

# 行政院國家科學委員會專題研究計畫 期中進度報告

## 圖論中之分解、覆蓋與包裝問題(2/3)

計畫類別：個別型計畫

計畫編號：NSC92-2115-M-002-015-

執行期間：92年08月01日至93年07月31日

執行單位：國立臺灣大學數學系暨研究所

計畫主持人：張鎮華

報告類型：精簡報告

報告附件：國外研究心得報告

出席國際會議研究心得報告及發表論文

處理方式：本計畫可公開查詢

中 華 民 國 93 年 5 月 26 日

**Midterm Report for the National Science Council Project**  
**Project Title: Decomposition, Covering and Packing in Graphs (2/3)**  
**Project Number: NSC 92-2115-M-002-015**  
**Project Duration: August 1, 2003 to July 31, 2004**  
**Project Investigator: Gerard Jennhwa Chang**  
**Department of Mathematics, National Taiwan University**  
**Report Date: May 26, 2004**

This is the second year of the whole project, which is for three years from August 1, 2002 to July 31, 2005. During this year, our results are on coloring, group testing and path partition of graphs. Five papers are finished and submitted to journals. Below are the list and the abstracts of these papers.

- [131] **G. J. Chang**, C. Lu, and S. Zhou, “Consecutive 2-distance colouring of Cayley graphs on abelian groups,” submitted. (NSC92-2115-M002-015) (Zhou-DM)
- [137] J. S.-t. Juan and **G. J. Chang**, “Adaptive group testing for consecutive positives,” (NSC92-2115-M002-015 and NCTS) (Juan-SIDAM044293-1)
- [139] J.-J. Pan and **G. J. Chang**, “Isometric-path numbers of graphs,” submitted. (NSC92-2115-M002-015) (DM12399)
- [146] J.-J. Pan and **G. J. Chang**, “Induced-path partition on graphs with special blocks,” submitted. (NSC92-2115-M002-015 and NCTS) (TCSA2374)
- [149] **G. J. Chang**, A note on equitable colorings of forests, submitted. (NSC92-2115-M002-015 and NCTS-2004-005) (JCTB5538)

**[131] Consecutive 2-distant colouring of Cayley graphs on abelian groups**

A consecutive 2-distant colouring of a graph  $\Gamma$  is an assignment  $c$  of non-negative integers to the vertices of  $\Gamma$  such that  $|c(v) - c(w)| \geq 2$  for each pair  $v, w$  of adjacent vertices, and the integers used are consecutive. Whenever such a colouring exists, define  $\text{csp}(\Gamma)$  to be the minimum difference (over all  $c$ ) between the largest and smallest integers used. Motivated by the channel assignment problem for cellular radio communication systems, in this paper we propose an approach to studying the consecutive 2-distant colouring problem for Cayley graphs over finitely generated

abelian groups. We give sufficient conditions for the existence of consecutive 2-distant colourings of such graphs, and obtain upper bounds for the minimum span  $\text{csp}(\Gamma)$ .

### [137] Adaptive group testing for consecutive positives

This paper studies adaptive group testing for consecutive positives. Suppose  $V_n = \{v_1, v_2, \dots, v_n\}$  is a linearly ordered set with  $v_1 \prec v_2 \prec \dots \prec v_n$ , in which each item has an associated *state* positive or negative. The set  $V_n$  is known to have the *d-consecutive positive property*, namely it contains at most  $d$  positive items which form a consecutive set under the order  $\prec$ . The goal of this paper is to determine the minimum number of test, denoted by  $M(C_{d,n})$  needed to identify these  $d$  consecutive positive items from  $V_n$  by using an adaptive group testing method. In particular, we give closed lower and upper bounds for  $M(C_{d,n})$ . Exact values of  $M(C_{2^s,n})$  and  $M(C_{3,n})$  are

### [139] Isometric path numbers of graphs

An isometric path between two vertices in a graph  $G$  is a shortest path joining them. The isometric path number of  $G$ , denoted by  $\text{ip}(G)$ , is the minimum number of isometric paths needed to cover all vertices of  $G$ . In this paper, we determine exact values of isometric path numbers of complete  $r$ -partite graphs and Cartesian products of 2 or 3 complete graphs.

### [146] Induced-path partition on graphs with special blocks

In a graph, an induced path is a path in which two vertices are adjacent only for those with consecutive indices. An induced-path partition of a graph is a collection of vertex-disjoint induced paths that cover all vertices of the graph. The induced-path-partition problem is to find the induced-number  $\rho(G)$  of a graph  $G$  which is the cardinality of an induced-path partition of  $G$ . The purpose of this paper is to present a linear-time algorithm for the induced-path-partition problem on graphs whose blocks are complete graphs, cycles or complete bipartite graphs.

### [149] A note on equitable colorings of forests

This note gives a short proof on characterizations of a forest to be equitably  $k$ -colorable.