行政院國家科學委員會專題研究計畫 成果報告

活動邊界的均曲率流動
研究成果報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 95-2115-M-002-016-
執 行 期 間 ：95 年 08 月 01 日至 96 年 10 月 31 日
執 行 單 位 ：國立臺灣大學數學系暨研究所

計畫主持人：王藹農

計畫參與人員：此計畫無參與人員：無

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 97 年 03 月 10 日

An Unbounded Extremal Domain of the Constant Mean Curvature Equation

by

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We describe in this note an unbounded domain which is a generalization of "extremal domains" originally defined only for bounded domains. Let $\Omega = \{-f(x) - 1 < y < f(x) = \frac{c}{1+x^2}\}$ where c will be determined by the existence functions $u(x, y) \pm const$ satisfying

$$divTu = div\frac{Du}{\sqrt{1+|Du|^2}} = 2 \quad in\ \Omega \tag{1}$$

$$Tu \cdot \nu = \cos\gamma = 1 \quad on\ \partial\Omega$$

$$|Du| \to \infty \quad as\ x \to +\infty$$

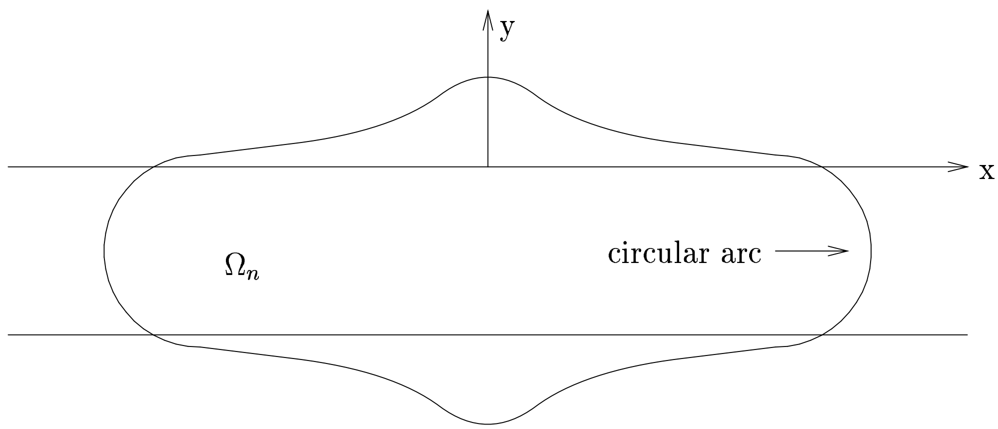
The existence of such u entrainent the non-existence of any bigger domain hosting the same equation (1). In fact u will be constructed as the decreasing limit of u_n defined on a smoothly truncated domain Ω_n (see figure) satisfying

$$divTu_n = H_n > 2 \quad in\ \Omega_n$$

$$Tu_n \cdot \nu = 1 \quad on\ \partial\Omega_n$$

and normalized by $u_n(0, -1/2) = 0$.

[1]partially supported by NSC 83-0208-M-002-31



By Stokes' theorem

$$H_n\left(\int_0^x f \ + \ \frac{x}{2}+\frac{\pi}{4}(\frac{1}{2}+f(x))^2\right)=\int_0^x\sqrt{1+f'^2} \ + \ \frac{\pi}{2}(\frac{1}{2}+f(x)) \qquad (2)$$

As $x\to\infty$ we require $H_n\to 2$ and get

$$\lim_{x\to\infty}\left(2\int_0^x f \ + \ x \ + \ \frac{\pi}{8}-\int_0^x\sqrt{1+f'^2} \ - \ \frac{\pi}{4}\right)=0$$

So it amounts to solve

$$2c\frac{\pi}{2}-\frac{\pi}{8}-\int_0^\infty(\sqrt{1+f'^2} \ - \ 1)=0$$

Since $\sqrt{1+f'^2}-1<c^2\frac{2x^2}{(1+x^2)^4}$, and $\int_0^\infty\frac{x^2}{(1+x^2)^4}dx=\frac{1}{16}\frac{\pi}{2}$, such a c does exist and it remains to show $H_n>2$. But this follows easily from the fact that in (2), with c determined as above and x sufficiently large,

$$\frac{\partial}{\partial x}\left[2\int_0^x f+\frac{\pi}{2}(\frac{1}{2}+f(x))^2-\int_0^x(\sqrt{1+f'^2} \ - \ 1) \ - \ \frac{\pi}{2}(\frac{1}{2}+f(x))\right]>0$$