附件一

行政院國家科學委員會補助專題研究計畫 ☑成果報告 □期中進度報告

（計畫名稱）

正常極小曲面的模空間

計畫類別：☑個別型計畫　□整合型計畫

計畫編號：NSC 90－2115－M－002－011－

執行期間：90 年 8 月 1 日至 91 年 10 月 31 日

計畫主持人：王藹農

共同主持人：

計畫參與人員：

成果報告類型(依經費核定清單規定繳交)：□精簡報告　☑完整報告

本成果報告包括以下應繳交之附件：

□赴國外出差或研習心得報告一份

□赴大陸地區出差或研習心得報告一份

□出席國際學術會議心得報告及發表之論文各一份

□國際合作研究計畫國外研究報告書一份

處理方式：除產學合作研究計畫、提升產業技術及人才培育研究計畫、列管計畫及下列情形者外，得立即公開查詢

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執行單位：台大數學系

中　華　民　國　92 年　元 月　30　日

REPRESENTING RIEMANN'S MINIMAL SURFACE BY WEIERSTRASS ζ FUNCTION

Ai-Nung Wang

It is more desirable to represent Weierstrass data of a minimal surface by Weierstrass ζ function rather than by complex integration. For example, Alfred Gray [1] figured out that the Costa minimal surface is given explicitly by

$$x = \frac{1}{2}\Re\Big\{ -\zeta(u+iv) + \pi u + \frac{\pi^2}{4e_1} + \frac{\pi}{2e_1}[\zeta(u+iv-\frac{1}{2}) - \zeta(u+iv-\frac{1}{2}i)]\Big\}$$

$$y = \frac{1}{2}\Re\Big\{ -i\zeta(u+iv) + \pi v + \frac{\pi^2}{4e_1} - \frac{\pi}{2e_1}[i\zeta(u+iv-\frac{1}{2}) - i\zeta(u+iv-\frac{1}{2}i)]\Big\}$$

$$z = \frac{1}{4}\sqrt{2\pi}\log\left|\frac{\mathfrak{P}(u+iv)-e_1}{\mathfrak{P}(u+iv)+e_1}\right|$$

For Riemann's minimal surface parametrized by unit square,

$$g = z = \mathfrak{P}(u)$$

$$w = \mathfrak{P}' = \frac{dz}{du} = \frac{1}{\sqrt{z(1-z)(1+z)}}$$

$$\eta = \frac{du}{\mathfrak{P}} = \frac{1}{z}\frac{dz}{w}$$

therefore

$$x_1 = \Re\int \frac{1}{2}(1-g^2)\eta = \Re\int \frac{1}{2}\Big[\frac{1}{\mathfrak{P}(u)} - \mathfrak{P}(u)\Big]du$$

$$x_2 = \Re\int \frac{i}{2}(1+g^2)\eta = \Re\int \frac{i}{2}\Big[\frac{1}{\mathfrak{P}(u)} + \mathfrak{P}(u)\Big]du$$

$$x_3 = \Re\int g\eta = \Re u$$

To figure out $\int du/\mathfrak{P}$, we note

$$\int du/\mathfrak{P} = \int \frac{1}{z}\frac{dz}{w} = \int \frac{1}{z}\frac{dz}{\sqrt{z(1-z)(1+z)}}$$

let $z \to \frac{1}{z}$,

$$\int z\frac{-dz/z^2}{\sqrt{1/z(1-1/z)(1+1/z)}} = \int z\frac{-dz}{\sqrt{z(z-1)(z+1)}} = i\int z\frac{dz}{w}$$

that is the Weierstrass ζ function. Similar formulae can be derived for Toubiana's generalizations [1] of riemann minimal surface, which are also bounded by two straight lines but have self intersections.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

REFERENCES

[1] Gray (Ferguson et al. 1996, Gray 1997), *http://mathworld.wolfram.com/CostaMinimalSurface.html.*

[2] E. Toubiana, *On minimal surfaces of riemann.*, Comment. Math. Helvetici **67** (1992), 546-570.