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代數纖業上的雙有理幾何(1/3)

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一、中文摘要

我們利用之前的研究成果刻劃了所有 $P_3=4, q=\dim X$ 的代數流形。

關鍵詞：線性系、重正典映射、傅利葉-向井變換、非正規流形、可交換流形。

Abstract

We give a complete characterization of complex projective varieties of $P_3=4$, $q=\dim X$.

Keywords: linear series, pluricanonical map
Fourier-Mukai transform, irregular varieties, abelian variety.

二、緣由與目的

Let X be a smooth complex projective variety. It is very hard to classify such varieties in terms of their birational invariants if the dimension of X if greater or equal than three.

Surprisingly, when X has non-zero holomorphic 1-forms, i.e. an irregular varieties, it is sometimes possible to achieve classification results in any dimension. In [Ka], Kawamata showed that: If X is a smooth complex projective variety with $\kappa(X)=0$ and $q(X)=\dim(X)$ then X is birational to an abelian variety. Subsequently, Kollár proved an effective version of this result (cf. [Ko2]): If X is a smooth complex projective variety with $P_m(X)=1$ for some $m\geq 4$, then the Albanese morphism alb :X→ Alb (X) is surjective. If moreover, $q(X)=\dim(X)$, then X is birational to an abelian variety.

These results where further refined and expanded as follows:

Thorem 1.(cf. [CH1], [CH3], [HP], [Hac2])

If $P_m(X)=1$ for some $m\geq 2$ or if $P_3(X)\leq 3$, then the Albanese morphism alb :X→ Alb (X) is surjective.

If moreover $q(X)=\dim(X)$, then if $P_m(X)=1$ for some $m\geq 2$, then X is birational to an abelian variety.

If $P_3(X)=2$, then $\kappa(X)=1$ and X is a double cover of its Albanese variety.

If $P_3(X)=3$, then $\kappa(X)=1$ and X is a bi-double cover of its Albanese variety.

三、結論與討論

In this project, we prove a similar result for varieties with $P_3(X)=4$ and $q(X)=\dim(X)$. We start by considering the following examples:

Example 1.

Let G be a group acting faithfully on a curve C and acting faithfully by translations on an abelian variety K, so that C/G=E is an elliptic curve and $\dim H^0(C, \omega^3\})^G=4$.

Let G act diagonally on $K\times C$, then $X:=K\times C/G$ is a smooth projective variety with $\kappa(X)=1$, $P_3(X)=4$ and $q(X)=\dim(X)$.

Example 2.

Let $q: A \rightarrow S$ be a surjective morphism with connected fibers from an abelian variety of dimension ngeq 3 to an abelian surface.

Let L be an ample line bundle on S with $h^0(S,L)=1$, chooise appropriate $P\in \mathrm{Pic}^0(A)$ with $P\notin$ notin $\mathrm{Pic}^0(S)$ and $P^2\in \mathrm{Pic}^0(S)$.

For D an appropriate reduced divisor in $|2L+2P|$, there is a degree 2 cover alb :X→ Alb. One sees that $P_i(X)=1,4,4$ for $i=1,2,3$.

Example 3.

Let $q:\mathrm{Alb}\rightarrow E_1\times E_2$ be a surjective morphism from an abelian variety to the product of two elliptic curves, $P,Q\in \mathrm{Pic}^0(\mathrm{Alb})$ such that P,Q generate a subgroup of $\mathrm{Pic}^0(\mathrm{Alb})/\mathrm{Pic}^0(E_1\times E_2)$ which is isomorphic to $(\mathbb{Z}_2)^2$. Then one has double covers $X_i\rightarrow$ Alb .

The corresponding bi-double cover satisfies that $P_i(X)=1,4,4$ for $i=1,2,3$.

We prove the following:

Theorem 2.

Let X be a smooth complex projective variety with $P_3(X)=4$, then the Albanese morphism alb :X → Alb is surjective (in particular $q(X)\leq \dim(X)$).

If moreover, $q(X)=\dim(X)$, then $\kappa(X)\leq 2$ and we have the following cases:

If $\kappa(X)=2$, then X is birational either to a double cover or to a bi-double cover of Alb as in Examples 2 and 3.

If κ (X)=1, then X is birational to the quotient (K×C)/G where C is a curve, K is an abelian variety, G acts faithfully on C and tilde{K}. One has that either $P_2(X)=2$ and C→ C/G is branched along 2 points or $P_2(X)=4$ and C→C/G is branched along 4 points. See Example 1.

The main technique involve here is the Fourier-Mukai transform (cf. [M]), geometry of cohomological support loci (cf. [Si]), generic vanishing theoprem (cf. [EL], [CH2]), and also Kawamata-Viehweg vanishing theorems.

四、參考文獻

[CH1] Chen, Jungkai A.; Hacon, Christohper D., *Characterization of abelian varieties*. Invent. Math. **143** (2001), 435-447.

[CH2] Chen, Jungkai A.; Hacon, Christohper D., *Pluricanonical maps of varieties of maximal Albanese dimension.* Math. Ann. **320** (2001), 367-380.

[CH3] Chen, Jungkai A.; Hacon, Christohper D., *On algebraic fiber spaces over varieties with maximal Albanese dimension*, Duke Math J. **111**(2002), 159-175.

[EL] L. Ein, R. Lazarsfeld, *Singularities of theta divisors, and birational geometry of irregular varieties,* Jour. AMS **10**, (1997), 243-258.

[Hac2] C. Hacon, *Varieties with P_3=3 and q=dim (X)*, to appear in Math. Nach.

[HP] Hacon, C., D., Pardini, R., *On the birational geometry of varieties of maximal albanese dimension*, Jour. Reine Angew. Math. **546** (2002).

[Ka] Y. Kawamata, *Minimal models and the Kodaira dimension of algebraic fiber spaces*, Jour. Reine Angew. Math. **363** (1985), 1--46.

[Ko] J. Kollár, *Shafarevich Maps and Automorphic Forms,* Princeton University Press, 1995.

[M] S. Mukai, *Duality between D(X) and D(hat {X}) with its application to Picard sheave*, Nagoya math. J. **81** (1981), 153--175.

[Si] Simpson, C., *Subspaces of moduli spaces of rank one local systems*, Ann. Sci. École Norm. Sup. (4) **26** (1993), 361--401.