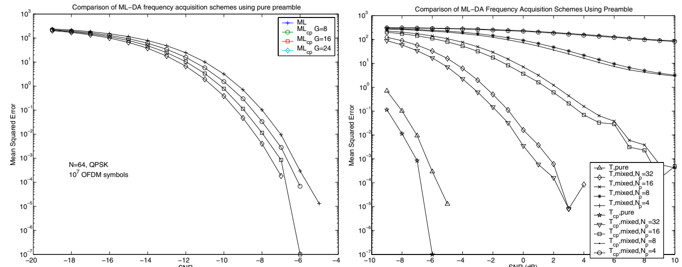
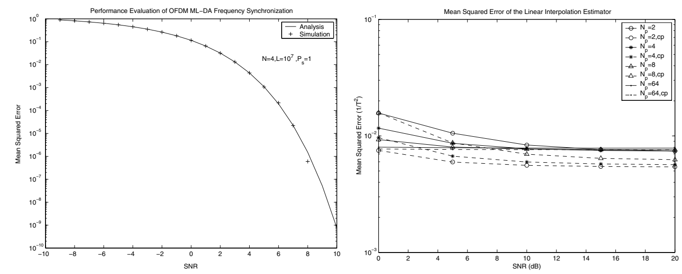


Fig. 3. The third scenario of the proposed tracking schemes.



(a) Time domain approaches using pure preamble. (b) Time domain approaches using Preamble.

Fig. 4. Comparison of the MSE of the acquisition schemes using preamble.



(a) Acquisition: time domain approach without cyclic prefix using pure preamble. (b) Tracking: time domain approaches using preamble.

Fig. 5. Comparison of the MSE of the acquisition and tracking schemes.

assumption in tracking schemes remains the same except that the CFO has a continuous value within the range of [-1/4,1/4]. And the open-loop tracking by linear interpolation, i.e., the first scenario in Section IV is employed. The QPSK modulation is adopted for both data and pilot symbols, N and Nu are set to be 64 and N_p is generally set to be 16 if not specifically defined. Then, we simulate the mean-squared error (MSE) of the proposed acquisition and tracking schemes and each simulation point uses 10^7 OFDM symbols. Again, the performance evaluation of the frequency domain approach is omitted here due to its equivalence.

The comparison of the MSE of the time domain approaches using pure preamble with different cyclic prefix length is illustrated in Figure 4-(a). From Figure 4-(a), we can find that an incremental cyclic prefix length of 8 samples gives about an 0.5 dB gain.

The comparison of the MSE of the proposed acquisition schemes using different preambles are illustrated in Figure 4-(b). From Figure 4-(b), the acquisition schemes using pure preamble outperform the same schemes using mixed preamble with $N_p = 32$ about 8 dB. The performance differences between the acquisition schemes using mixed preamble are about 4 dB. This result is reasonable and conform to our expectation.

The validness of the simulation results in Figure 4-(a) and Figure 4-(b) are justified by Figure 5-(a) in which the results of computer simulation conform to that of mathematical analysis. The selected algorithm is the time domain acquisition using pure preamble with $N = 4$. The detail derivation is omitted here due to the lack of space.

The comparison of the MSE of the tracking schemes are illustrated in Figure 5-(b). We can observe that when pure preamble is used, the performance is insensitive to the decay of SNR. Because when SNR> 0, the acquisition result sent to linear interpolation is extremely accurate, hence, the performance is dominated by the limit of linear interpolation itself. When mixed preamble is used, the performance is sensitive to the decay of SNR. The effectiveness of the tracking schemes without cyclic prefix is proportional to the size of N_p while that with cyclic prefix is inversely proportional to N_p. The main reason is because when cyclic prefix is considered, the linearity of phase response in Figure 2-(a) will be destroyed when N_p grows. And this loss of linearity dominate the performance.

VI. Conclusion

In this paper, three kinds of data-aided maximum likelihood frequency estimation algorithm based on generalized data-aided signal model are proposed. The main differences of them lies in the processing domain and the observation space. The proposed algorithms have a wide acquisition range and can systematically estimate both the integer part and the fractional of the frequency offset. Finally, We justify the effectiveness of the proposed acquisition and tracking schemes by simulation and analysis. The acquisition schemes using pure preamble are shown to be quite accurate even with a negative signal-to-noise ratio while the open loop tracking schemes have moderate performance and fairly low complexity.

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