

行政院國家科學委員會專題研究計畫 期中進度報告

均曲率流及高餘維極小子流形(1/3) 期中進度報告(精簡版)

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Mean Curvature Flow and Minimal Submanifolds of Higher Codimension

均曲率流及高餘維極小子流形(1/3)(96-2628-M-002-016-) 期中報告

台大數學系 李瑩英 2008. 05. 26

One potential approach to the construction of special Lagrangians is the mean curvature flow- as the negative gradient flow of the volume functional. Being Lagrangian in an Einstein manifold is preserved under mean curvature flow when the solutions are smooth. However, singularities may occur and we do not have long time existence of the solutions. It is thus desirable to understand these singularities or find some weak formulation of mean curvature flow, which allows the appearance of singularities.

Brakke flow is one of such generalizations. I made a lot of efforts in constructing general solutions for Brakke flow in the Lagrangian category. In 2005, I came up with an important observation that it is impossible to define generalized Lagrangian mean curvature flow without any further restriction on the initial data. The observation is based on Wolfson's K3 example, on which exists no special Lagrangian.

Never the less in this project, we construct special eternal solutions for Lagrangian Brakke flow without mass loss. Recall that an eternal solution is a solution that is defined for all time. Our solutions are Hamiltonian stationary for every time slice and are self-shrinkers for negative t , and self-expanders for positive t . The slice at time zero is a Hamiltonian stationary cone. In dimension two, the results resolve pairs of Schoen-Wolfson cones and can show that only $C_{\{2,1\}}$ is infinitesimal volume minimizing. It is conjectured that only $C_{\{2,1\}}$ cone is volume minimizing. The importance of the problem is that it determines the cone singularities of Lagrangian minimizers. Our result is the first one that can distinguish $C_{\{2,1\}}$ cone from all other cones. Recall that Schoen-Wolfson cones occur as the cone singularities in Schoen and Wolfson's study on Lagrangian minimizers and can be considered as the obstructions to the existence of special Lagrangians.

We also generalize the results to higher dimension and in particular obtain higher dimensional Hamiltonian stationary cones with different topology, which generalize 2-d Schoen-Wolfson cones. Other eternal solutions to Lagrangian Brakke flow without mass loss, which are not necessarily Hamiltonian stationary, are also constructed.

At the same time, I make a lot of efforts on international cooperation and training students. The above mentioned results are done with M.T. Wang from Columbia University. Other important and interesting examples on self-similar and translating solutions are constructed in a joint work with D. Joyce from Oxford University and M.P. Tsui from University of Toledo. Currently, I just start a new joint project with R. Schoen in Stanford University. Except strengthening these already exist cooperation, I also hope to develop closer connections with other mathematicians, like A. Neves, M. Haskins, N. Lei, and J. Wolfson.

A few regular student seminars are arranged. One is mainly on general relativity, one on Ricci flow, and one studying the book by Schoen and Yau. We also plan to go through the literature on mean curvature flow. It is a great pleasure to see the progress and devoting of the students. Some activities take place without me. Because of the time constrain, they sometimes meet in the late evening, and once even met at 6 am in the morning to seminar geometric measure theory. I also organize a weekly geometry seminar, which is usually on Tuesday afternoon. Last summer, we organize one international conference on geometric analysis. We will host “the Ninth Pacific Rim Geometry Conference” this December. I sincerely wish that with these efforts, another research team can be formed in the near future and we have more cooperation within Asia and worldwide.

赴國外研究心得報告

計畫編號	96-2628-M-002-016
計畫名稱	均曲率流及高餘維極小子流形(1/3)
出國人員姓名 服務機關及職稱	李瑩英 教授，台大數學系
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國外研究機構	UBC

工作記要：

我本次的出國行程是在 2007 年 12 月 7 日至 16 日,先到加拿大溫哥華的 UBC 訪問陳競一教授,之後到 Banff 參加一個 5 天的研討會「Minimal submanifolds and related problems」。

陳競一教授是我在史丹福讀博士時的同學,我們不但都選擇 R. Schoen 教授為指導老師,同時一直在相同的辦公室,所以相當熟悉。可惜畢業後聯絡及討論的機會不多,他也是這次我所要參加的 Banff 研討會之主辦者之一。田剛教授和我同一天到 UBC 訪問陳競一,然後一起到 Banff 參加研討會,我以前曾到紐約及波士頓訪問田剛多次,感覺十分投緣,但已經將近 10 年沒碰面,加上這些年來他和丘成桐之間的種種紛擾,雖然我們還是十分和氣地相處,但總是生疏不少,也沒機會深入討論數學。另一個使我無法放鬆專注於討論的原因,是到了 UBC 後,我才知道我在 Banff 的演講被安排在第 1 天,而出國前因為太忙,根本無暇準備。

Banff 是一個非常知名的數學中心,由於位於加拿大的國家公園內,風景秀麗,而且活動規劃得很好,大家都非常喜歡參加這裡的活動。每週一個 5 天的研討會,週日進駐,週五下午離開,大家都住在中心,三餐在同一餐廳,自助式用餐,有非常多的時間討論及互動。所以活動常在 1,2 年前即已排滿,其中心主任在我來訪問前,曾到台大數學科學中心訪問,我在 UBC 時又見到他,大家一起吃晚餐聊天,我很訝異發現,雖然有這麼龐大的業務,他並不需要常常待在 Banff,一個設計良好的制度,真得很重要,唯有如此才能長久,也唯有能兼顧自己的研究,才會吸引人從事這項行政工作。

由於是一個專門的主題,與會 30 多人的興趣均十分相近,有些是十分熟悉的朋友,有些雖然以前沒碰過,但對其名字及工作,卻是早已耳熟能詳。和與會的人都接觸了不少,其中討論較多數學的是 R. Schoen 及 Yuan Yu,他們兩個也都是主辦者,我與 R. Schoen 開始一個新的合作計畫,所以趁這個機會交換一下想法,Yuan Yu 作了一些關於 Special Lagrangian equation 很好的工作,可是我未曾仔細讀過,所以找了一個晚上請他解釋給我聽。我對自己的演講不太滿意,我帶來的電腦初次使用,操作及其呈現效果不熟悉,所以弄得手忙腳亂,還調了演講的時間處理,而自己的英文真應該好好加強,由於這麼多狀況加上緊張,講得詞不達義,七零八落。不過多認識及熟悉許多人,還是很值得高興,我還希望能多和其中一些人討論,很可惜時間不夠。

我非常喜歡 Banff 的環境及活動形式,雖然天氣真得很冷,冰天雪地,大家都全副武裝,但週遭昂然而立的大山,加上雪白的世界,帶給人們的感覺,卻是壯闊與寧靜。我在 2008 年 2 月也受邀參加 Banff 另一個研討會,我原先已經答應參加,可惜之後因為生病,而且太忙,只好取消行程。