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Report on "Complex geometrical optics solutions and electrical impedance tomography (1/3) (96-2628-M-002-009)"

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This is a report on the NSC grant 96-2628-M-002-009 entitled "Complex geometrical optics solutions and electrical impedance tomography (1/3)". The main theme of this grant is to investigate some problems related to the electric impedance tomography. My aim is to develop reliable numerical algorithm in reconstructing unknown object from observable data such as boundary measurements. The focal point is that the complex geometrical optics solutions. I am grateful to the grant support from NSC. Now I list several results benefited from this grant.

1. With Hideki Takuwa and Gunther Uhlmann, we construct complex geometrical optics solutions with general phase functions for the second order elliptic equation in two dimensions. We then use these special solutions to treat the inverse problem of reconstructing embedded inclusions by boundary measurements.

In previous works, special complex geometrical optics (CGO) solutions for certain isotropic systems in the plane were constructed and their applications to the object identification problem were also demonstrated theoretically and numerically. Those systems include the conductivity equation, the elasticity equations, and the Stokes equations, all with isotropic inhomogeneous coefficients. In this work we extend the method to scalar equations with anisotropic inhomogeneous coefficients in the plane. We shall focus on the conductivity equation:

$$L_\gamma u =: \sum_{i,j=1}^{2} \partial_{x^j}(\gamma^{ij}(x)\partial_{x^i}u) = 0 \quad \text{in} \quad \Omega, \tag{1}$$

where Ω is a bounded open set in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$ and the matrix $\gamma(x) = [\gamma^{ij}(x)]$ is symmetric and there exists $C_0 > 0$ such that

$$\gamma(x) \geq C_0 I \quad \text{for all} \quad x \in \Omega.$$

The precise regularity of γ will be specified later on. Our method can treat the equation with L_γ as the leading order without essential modification. The main aim here is to construct solutions to (1) with general complex phases. These types of solutions are very useful in the reconstruction of objects embedded in Ω by boundary measurements.

The key step in treating (1) is to transform it to an isotropic equation using isothermal coordinates, which are closely related to quasiconformal mappings. This idea is crucial in studying the Calderón problem in two dimensions. This work is currently under review for journal publication.

2. With Ching-Lung Lin and Gen Nakamura, we study the local behavior of a solution to second order elliptic operators with sharp singular coefficients in lower order terms. One of the main results is the bound on the vanishing order of the solution, which is a quantitative estimate of the strong unique continuation property. Our proof relies on Carleman estimates with carefully chosen phases. A key strategy in the proof is to derive doubling inequalities via three-sphere inequalities. Our method can also be applied to certain elliptic systems with similar singular coefficients.