行政院國家科學委員會補助專題研究計畫成果報告

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行政院國家科學委員會專題研究計畫成果報告

計畫編號：NSC 89-2115-M-002-020
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中文摘要

Colding 與 Minicozzi 簡化了廣義尼采猜測的證明 談迺 Meeks 與 Rosenberg 證出 helicoid 猜測 兩者皆需用到

關鍵詞：單邊的 Reifenberg 條件

Abstract

Both in the proofs of generalized Nitsche conjecture and helicoid conjecture, one needs

Keywords: one-sided Reifenberg condition

ONE-SIDED REIFENBERG CONDITION

AI-NUNG WANG

The most general form of the one-sided Reifenberg condition is given by Tobias H. Colding and William P. Minicozzi II in [1]:

Definition: A subset Γ of M^n is said to satisfy the (δ, r_0)-one-sided-Reifenberg condition at $x \in \Gamma$ if for every $0 < \sigma \leq r_0$ and every $y \in B_{r_0-\sigma}(x) \cap \Gamma$, there corresponds a connected hypersurface,$L^{n-1}_{y,\sigma}$, with $\partial L_{y,\sigma} \subset \partial B_\sigma(y)$,

$$B_{\delta\sigma}(y) \cap L_{y,\sigma} \neq \phi,$$

$$\sup_{B_\sigma(y)\cap L} |A_L|^2 \leq \delta^2\sigma^{-2},$$

and such that the connected component of $B_\sigma(y) \cap \bar{\Gamma}$ through y lies on one side of $L_{y,\sigma}$.

In practical use, it is formulated as follows: cf. [2],[3]

Theorem: There exists an $\epsilon > 0$ such that the following holds. Let $y \in \mathbb{R}^3$, $r > 0$ and $\Sigma \subset B_{2r}(y) \cap \{x_3 > x_3(y)\} \subset \mathbb{R}^3$ be a compact embedded minimal disk with $\partial\Sigma \subset \partial B_{2r}(y)$. For any connected component Σ' of $B_r(y) \cap \Sigma$ with $B_{\epsilon r}(y) \cap \neq \phi$, one has $sup_{\Sigma'}|A_{\Sigma'}|^2 \leq r^{-2}$.

We indicate the crucial steps of its proof from a lecture note at MSRI by Minicozzi.

Lemma: Given $C, \exists\epsilon > 0$ so that if $\mathcal{B}_{9s} \subset \Sigma$ is an embedded minimal disk

$$\int_{\mathcal{B}_{9s}} |A|^2 \leq C \quad and \quad \int_{\mathcal{B}_{9s}\setminus\mathcal{B}_s} |A|^2 \leq \epsilon$$

then $sup_{\mathcal{B}_s}|A|^2 \leq s^{-2}$

Corollary: Given $C_1, \exists C_2$ so that if $\mathcal{B}_{2s} \subset \Sigma$ is an embedded minimal disk with $\int_{\mathcal{B}_{2s}|A|^2} \leq C_1$, then $sup_{\mathcal{B}_s}|A|^2 \leq C_2 s^{-2}$.

Proof of the Lemma: By an estimate of Choi and Schoen

$$\sup_{\mathcal{B}_{8s}\setminus\mathcal{B}_{2s}} |A|^2 \leq C_1\epsilon s^{-2}$$

We will show that $\int_{\mathcal{B}_{9s}} |A|^2 \leq C$ implies

$$L(2s) \leq 4\pi s + Cs$$

Typeset by $\mathcal{AMS}$-TEX

indeed,

$$L'(t) = \int_{\partial\mathcal{B}_t} \kappa_g = 2\pi - \int_{\mathcal{B}_t} K = 2\pi + \frac{1}{2}\int_{\mathcal{B}_t} |A|^2,$$

$$L(t) = 2\pi t + \frac{1}{2}\int_0^t ds\Big[\int_{\mathcal{B}_s} |A|^2\Big],$$

$$2(Area - \pi r_0^2) = \int_0^{r_0} dt \int_0^t ds\Big[\int_{\mathcal{B}_s} |A|^2\Big]$$

$$= \int_0^{r_0} ds \int_s^{r_0} dt\Big[\int_{\mathcal{B}_s} |A|^2\Big]$$

$$= \int_0^{r_0} ds\Big[(r_0 - s)\int_{\mathcal{B}_s} |A|^2\Big]$$

$$= \frac{(r_0-s)^2}{-2}\int_{\mathcal{B}_s} |A|^2\Big|_{s=0}^{r_0} + \frac{1}{2}\int_0^{r_0} (r_0-s)^2\Big[\int_{\partial\mathcal{B}_s} |A|^2\Big] ds$$

$$= 0 + \frac{r_0^2}{2}\int_0^{r_0} (1 - \frac{s}{r_0})^2 |A|^2$$

In particular, $\forall x, x' \in \mathcal{B}_{8s}\backslash\mathcal{B}_{2s}$ can be joined by a path of length $\leq C_2(1+C)s$. Since $|\nabla\vec{n}| = |A|$, we conclude that over $\mathcal{B}_{8s}\backslash\mathcal{B}_{2s}$ it is a graph (at least locally).

If $x \in \partial\mathcal{B}_{2s}$, let γ_s = outward normal geodesic, then $|\kappa_g^{\mathbb{R}^3}| \leq \sqrt{C_1\epsilon}/s$, so it stays close to its initial tangent ray. In particular dist(end points of γ_x) is almost 6s. Since it is a graph with small gradient, the cylinder $\{x_1^2 + x_2^2 = (2s)^2\} \cap \mathcal{B}_{8s}$ does not intersect $\partial\mathcal{B}_{8s}$. Therefore we get a collection of graphs. Finally using embeddedness we see the the intersection is a collection of disjoint embedded circles, and by maximum principle we know one of these bounds a disk containing 0 in Σ. The proof is finished by recalling

Theorem: (Rado) If Σ is minimal and $\partial\Sigma$ is a graph over boundary of convex domain, then Σ is a graph.

References

[1] Colding, T. H., and Minicozzi, W. P., II, *Minimal Surfaces*, Courant Lecture Notes in Mathematics **4** (1999).

[2] Colding, T. H., and Minicozzi, W. P., II, *Complete properly embedded minimal surfaces in* $\mathbb{R}^3$, Duke. Math. Jour. **107** (2001), 421-426.

[3] William Meeks III and Harold Rosenberg, *The uniqueness of the helicoid and the geometry of properly embedded minimal suraces with finite topology*, pre-print.