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Adaptive Backstepping Tracking Control of the Stewart Platform

Chin-I Huang[1], *student member*, *IEEE* and **Li-Chen Fu**[1,2], *Fellow*, *IEEE*
Department of Electrical Engineering[1]
Department of Computer Science and Information Engineering[2]
National Taiwan University, Taipei, Taiwan, R.O.C.
E-mail: lichen@csie.ntu.edu.tw

Abstract—**This paper presents an adaptive backstepping control approach for the motion control of a Stewart platform. The control scheme is proposed given that the overall system parameters are subject to uncertainties while only the positions and velocities of links are measurable. To achieve high performance tracking control of a 6 DOF Stewart platform normally requires the full knowledge of the system dynamics. In this paper, some important properties of the dynamics of the Stewart platform have been derived and exploited to develop an adaptive backstepping controller which can drive the motion tracking error to zero asymptotically. Stability analysis based on Lyapunov theory is performed to guarantee that the controller design is stable. Finally, the experimental results confirm the effectiveness of our control design.**

I. Introduction

In recent years, the parallel link manipulators have attracted much attention and many studies have been done on the kinematics or static analysis of the parallel link manipulators [12]. Generally speaking, the parallel link manipulators provide better accuracy, higher rigidity, higher load-to-weight ratio, and more uniform load distribution than the serial manipulators. Such advantages of fully parallel manipulators [1] originate from the fact that the actuators act in parallel sharing the common payload. The Stewart platform manipulator is a 6DOF mechanism with two bodies connected together by six extensible legs [1, 2]. This closed-loop structure makes the manipulator system far more rigid in proportion to size and weight than any serial link robot, and yields a force-output-to-manipulator-weight ratio more than one order of magnitude greater than serial link robot. Practical usage of the Stewart platform manipulator has generally been in the area of low speed and large payload conditions such as motion base of the classical automobile or flight simulator, and motion bed of a machine tool [4]. For the design and the control of the Stewart platform manipulators, dynamics analysis is a crucial step. In recent years, many research works have been conducted on the dynamics of the Gough–Stewart platform manipulator [3–11]. Several methods such as the Lagrange equation, Newton–Euler equation and principle of virtual work are proposed to derive dynamic equations of the Gough–Stewart platform. The Lagrange formulation is well structured and can be expressed in closed form, but a large amount of symbolic computation is needed to find partial derivatives of the Lagrangian in this method. The Newton–Euler approach requires computation of all constraint forces and moments between the links. However, these computations are not necessary for the simulation and control of a manipulator.

The method of virtual work is an efficient approach to derive dynamic equations for the inverse dynamics of the Stewart platform [8, 9]. However, for the forward dynamics, the method of virtual work is not straightforward because of the complicated velocity transform between the joint-space and task-space.

In this paper, an approach based on a sliding-mode control technique has been successfully developed for motion control of the Stewart platform system with having parametric uncertainty. These schemes are designed to guarantee practical robustness and stability. The remainder of this article is organized as follows: The kinematics and dynamics models for the Stewart platform are discussed in Section 2. In Section 3, 4 the Smooth Projection Algorithm and the adaptive backstepping controller for a Stewart platform system is developed and the stability analysis is conducted, respectively. Section 5 shows some experimental results on controlling a realistic Stewart platform. Finally, some conclusions are made in Section 6.

II. Kinematics and Dynamics of a Stewart Platform

The Stewart Platform is a parallel manipulator [1, 2]. It has a lower base platform and an upper payload platform connected by six extensible legs with ball joints at both ends. In the following subsections, we first make the inverse kinematics analysis of the Stewart platform, and then derive its dynamics.

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A. Inverse and Forward Kinematics Analysis of a Stewart Platform

The Inverse and Forward Kinematics Analysis of Stewart platform please refer to reference [17].

B. Dynamics Analysis of Stewart Platform

To design a system with high operational performance, a sound control method is crucially needed. However, to control the Stewart Platform system well is very challenging due to the high nonlinearity in system dynamics, system uncertainties, and complex kinematics. In general, the dynamic equations [4] of the Stewart Platform system can be written as:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=Kz_1 \tag{1a}$$

$$J_m\ddot{q}_m+B_m\dot{q}_m+Kz_1=\tau \tag{1b}$$

and $z_1=q_m-q$,

where $M(q)$ is an 6×6 inertia matrix, which is a symmetric and positive definite for all $q\in R^6$; $C(q,\dot{q})\dot{q}$ is the Coriolis/Centripetal vector; $G(q)$ and τ are 6×1 vectors containing gravity torques and input torques, respectively; $q_m\in R^6$ is generalized coordinates of the actuator subsystem; and Kz_1 are elastic force. We can see this system like a flexible joint manipulators system. Some pertaining properties are given below.

Property 1: $C(q,\dot{q})$ and $\dot{M}(q)$ are bounded functions if q and $\dot{q}$ are bounded. $\dot{C}(q,\dot{q})$ is a bounded function if q, $\dot{q}$ and $\ddot{q}$ are bounded.

Property 2: M is a symmetric and positive definite matrix. Moreover, for an appropriate choice of C, $\dot{M}-2C$ can be a skew-symmetric matrix, which means that $x^T(\dot{M}-2C)x=0,\ \forall x\in R^n$. This property is well known in the robotics literature.

III. Smooth Projection Algorithm

The smooth projection algorithms are studied or used in [14, 15]. For our application, a similar convex domain defined in Lozano and Brogliato [15] is considered and is approximated by a new convex set with a proper choice of a convex function. Then the smooth projection algorithm is appropriately defined such that the singularity of estimation will be avoided in the adaptive control and thus the backstepping design can proceed smoothly.

First, using the facts $\theta_{ki}\geq \underline{k}_i$ ($\underline{k}_i$ is the lower bound of stiffness coefficient), the convex domain of parameters θ_{ki}, θ_{d1}, and θ_{d2} will be defined in a general form. Let us define a subspace D spanned by the known function $d(q)$ as q ranges over R^n and expressed as

$$D=\left\{d\in R^h \mid d=d(q),\ \forall q\in R^n\right\}$$

in which the corresponding function $d(\cdot)$ of θ_{ki}, θ_{d1}, and θ_{d2} are 1, $d_1(q)$, and $d_2(q)$, respectively. The region of true parameters is defined in

$$\Pi=\left\{\theta\in R^h \mid d^T\theta\geq\underline{\theta},\ \forall d\in D\right\}$$

where θ is the concerned parameters, such as θ_{ki}, θ_{d1}, and θ_{d2}, with a known constant $\underline{\theta}$ is the lower bound of θ_{ki}. To avoid the singularity due to parametric estimations, the region of possible values of estimated parameters is defined by

$$\Pi_a=\left\{\hat{\theta}\in R^h \mid d^T\hat{\theta}\geq\underline{\theta}-\delta,\ \forall d\in D\right\}$$

where $\delta>0$ is a thickness about the neighborhood of $\underline{\theta}$ and satisfies $\underline{\theta}-\delta>0$. For the smooth projection of parameter estimation, the convex domain Π concerning $\hat{\theta}$ will be arbitrarily close to a new compact convex set constructed by

$$\Pi_\ell=\left\{\hat{\theta}\in R^h \mid \ell(\hat{\theta})\leq 1\right\}$$

where the smooth function $\ell(\cdot)$ from Π_a to R satisfies the following conditions:

(1) The true parameter vector θ satisfies that $\ell(\theta)\leq 0$, which means θ lies in Π and also be restricted in the interior of Π_ℓ.

(2) The set $\left\{\hat{\theta}\in R^h \mid \ell(\hat{\theta})\leq p\right\}$ is convex and contains Π for each real number p in $[0,1]$.

(3) The vector $p_\ell(\hat{\theta})\equiv \partial\ell/\partial\hat{\theta}(\hat{\theta})$ is nonzero for all $\hat{\theta}$ satisfying $\ell(\hat{\theta})\in[0,\ 1]$.

From the definition of a convex Π_ℓ, an example of $\ell(\cdot)$ is given as

$$\ell(\hat{\theta})=\frac{\underline{\theta}^\mu-(d^T\hat{\theta})^\mu}{\underline{\theta}^\mu-(\underline{\theta}-\delta)^\mu},\ \text{for}\ \mu\geq 2.$$

It also noted that $\ell(\hat{\theta})=1$ when $d^T\hat{\theta}=\underline{\theta}-\delta$. In this case, the defined convex set Π_ℓ is concerning as a union of the set Π and a boundary layer $O(\delta)$ around it.

The smooth projection algorithm is then constructed by

$$\text{Proj}(\hat{\theta},\ \psi)=\begin{cases}\psi & \text{if } \ell(\hat{\theta})\leq 0\\ \psi & \text{if } \ell(\hat{\theta})\geq 0 \text{ and } p_\ell^T\psi\leq 0\\ \left(I_h-\ell(\hat{\theta})\dfrac{p_\ell p_\ell^T}{\|p_\ell\|^2}\right)\psi & \text{otherwise}\end{cases}$$

with the initial condition $\hat{\theta}(0)$ chosen in Π_ℓ. This results the following useful properties:

(M1) $\text{Proj}(\hat{\theta},\ \psi)$ is Lipschitz continuous;

(M2) If $\psi(t)$ is continuously differentiable and $\dot{\hat{\theta}}=\text{Proj}(\hat{\theta},\ \psi)$, $\hat{\theta}(0)\in\Pi_\ell$, then the solution $\hat{\theta}(t)$

remains in Π_ℓ;

(M3) $\text{Proj}(\hat{\theta},\ \psi)^T \text{Proj}(\hat{\theta},\ \psi) \le \psi^T \psi$;

(M4) $-(\theta-\hat{\theta})^T \text{Proj}(\hat{\theta},\ \psi) \le -(\theta-\hat{\theta})^T \psi$, $\forall\ \theta \in \Pi,\ \hat{\theta} \in \Pi_\ell$.

These properties will be used later in the stability analysis.

IV. Adaptive Backstepping Control for Stewart Platform

In this section, a two-stage approach, or namely the integrator backstepping approach is applied to design the adaptive controller.

First stage: For the rigid manipulator subsystem, design a virtual control input z_{1d} such that $q \to q_d$ and $\dot{q} \to \dot{q}_d$ as $t \to \infty$ where $q_d(t)$ is the desired trajectory.

Second stage: Design the actual control input τ to drive the variable z_1 converges to z_{1d}.

All the system parameters are unknown in the design phase, whereas only positions and velocities of the manipulator are measurable.

A. Design a Virtual Control Input z_{1d} for the First Stage

Define $e = q - q_d$ and $q_r = \dot{q}_d - \Lambda e$, where $e,\ q_r$ denote the motion error and an auxiliary signal vector, respectively. In addition, define an exponentially stable manifold s be the measurement of motion tracking error, denoted by $s = \dot{e} + \Lambda e = \dot{q} - q_r$. If the error measure s is driven to zero, then the position and velocity tracking error will both converge to zero in an exponential behavior. To this end, the error dynamics in terms of the error measure is written as:

$$\begin{aligned} M\dot{s} &= M\ddot{q} - M\dot{q}_r \\ &= \tilde{K}z_1 - Cs + \hat{K}\tilde{z}_1 - Y(\cdot)\theta_r + \hat{K}z_{1d} \end{aligned} \tag{2}$$

where $Y(\cdot) = M\dot{q}_r + Cq_r + G$; $\hat{K} = diag\{\hat{\theta}_{ki}\}$ and $\tilde{K} = K - \hat{K}$ are the estimated flexibility constants and its error, respectively; and $\tilde{z}_1 = z_1 - z_{1d}$ where z_{1d} is the virtual control input. The virtual control law z_{1d} is set as

$$z_{1d} = \hat{K}^{-1}\left(Y_d\hat{\theta}_r - \left((K_s + \|e\|^2 K_{sp})s + K_p e\right)\right), \tag{3}$$

where the control gains $K_s,\ K_{sp},\ K_p$ are positive-definite matrix. Then substituting (3) into (2) results in the error dynamics in the form:

$$M\dot{s} + Cs + (K_s + \|e\|^2 K_{sp})s + K_p e = -\bar{Y}_D\tilde{\theta}_{rk} - \Delta Y\theta_r + \hat{K}\tilde{z}_1. \tag{4}$$

In above equation, we have some notions as $\tilde{\theta}_{rk}^T \equiv \begin{bmatrix}\tilde{\theta}_r^{\ T} & \tilde{\theta}_k^{\ T}\end{bmatrix}$ with $\tilde{\theta}_r = \theta_r - \hat{\theta}_r$ and $\tilde{\theta}_k = \theta_k - \hat{\theta}_k$, $\bar{Y}_D \equiv \begin{bmatrix}Y_d(\cdot) & -D_z(\cdot)\end{bmatrix}$, $\Delta Y \equiv Y(\cdot) - Y_d(\cdot)$.

Based on a similar argument in [Sadegh and Horowitz, 16], the compensation matrix $\Delta Y(\cdot)$ is proven to be bounded above through the following inequality:

$$\|\Delta Y\theta_r\| \le \rho_1\|s\| + \rho_2\|s\|\|e\| + \rho_3\|e\| + \rho_4\|e\|^2 \tag{5}$$

where $\rho_i,\ i = 1,\cdots,4$ are positive constants which depend on the desired trajectories, i.e., q_d, $\dot{q}_d$, $\ddot{q}_d$, the upper bounds of system parameters, and control gain Λ.

To select the adaptation laws for $\hat{\theta}_r$ and $\hat{\theta}_k$, let us consider a Lyapunov function candidate:

$$V_1 = \tfrac{1}{2}s^T M s + \tfrac{1}{2}e^T K_p e + \tfrac{1}{2}\tilde{\theta}_r^T\Gamma_r^{-1}\tilde{\theta}_r + \tfrac{1}{2}\tilde{\theta}_k^T\Gamma_k^{-1}\tilde{\theta}_k,$$

where Γ_r and Γ_k are symmetric positive-definite matrices. The time derivative of V_1 along the error dynamics (4) is

$$\begin{aligned} \dot{V}_1 = &-s^T K_s s - \|e\|^2 s^T K_{sp} s - s^T K_p e + e^T K_p \dot{e} + \tilde{\theta}_r^T\left(\Gamma_r^{-1}\dot{\tilde{\theta}}_r + Y_d^T s\right) \\ &+ \tilde{\theta}_k^T\left(\Gamma_k^{-1}\dot{\tilde{\theta}}_k + D_z s\right) - s^T\Delta Y\theta_r + s^T\hat{K}\tilde{z}_1. \end{aligned}$$

Then the update law of $\hat{\theta}_r$ is chosen as

$$\dot{\hat{\theta}}_r = -\Gamma_r Y_d^T s. \tag{6a}$$

According that K is a diagonal matrix with strictly positive entries and $\hat{K}$ is required to be invertable, i.e., $\hat{\theta}_{ki} \ne 0$, for every $t \ge 0$, the adaptive law of $\hat{\theta}_k$ is set by using the smooth projection:

$$\dot{\hat{\theta}}_{ki} = \text{Proj}\left(\hat{\theta}_{ki},\ \left(\Gamma_k D_z s\right)_i\right), \quad \text{with } \hat{\theta}_{ki}(0) \in \Pi_{ki} \tag{6b}$$

where the smooth projection algorithm, $\text{Proj}\left(\hat{\theta}_{ki},\ \omega_i\right)$ with $\omega = \Gamma_k D_z s$, is given by the corresponding proper convex function $\ell_{ki}\left(\hat{\theta}_{ki}\right)$ in Section 3, and the initial condition $\hat{\theta}_{ki}(0)$ are set in the interior of Π_{ki}. The property (M4) of a smooth projection for (6b) can be presented as

$$\begin{aligned} \tilde{\theta}_k^T\left(\Gamma_k^{-1}\dot{\tilde{\theta}}_k + D_z s\right) &= -\sum_{i=1}^{n}\Gamma_{ki}^{-1}\tilde{\theta}_{ki}\left(\dot{\hat{\theta}}_{ki} - \omega_i\right) \\ &= -\sum_{i=1}^{n}\Gamma_{ki}^{-1}\tilde{\theta}_{ki}\left(\text{Proj}(\hat{\theta}_{ki},\omega_i) - \omega_i\right) \le 0, \end{aligned} \tag{7}$$

Consequently, $\dot{V}_1$ is reduced to

$$\dot{V}_1 = -s^T K_s s - \|e\|^2 s^T K_{sp} s - e^T\Lambda K_p e - s^T\Delta Y\theta_r + s^T\hat{K}\tilde{z}_1. \tag{8}$$

Since the compensation matrix $\Delta Y(\cdot)$ satisfies (5), the time derivative of V_1 in (8) leads to

$$\begin{aligned} \dot{V}_1 &\le -\underline{k}_s\|s\|^2 - \underline{k}_{sp}\|e\|^2\|s\|^2 - \underline{k}_p\|e\|^2 + \rho_1\|s\|^2 + \rho_2\|s\|^2\|e\| + \rho_3\|e\|\|s\| + \rho_4\|s\|\|e\|^2 + s^T\hat{K}\tilde{z}_1 \\ &\le -(\underline{k}_s - \rho_1)\|s\|^2 - \underline{k}_{sp}\|e\|^2\|s\|^2 - \underline{k}_p\|e\|^2 + \rho_2\|s\|^2\|e\| + \rho_3\|e\|\|s\| + s^T\hat{K}\tilde{z}_1 \\ &\quad -\rho_4\|e\|^2\left(\tfrac{1}{2} - \|s\|\right)^2 + \tfrac{1}{4}\rho_4\|e\|^2 + \rho_4\|e\|^2\|s\|^2 \\ &\le -x^T Q x + s^T\hat{K}\tilde{z}_1 \end{aligned} \tag{12}$$

where $\underline{k}_s$, $\underline{k}_{sp}$, and $\underline{k}_p$ are the minimum norm of K_s, K_{sp}, and ΛK_p, respectively, and

$$x = \begin{bmatrix}\|s\| \\ \|e\| \\ \|e\|\|s\|\end{bmatrix} \quad Q = \begin{bmatrix}\underline{k}_s - \rho_1 & \tfrac{1}{2}\rho_3 & \tfrac{-1}{2}\rho_2 \\ \tfrac{-1}{2}\rho_3 & \underline{k}_p - \tfrac{1}{4}\rho_4 & 0 \\ \tfrac{-1}{2}\rho_2 & 0 & \underline{k}_{sp} - \rho_4\end{bmatrix} \equiv \begin{bmatrix}Q_{11} & Q_{12} \\ Q_{21}^T & Q_{22}\end{bmatrix}.$$

If the control gain can be suitably chosen such that $Q_{11} > 0$ and $Q_{22} - Q_{12}Q_{11}^{-1}Q_{12}^T > 0$, i.e., matrix Q is positive-definite, then e and s are uniformly asymptotically stable once $\tilde{z}_1$ is steered to zero. Hence the first stage is completed. The control problem has been transferred into an equivalent control problem for stabilization of $\tilde{z}_1$.

B. Regulate z_1 Converge to z_{1d} for the Second Stage

To stabilize $\tilde{z}_1$, we differentiate $\tilde{z}_1$ to yield

$$\dot{\tilde{z}}_1 = \dot{z}_1 - \dot{z}_{1d}, \tag{10}$$

where z_{1d} is defined in (3). Since the virtual control input z_{1d} is composed of the integral positions and velocities of links, its time derivative $\dot{z}_{1d}$ depends not only on q and $\dot{q}$, but also $\ddot{q}$. To implement $\dot{z}_{1d}$ using only measurable quantities, such as positions and velocities, the right-hand side of equation $\ddot{q} = M^{-1}(Kz_1 - C(q,\dot{q})\dot{q} - G(q))$ is substituted to $\ddot{q}$ appearing in $\dot{z}_{1d}$. Hence it become possible to avoid using the link acceleration. It is also noted that the desired trajectory compensation $Y_d(\cdot)$ is introduced in z_{1d}.

The first and the second time derivative of $Y_d(\cdot)$ are all exactly-known and is beneficial to the replacement of $\ddot{q}$. Since $\det(M)$ appears in the denominator, care must be taken to express $\dot{z}_{1d}$ as an LP form. We multiply both sides of (10) by $\det(M)$ and obtain

$$\det(M)\dot{\tilde{z}}_1 = \det(M)\dot{z}_1 - \det(M)\dot{z}_{1d}. \tag{11}$$

This ensures that $\det(M)\dot{z}_{1d}$ is an LP form and depends on reasonable measurements q, $\dot{q}$, and z_1.

Since the actual control input does not appear in (11), the integrator backstepping procedure continues to introduce a new virtual control input z_{2d} and define

$$\dot{z}_1 = \tilde{z}_2 + z_{2d}. \tag{12}$$

In light of stability analysis using Lyapunov method, adding and subtracting $\frac{1}{2}(\frac{d}{dt}\det(M))\tilde{z}_1$ to the right-hand side of (11), the error dynamics of $\tilde{z}_1$ in (11) is represented as follows

$$\det(M)\dot{\tilde{z}}_1 = (d_1^T\theta_{d1})z_{2d} - \tfrac{1}{2}(\tfrac{d}{dt}\det(M))\tilde{z}_1 \tag{13}$$

where $Y_{z1}\theta_{z1} = \det(M)\dot{z}_{1d} - \frac{1}{2}(\frac{d}{dt}\det(M))\tilde{z}_1$ and $Y_{z1}(\cdot)$ is a known function of q, $\dot{q}$, and z_1; θ_{z1} is an unknown parametric vector with appropriate dimension. Therefore, the virtual control law z_{2d} is designed in the form:

$$z_{2d} = (d_1^T\hat{\theta}_{d1})^{-1}(Y_{z1}\hat{\theta}_{z1} - K_{z1}\tilde{z}_1 - \hat{K}s) \tag{14}$$

with the estimated parameters $\hat{\theta}_{d1}$ and $\hat{\theta}_{z1}$; the positive definite control matrix gains K_{z1}; and an additional term $\hat{K}s$ for the purpose to eliminate the cross-term that appeared in (9). Let $\tilde{\theta}_{d1} = \theta_{d1} - \hat{\theta}_{d1}$, $\tilde{\theta}_{z1} = \theta_{z1} - \hat{\theta}_{z1}$ denote the estimated parameter errors. The error dynamics (13) becomes

$$\det(M)\dot{\tilde{z}}_1 = (d_1^T\tilde{\theta}_{d1})z_{2d} - (\tfrac{1}{2}(\tfrac{d}{dt}\det(M))I_n + K_{z1})\tilde{z}_1 - Y_{z1}\tilde{\theta}_{z1} - \hat{K}s + \det(M)\tilde{z}_2. \tag{15}$$

To select update laws of $\hat{\theta}_{d1}$ and $\hat{\theta}_{z1}$, let us consider a Lyapunov-like function

$$V_2 = V_1 + \tfrac{1}{2}\det(M)\tilde{z}_1^T\tilde{z}_1 + \tfrac{1}{2}\tilde{\theta}_{d1}^T\Gamma_{d1}^{-1}\tilde{\theta}_{d1} + \tfrac{1}{2}\tilde{\theta}_{z1}^T\Gamma_{z1}^{-1}\tilde{\theta}_{z1} \tag{16}$$

where Γ_{d1}, Γ_{z1} are positive-definite symmetric matrices. From the error dynamics of $\tilde{z}_1$ in (15), the time derivative of V_2 is

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \det(M)\tilde{z}_1^T\dot{\tilde{z}}_1 + \tfrac{1}{2}(\tfrac{d}{dt}\det(M))\tilde{z}_1^T\tilde{z}_1 + \tilde{\theta}_{d1}^T\Gamma_{d1}^{-1}\dot{\tilde{\theta}}_{d1} + \tilde{\theta}_{z1}^T\Gamma_{z1}^{-1}\dot{\tilde{\theta}}_{z1} \\ &\le -x_1^TQx_1 - \tilde{z}_1^TK_{z1}\tilde{z}_1 + \tilde{\theta}_{d1}^T(\Gamma_{d1}^{-1}\dot{\tilde{\theta}}_{d1} + \tilde{z}_1^Tz_{1d}d_1) + \tilde{\theta}_{z1}^T(\Gamma_{z1}^{-1}\dot{\tilde{\theta}}_{z1} - Y_{z1}^T\tilde{z}_1) \\ &\quad + \det(M)\tilde{z}_1^T\tilde{z}_2\end{aligned}$$

The update laws is then given by

$$\dot{\hat{\theta}}_{z1} = -\Gamma_{z1}Y_{z1}^T\tilde{z}_1 \tag{17a}$$

$$\dot{\hat{\theta}}_{d1} = \mathrm{Proj}(\hat{\theta}_{d1}, \chi_1),\ with\ \chi_1 = \tilde{z}_1^Tz_{2d}\Gamma_{d1}d_1, \hat{\theta}_{d1}(0) \in \Pi_{\chi 1} \tag{17b}$$

where $\Pi_{\chi 1}$ is a convex set for a chosen convex function of $\hat{\theta}_{d1}$ defined in Section 3.

According to the smooth projection (17b), $\hat{\theta}_{d1}$ satisfies

$$-\tilde{\theta}_{d1}^T\Gamma_{d1}^{-1}(\dot{\hat{\theta}}_{d1} - \chi_1) = -\tilde{\theta}_{d1}^T\Gamma_{d1}^{-1}(\mathrm{Proj}(\hat{\theta}_{d1}, \chi_1) - \chi_1) \le 0. \tag{18}$$

This yields that $\dot{V}_2$ hold the condition

$$\dot{V}_2 \le -x_2^T\begin{bmatrix} Q & 0 \\ 0 & K_{z_1}\end{bmatrix}x_2 + \det(M)\tilde{z}_1^T\tilde{z}_2 \tag{19}$$

with $x_2^T = \begin{bmatrix} x^T & \tilde{z}_1^T \end{bmatrix}$.

Thus, the error signal $\tilde{z}_2$ is required to be stabilized such that $\dot{V}_2 \le 0$.

C. Design the Control τ (Continuing the Second Stage)

Differentiating $\tilde{z}_2$ leads to

$$\dot{\tilde{z}}_2 = \ddot{z}_1 - \dot{z}_{2d} = J_m^{-1}(\tau - Kz_1 - B_m\dot{q}_m) - (\dot{q} + \ddot{q}) - \dot{z}_{2d} \tag{20}$$

To stabilize $\tilde{z}_2$, the time derivative of z_{2d} is investigated as a function of $\ddot{z}_{1d}$, $\dot{s}$, $\dot{\tilde{z}}_1$, and $\dot{\hat{\theta}}_{d1}$. Since the smooth projection is applied in the update law of $\hat{K}$, the second derivative z_{1d} is well defined. Similar to the argument of stabilization of $\tilde{z}_1$, pre-multiplying $\tilde{z}_2$ by $\det(M)$, which results that the parametric uncertainties with LP property can be compensated by using measurable signals. In light of this, the new strategy for linear parameterized is to multiply both sides of (20) by $(\det(M))^2J_m$ and yield

$$(\det(M))^2J_m\dot{\tilde{z}}_2 = (d_2^T\theta_{d2})\tau - \tfrac{1}{2}(\tfrac{d}{dt}(\det(M))^2)J_m\tilde{z}_2 - \det(M)\tilde{z}_1 - Y_{z2}\theta_{z2}$$

with

$$Y_{z2}\theta_{z2} = (det(M))^2(Kz_1 + B_m\dot{q}_m + J_m\dot{q} + J_m\dot{z}_{2d}) - (\tfrac{1}{2}(\tfrac{d}{dt}(\det(M))^2)J_m\tilde{z}_2$$
$$+\det(M)(J_m M_a(Kz_1 - C\dot{q} - G) - \tilde{z}_1)$$

Meanwhile, we have added and subtracted $\frac{1}{2}(\frac{d}{dt}(\det(M))^2)J_m\tilde{z}_2$ and $\det(M)\tilde{z}_1$ for eliminating cross-term based on the same approach for stabilization of $\tilde{z}_1$. Accordingly, the actual control law is set as follows

$$\tau = (d_2^T\hat{\theta}_{d2})^{-1}(Y_{z2}\hat{\theta}_{z2} - K_{z2}\tilde{z}_2) \quad (21)$$

and the estimated parameters are turned by

$$\dot{\hat{\theta}}_{z2} = -\Gamma_{z2}Y_{z2}^T\tilde{z}_2 \quad (22a)$$

$$\dot{\hat{\theta}}_{d2} = \text{Proj}(\hat{\theta}_{d2}, \chi_2),\ with\ \chi_2 = \Gamma_{d2}\tilde{z}_2^T\tau d_2, \hat{\theta}_{d2}(0) \in \Pi_{\chi 2} \quad (22b)$$

which result in the error dynamics

$$(\det(M))^2 J_m\dot{\tilde{z}}_2 = (d_2^T\theta_{d2})\tau - (\tfrac{1}{2}(\tfrac{d}{dt}(\det(M))^2)J_m + K_{z2})\tilde{z}_2 - \det(M)\tilde{z}_1 - Y_{z2}\tilde{\theta}_{z2}. \quad (23)$$

Consider the finial Lyapunov candidate function as

$$V = V_2 + \tfrac{1}{2}(\det(M))^2\tilde{z}_2^T J_m\tilde{z}_2 + \tfrac{1}{2}\tilde{\theta}_{d2}^T\Gamma_{d2}^{-1}\tilde{\theta}_{d2} + \tfrac{1}{2}\tilde{\theta}_{z2}^T\Gamma_{z2}^{-1}\tilde{\theta}_{z2}\ .(24)$$

After differentiating above equation with respect to time along the error dynamics (23) and accompanying with the results in (19), it results in

$$\dot{V} \le -\underline{e}^T P\underline{e} \quad (25)$$

with $\underline{e}^T = \begin{bmatrix} x^T & \tilde{z}_1^T & \tilde{z}_2^T \end{bmatrix}$ and $P = diag\{Q, K_{z1}, K_{z2}\}$.

In the derivation, the property (M4) of smooth projection (22b) is used again like the presentation in (18). Since (25) is without containing perturbation terms, it means that the integrator backstepping procedure is completely accomplished. Thus the stability results of overall tracking systems are in turn to be addressed.

Theorem : Consider the Stewart Platform system (1a) with actuator dynamics (1b). Using the control law (21) with virtual control inputs designed in (7) and (14) and the update laws defined in (6), (20), and (22), the following statement will be truly satisfied provided that the control gains are proper chosen.

(A1) All signals in the overall dynamic systems are uniformly bounded.

(A2) $q(t), \dot{q}(t)$ and $\ddot{q}(t)$ will asymptotically track $q_d(t), \dot{q}_d(t)$ and $\ddot{q}_d(t)$ as $t \to \infty$.

V. EXPERIMENTAL RESULTS

In this section, we make a series experiment on the adaptive controller design which is proposed in Section 4.

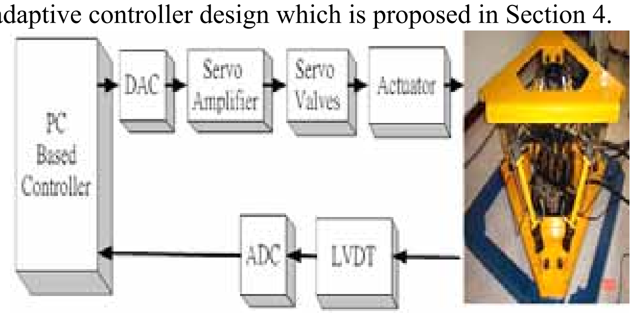

Fig. 1 The Experimental System Configuration Diagram

Figure 1 show the experimental system configuration of the Stewart platform system in this study. The motion control system computer runs a drive logic to control a hydraulic system that drive a 6 degree-of-freedom Stewart platform for creating realistic motion cue. The experiment are done with 6 DOF hydraulic Stewart platform system, and is manufactured by SGD Co.. The hydraulic servo valve in the Stewart platform uses MOOG's J076-104. The D/A card uses Adventech's 726 and A/D card uses Adventech's 818H. The Detailed parameters and specification will be found in Table 1.

Table 1: Specification and parameter of the Stewart platform

Motion	
Degree Of Freedom	6(Heave,Sway,Surge,Pitch,Roll,Yaw)
Net Loading	500kg
Accelebration	$\pm 1g$
Angular Acceleration	$\pm 60\circ/\sec^2$
Control	Servo Class Hydraulic Actuating System
Heave	$0mm \sim 221.518mm$
Sway	$\pm 211.198mm$
Surge	$\pm 244.758mm$
Pitch	$\pm 12.960^\circ$
Roll	$\pm 10.821^\circ$
Yaw	$\pm 18.474^\circ$
Motion Platform	
Dimension(L x W x H)	$1350x1200x760mm$
Net Weight	600kg
Hydraulic System	
Dimension(L x W x H)	$1250x1250x690mm$
Net Weight	800 kg
Electricity Power	$380V/660V/3\phi/50/60Hz$
Rated Motor Power	7.5HP
Rated (Max.) Operation Pressure	4.5(10)Mpa
System Flow Rate	65 $liter$/min
Oil Type	ISO VG46
Oil Operation Temperature	$10^\circ \sim 50^\circ C$
Cooler	
Dimension (L x W x H)	$550x450x470mm$
Cooling Capacity	6520 cal/hr @$35^\circ C$

A. Results of Adaptive Backstepping Controller Design for circular motion tracking

The desired trajectories for circular motion tracking case are shown in Table 2.

Table 2 Circular Motion Trajectories

Heave	0-10-0 cm
Sway	10sin(ft); f=rad/sec
Surge	10cos(ft); f=rad/sec
Pitch	0 degree
Roll	0 degree

Yaw	0 degree

The desired circular trajectories on x-y plant are shown in Fig 2. The desired heave, pitch, roll and yaw trajectories are shown in Fig.3. The every desired and real trajectory of the leg is shown in Fig. 4-9.

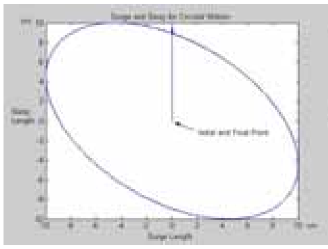

Fig. 2 Desired Circular Trajectory

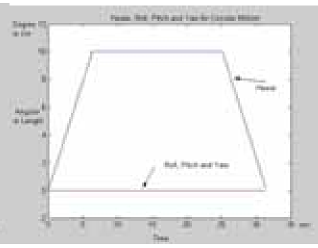

Fig. 3 Desired z,α,β,γ Trajectory

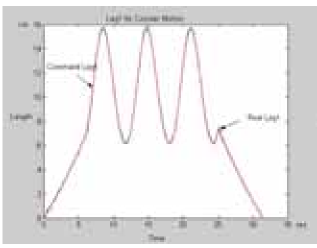

Fig. 4 The Leg1 Trajectories

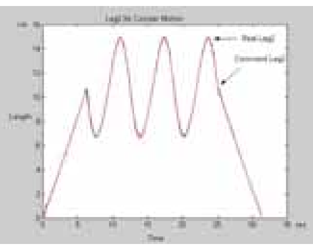

Fig. 5 The Leg2 Trajectories

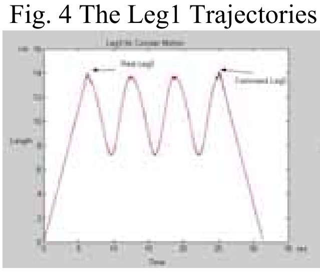

Fig. 6 The Leg3 Trajectories

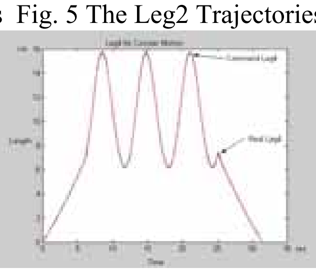

Fig. 7 The Leg4 Trajectories

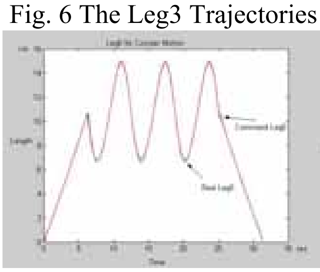

Fig. 8 The Leg5 Trajectories

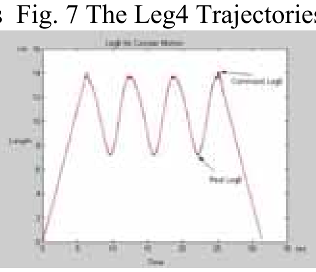

Fig. 9 The Leg6 Trajectories

VI. CONCLUSIONS

In this paper we present an adaptive backstepping control approach for the motion control of a Stewart platform. The control scheme is proposed given that the overall system parameters are subject to uncertainties while only the positions and velocities of links are measurable. To achieve high performance tracking control of a 6 DOF Stewart platform normally requires the full knowledge of the system dynamics. In this paper, some important properties of the dynamics of the Stewart platform have been derived and exploited to develop an adaptive backstepping controller which can drive the motion tracking error to zero asymptotically. Stability analysis based on Lyapunov theory is performed to guarantee that the controller design is stable. Finally, the experimental results confirm the effectiveness of our control design.

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