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Report for the National Science Council Project
Project Title: Decomposition, Covering and Packing in Graphs (3/3)
Project Number: NSC 93–2115–M–002–003
Project Duration: August 1, 2004 to July 31, 2005
Project Investigator: Gerard Jennhwa Chang
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This is the last year of the project "Decomposition, Covering and Packing in Graphs", which is for three years from August 1, 2002 to July 31, 2005. During this year, our results are on group testing, profile minimization, Vogan diagram, domination, intersection graph, and minimal separator. Seven papers are finished and submitted to journals, with some of them accepted. Below are the list and the abstracts of these papers.

[129] J. S.-t. Juan and **G. J. Chang**, "Group testing in graphs," submitted to *J. Comb. Optimization* (minor revision). (NSC93-2115-M002-003)

[135] Y.-P. Tsao and **G. J. Chang**, "Profile minimization of compositions of graphs," submitted to *J. Comb. Optimization* (minor revision). (NSC93-2115-M002-003)

[138] **G. J. Chang** and M.-K. Chuah, "Vogan diagrams and the classification of real simple Lie Algebra," revision. (NSC93-2115-M002-003)

[148] Y.-P. Tsao and **G. J. Chang**, "Profile minimization on product of graphs," *Disc. Math.* (accepted). (NSC93-2115-M002-003)

[150] M. Zhao, L. Kang and **G. J. Chang**, "Power domination in graphs," *Disc. Math.* (accepted). (NSC93-2115-M002-003)

[151] B.-J. Li and **G. J. Chang**, "Uniquely multiple pseudointersectable connected line graphs," submitted to *Disc. Math.* (major revision). (NSC93-2115-M002-003)

[152] **G. J. Chang**, A. J. J. Kloks, J. P. Liu and S.-L. Peng (2005), "The PIGs full monty–A floor show of minimal separators," Lecture Notes in Computer Science 3404, 521-532. (NSC93-2115-M002-003)

[129] Group testing in graphs

This paper studies the group testing problem in graphs as follows. Given a graph $G = (V, E)$, determine the minimum number $t(G)$ such that $t(G)$ tests are sufficient to identify an unknown edge e with each test specifies a subset $X \subseteq V$ and answers whether the unknown edge e is in $G[X]$ or not. Damaschke proved that $\lceil \log_2 e(G) \rceil \leq t(G) \leq \lceil \log_2 e(G) \rceil + 1$ for any graph G, where $e(G)$ is the number of edges of G. While there are infinitely many complete graphs that attain the upper bound, it was conjectured by Chang and Hwang that the lower bound is attained by all bipartite graphs. Later, they proved that the conjecture is true for complete bipartite graphs. Chang and Juan verified the conjecture for bipartite graphs G with $e(G) \leq 2^4$ or $2^{k-1} < e(G) \leq 2^{k-1} + 2^{k-3} + 2^{k-6} + 19 \cdot 2^{\frac{k-7}{2}}$ for $k \geq 5$. This paper proves the conjecture for bipartite graphs G with $e(G) \leq 2^5$ or $2^{k-1} < e(G) \leq 2^{k-1} + 2^{k-3} + 2^{k-4} + 2^{k-5} + 2^{k-6} + 2^{k-7} + 27 \cdot 2^{\frac{k-8}{2}} - 1$ for $k \geq 6$.

[135] Profile minimization of compositions of graphs

The profile minimization problem arose from the study of sparse matrix technique. In terms of graphs, the problem is to determine the profile of a graph G which is defined as

$$P(G) = \min_f \sum_{v \in V(G)} \max_{x \in N[v]} (f(v) - f(x)),$$

where f runs over all bijections from $V(G)$ to $\{1, 2, \ldots, |V(G)|\}$ and $N[v] = \{v\} \cup \{x \in V(G) : xv \in E(G)\}$. This is equivalent to the interval graph completion problem, which is to find a super-graph of a graph G with as few number of edges as possible. The purpose of this paper is to study the profiles of compositions of two graphs.

[138] Vogan diagrams and the classification of real simple Lie algebra

A Vogan diagram is a Dynkin diagram with an involution, such that the vertices fixed by the involution can be painted or unpainted. The Vogan diagrams are convenient tools in representing real simple Lie algebras. In this paper, we use them to classify the real simple Lie algebras by easy combinatorial methods.

[148] Profile minimization on product of graphs

The profile minimization problem arose from the study of sparse matrix technique. In terms of graphs, the problem is to determine the profile of a graph G which is defined as

$$P(G) = \min_f \sum_{v \in V(G)} \max_{x \in N[v]} (f(v) - f(x)),$$

where f runs over all bijections from $V(G)$ to $\{1, 2, \ldots, |V(G)|\}$ and $N[v] = \{v\} \cup \{x \in V(G) : xv \in E(G)\}$. The main result of this paper is to determine the profiles of $K_m \times K_n$, $K_{s,t} \times K_n$ and $P_m \times K_n$.

[150] Power domination in graphs

The problem of monitoring an electric power system by placing as few measurement devices in the system as possible is closely related to the well-known domination problem in graphs. In 1998, Haynes et al. considered the graph theoretical representation of this problem as a variation of the domination problem. They defined a set S to be a power dominating set of a graph if every vertex and every edge in the system is monitored by the set S (following a set of rules for power system monitoring). The power domination number $\gamma_P(G)$ of a graph G is the minimum cardinality of a power dominating set of G. In this paper, we present upper bounds on the power domination number for a connected graph with at least three vertices and a connected claw-free cubic graph in terms of their order. The extremal graphs attaining the upper bounds are also characterized.

[151] Uniquely multiple pseudointersectable connected line graphs

A multiple pseudorepresentation of a graph $G = (V, E)$ is an indexed family $\mathcal{F} = \{F_v\}_{v \in V}$ of (not necessarily distinct) subsets of some set S such that $xy \in E$ if and only if $x \neq y$ and $|F_x \cap F_y| = 1$. The multiple pseudointersection number $i_{\pm}(G)$ of a graph G is the minimum size of S on which there is a multiple pseudorepresentation for G. If furthermore that any two such multiple pseudorepresentations are isomorphic, then G is called uniquely multiple pseudointersectable. Bylka and Komar [?], partitioned connected line graphs into three classes $\mathcal{L}_1$, $\mathcal{L}_2$ and $\mathcal{L}_3$. They characterized graphs in $\mathcal{L}_1 \cup \mathcal{L}_3$ which are uniquely multiple pseudointersectable, and left three conjectures for the graphs in $\mathcal{L}_2$. The main effort of this paper is to prove these conjectures. A proof technique is also used to provide an alternative proof for the De Bruijn-Erdős Theorem.

[152] The PIGs full monty–A floor show of minimal separators

Given a class of graphs $\mathcal{G}$, a graph G is a *probe graph of* $\mathcal{G}$ if its vertices can be partitioned into two sets P (the probes) and N (non–probes), where N is an independent set, such that G can be embedded into a graph of $\mathcal{G}$ by adding edges between certain vertices of N. We show that the recognition problem of *probe interval graphs*, i.e., probe graphs of the class of interval graphs, is in P.