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Soap Bubble on a Funnel

by

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In this article we indicate a one dimensional analogue of a soap bubble moving on a funnel like surface and propose a mathematical formulation for the original two dimensional problem. A close soap bubble in $\mathbf{R^3}$ tends to minimize its area due to its surface tension, while the enclosed volume is kept constant. A soap bubble with its boundary moving on a fixed surface is further assumed to intersect the fixed surface perpendicularly. In both cases, the evolution equation for the soap bubble is the normalized mean curvature motion.

Let X be a point on the soap bubble, ν be the inward unit normal. If h is the mean curvature and $\bar{h} = \int h/area$, then the motion is described by the equation

$$\frac{\partial}{\partial t}X = (h - \bar{h})\nu$$

Let H be the mean curvature of the fixed surface. If the gradient ∇H is nowhere zero, it is intuitively clear that the soap bubble will move in the gradient direction to larger H region. Mathematically we would formulate this as a conjecture:

$$\oint \nabla H \cdot (h - \bar{h})\nu > 0$$

Though we are not able to prove this inequality, its 1-dimensional analogue can be shown to be true under certain assumption. First we observe

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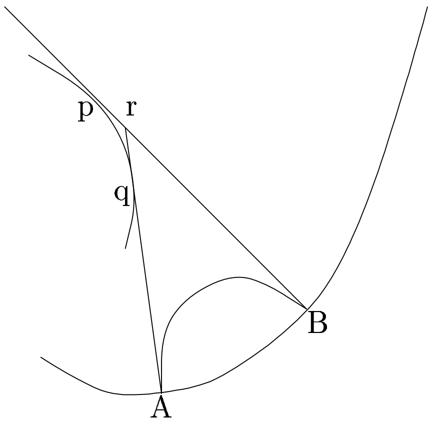
that if the curvature K of the fixed curve is decreasing in the following picture, it is impossible to find a circular arc AB intersecting it perpendicularly. Because if the center of curvature at A is q, and that at B is p, then $\overline{Ar} = \overline{Br}$ leads to a contradiction.



If we assume the curvature κ of the bubble is monotone, we claim $\kappa(A) < \kappa(B)$ and the bubble moves to the left. Indeed, by the above consideration, $\overline{Ar} > \overline{Br}$. If we set the coordinate origin at A and x-axis in the AB direction, then unit tangent $T = (x', y')$ and normal $N = (y', -x')$. By Frenet formula $N' = -\kappa T$, $x" = \kappa y'$, integration by parts gives

$$0 > x'(B) - x'(A) = \int_A^B x" = \int_A^B \kappa y' = -\int_A^B \kappa' y$$

Therefore κ must be increasing.