行政院國家科學委員會專題研究計畫　成果報告

殘留應力彈性方程式的 Carleman 型估計及其應用(2/2)

計畫類別： 個別型計畫
計畫編號： NSC92-2115-M-002-016-
執行期間： 92 年 08 月 01 日至 93 年 07 月 31 日
執行單位： 國立臺灣大學數學系暨研究所

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報告類型： 完整報告
報告附件： 國外研究心得報告
處理方式： 本計畫可公開查詢

中　華　民　國 93 年 12 月 13 日

Report on "Carleman type estimates for the elasticity system with residual stress and their applications(92-2115-M-002-016)"

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This is a report on the NSC grant 92-2115-M-002-016. The main theme of this grant is to discuss the Carleman estimates of elasticity systems and its applications to the unique continuation property and inverse problems. We not only studied the residual stress model but also investigated general elasticity systems. Now I list several results benefited from this grant. I am grateful to the grant support from NSC.

1. Wang, Jenn-Nan (with Nakamura, Gen), Unique continuation for an elasticity system with residual stress and its applications, *SIAM J. Math. Anal.*, Vol. 35, No. 2 (2003), 304-317.

In this paper we prove the unique continuation property for an elasticity system with small residual stress. The constitutive equation of this elasticity system differs from that of the isotropic elasticity system by $T + (\nabla u)T$, where T is the residual stress tensor. It turns out this elasticity system becomes anisotropic due to the existence of residual stress T. The main technique in the proof is Carleman estimates. Having proved the unique continuation property, we will study the inverse problem of identifying the inclusion or cavity.

2. Wang, Jenn-Nan (with Nakamura, Gen), Construction of oscillating-decaying solutions for anisotropic elasticity systems, *The Chinese Journal of Mechanics, Series A*. A special issue dedicated to the Seventieth Birthday of Professor Thomas Chi Tsai Ting, Vol. 19, No. 1 (2003), 119-125.

In this paper, we present a framework of constructing oscillating-decaying solutions for the general inhomogeneous anisotropic elasticity system. These oscillating-decaying solutions can be used in solving inverse problems concerning the identification of cavities or inclusions embedded in an elastic body.

3. Wang, Jenn-Nan (with Isakov, Victor and Nakamura, Gen), Uniqueness and stability in the Cauchy problem for the elasticity system with residual stress, *Contemporary Mathematics* (Inverse Problems: Theory and Applications), Vol. 333 (2003), AMS, 99-113.

In this paper we prove the uniqueness and stability in the Cauchy problem for the elasticity system with small residual stress.

4. Wang, Jenn-Nan (with Lin, Ching-Lung), Uniqueness in inverse problems for an elasticity system with residual stress, *Inverse Problems*, Vol. 19 (2003), 807-820.

In this paper we consider an elasticity system with residual stress. The constitutive equation of this elasticity system differs from that of the isotropic elasticity system by $R+(\nabla u)R$, where R is the residual stress tensor. This system is not isotropic due to the existence of the residual stress R. Thus, it is not possible to reduce the principal part of the system to uncoupled wave operators as we have for the isotropic elasticity system. Here we investigate inverse problems of identifying the force term or the density by a single measurement of lateral boundary. We establish uniqueness results by Carleman estimates when the residual stress is small.

5. Wang, Jenn-Nan (with Nakamura, Gen and Uhlmann, Gunther), Reconstruction of cracks in an anisotropic elastic medium, *J. Math. Pures Appl.*, Vol. 82 (2003), 1251-1276.

In this paper we give in two and three dimensions a reconstruction formula for determining cracks buried in an inhomogeneous anisotropic elastic body by making elastic displacement and traction measurements at the boundary. The information is encoded in the local Neumann-to-Dirichlet map. With the help of the Runge property, the local Neumann-to-Dirichlet map is connected to the so-called indicator function. This function can be expressed as an energy integral involving some special solutions, called reflected solutions. The heart of our method lies in analyzing the blow-up behavior at the crack of the indicator function, which is by no means an easy task for the inhomogeneous anisotropic elasticity system. To overcome the difficulties, we construct suitable approximations of the reflected solutions that capture the singularities of the reflected solutions. The indicator function is then analyzed by the Plancherel formula.

6. Wang, Jenn-Nan (with Nakamura, Gen), The limiting absorption principle for the two-dimensional inhomogeneous anisotropic elasticity system, to appear in *Transactions of AMS*.

In this work we establish the limiting absorption principle for the two-dimensional steady-state elasticity system in an inhomogeneous anisotropic medium. We then use the limiting absorption principle to prove the existence of a radiation solution to the exterior Dirichlet or Neumann boundary value problems for such a system. In order to define the radiation solution, we need to impose certain appropriate radiation conditions at infinity. It should be remarked that even though in this paper we assume that the medium is homogeneous outside of a large domain, it still preserves anisotropy. Thus the classical Kupradze's radiation conditions for the isotropic system are not suitable in our problem and new radiation conditions are required. The uniqueness of the radiation solution plays a key role in establishing the limiting absorption principle. To prove the uniqueness of the radiation solution, we make

use of the unique continuation property, which was recently obtained by the authors. The study of this work is motivated by related inverse problems in the anisotropic elasticity system. The existence and uniqueness of the radiation solution are fundamental questions in the investigation of inverse problems.

7. Wang, Jenn-Nan (with Nakamura, Gen and Uhlmann, Gunther), Unique continuation property for elliptic systems and crack determination in anisotropic elasticity, to appear in *Contemporary Mathematics* (Partial Differential Equations and Inverse Problems), AMS.

We give a reconstruction procedure to determine cracks embedded in an inhomogeneous anisotropic elastic body by making traction and displacement measurements at the boundary. The procedure depends on the unique continuation property for the system of anisotropic elasticity. We describe some recent results in this direction.

8. Wang, Jenn-Nan (with Lin, Ching-Lung), Strong unique continuation for the Lamé system with Lipschitz coefficients, to appear in *Math. Annalen*.

In this paper we prove the strong unique continuation property for the Lamé system with Lipschitz coefficients in the plane. The proof relies on reducing the Lamé system to a first order elliptic system and suitable Carleman estimates.

9. Wang, Jenn-Nan (with Nakamura, Gen and Uhlmann, Gunther), Oscillating-decaying solutions, Runge approximation property for the anisotropic elasticity system and their applications to inverse problems, to appear in *J. Math. Pures Appl.*

In this work we construct oscillating-decaying solutions for the general inhomogeneous anisotropic elasticity system. We also prove the Runge approximation property for the inhomogeneous transversely isotropic elasticity system. We then apply the oscillating-decaying solutions and the Runge approximation property to the inverse problem of identifying an inclusion or a cavity embedded in an elastic body with transversely isotropic medium.

10. Wang, Jenn-Nan (with Lin, Wen-Wei), Partial pole assignment for the vibrating system with aerodynamic effect, *Numer. Linear Algebra. Appl.*, Vol. 11, No. 1 (2004), 41-58.

The Partial Pole Assignment (PPA) problem is the one of reassigning a few unwanted eigenvalues of a control system by feedback to suitably chosen ones, while keeping the remaining large number of eigenvalues unchanged. The problem naturally arises in modifying dynamical behavior of the system. The PPA has been considered by several authors in the past for standard state-space systems and for quadratic matrix polynomials associated with second-order systems. In this paper, we consider the PPA for a cubic matrix polynomial arising from modelling of a vibrating system with aerodynamics effects and derive explicit formulas for feedback matrices in terms of the coefficient matrices of the polynomial. Our results generalize those of a quadratic matrix polynomial by Datta, Elhay, and Ram and is based on some new orthogonality relations for eigenvectors of the cubic matrix polynomial.

Besides playing an important role in our solution for the PPA, these orthogonality relations are of independent interests, and believed to be an important contribution to linear algebra in its own right.