行政院國家科學委員會專題研究計畫 期中進度報告

隨機粒子系統的研究(2/3)
期中進度報告(精簡版)

計 畫 類 別 ：個別型
計 畫 編 號 ：NSC 95-2115-M-002-004-
執 行 期 間 ：95 年 08 月 01 日至 96 年 09 月 30 日
執 行 單 位 ：國立臺灣大學數學系暨研究所

計 畫 主 持 人：張志中

處 理 方 式 ：本計畫可公開查詢

中 華 民 國 96 年 06 月 29 日

行政院國家科學委員會專題研究計畫期中進度報告

隨機粒子系統的研究(2/3)

STUDIES OF STOCHASTIC PARTICLE SYSTEMS (2/3)

計畫編號：NSC 95 – 2115 – M – 002 – 004
執行期限：95 年 8 月 1 日至 96 年 9 月 30 日
主持人：張志中　　台灣大學數學系
Email: ccchang@math.ntu.edu.tw

一、中文摘要

本年度計劃中我們主要探討了「推廣的對稱互斥過程」的大離差現象，以及「零域過程」中佔位時間的大離差估計。前者為非梯度形的粒子系統，後者當其粒子跳躍機率分佈為非對稱、但其期望值為零時亦為非梯度形的粒子系統，因此技術上困難許多。關於後者的研究，主要是建立了一個超指數估計，將一個位置的佔位時間聯繫到密度場的時間積分，而密度場的大離差估計已有前人的成果([5])可引用。

關鍵詞：推廣的對稱互斥過程、零域過程、佔位時間、大離差、超指數估計、非梯度形粒子系統

Abstract

This year we focus on the large deviations of the generalized symmetric exclusion process, and the occupation time large deviations of zero range processes. The former is a non-gradient model, so is the latter when its jump probability distribution is asymmetric with zero mean. This non-gradient nature makes the study much more difficult. The main result in the investigation of the zero range processes is that we are able to establish a super-exponential estimate, which relates the occupation time of one site with the time integral of the empirical density field, of which the large deviations have been derived ([5]).

Keywords: generalized symmetric exclusion process, zero range process, occupation time, large deviations, super-exponential estimate, non-gradient particle system

二、報告內容

（報告正文與參考文獻以英文撰寫，於次頁起始）
註：因尚未出國參訪國際學術會議或至國外出差，故此報告中未能繳交心得報告。

When supervising my Master students Hsuan-Chih Lin and Fong-Min Wu, we obtain some partial results on the occupation time large deviations of one dimensional mean zero asymmetric zero range processes and the large deviations of generalized symmetric exclusion process. Here we only state the main results of occupation time large deviations of one dimensional mean zero asymmetric zero range processes. Other results and details can be found in [1], [2], and [3].

Denote the configuration space by $\mathcal{X} = \mathbb{N}^{\mathbb{Z}}$. For an $\eta = \{\eta(x), x \in \mathbb{Z}\} \in \mathcal{X}$, $\eta(x) \in \mathbb{N}$ represents the number of particles at site $x \in \mathbb{Z}$ of the configuration η. A (one dimensional) zero range process is a continuous-time Markov process on $\mathcal{X}$ generated by

$$(Lf)(\eta) = \sum_{x,y\in\mathbb{Z}} \gamma(\eta(x))P(x,y)[f(\eta^{x,y}) - f(\eta)], \tag{1}$$

where f is a local function and $\eta^{x,y}$ stands for

$$\eta^{x,y}(z) \;=\; \begin{cases} \eta(z) & \text{if } z \neq x, y, \\ \eta(x) - 1 & \text{if } z = x, \\ \eta(y) + 1 & \text{if } z = y. \end{cases}$$

Here $P(\cdot,\cdot)$ is a transition probability on $\mathbb{Z}$ and $\gamma : \mathbb{N} \to \mathbb{R}_+$ is the rate at which particles jump. More precisely, if a site x has n particles one of the particles jump to site y at rate $\gamma(n)P(x,y)$.

Now we introduce the invariant measures of the process. Let $Z(\cdot) : \mathbb{R}_+ \to [0,\infty]$ be the partition function defined by

$$Z(\varphi) = 1 + \sum_{k\geq 1} \frac{\varphi^k}{\gamma(1)\cdots\gamma(k)},$$

which clearly is an increasing function for $\varphi \geq 0$. Let $\varphi*$ denote the radius of convergence of Z:

$$\varphi^* = \sup\{\varphi; Z(\varphi) < \infty\}.$$

We shall assume that

$$\lim_{\varphi\to\varphi^*} Z(\varphi) = \infty. \tag{2}$$

For $0 \leq \varphi < \varphi^*$, let $\overline{\nu}_\varphi$ be the translation invariant measure on X with marginals given by

$$\overline{\nu}_\varphi\{\eta; \eta(x) = k\} = \begin{cases} \frac{1}{Z(\varphi)} \frac{\varphi^k}{\gamma(1)\cdots\gamma(k)} & \text{if } k \geq 1\,, \\ \frac{1}{Z(\varphi)} & \text{if } k = 0\,. \end{cases}$$

Let $\rho(\varphi)$ be the density of particles of the measure $\overline{\nu}_\varphi$:

$$\rho(\varphi) = \overline{\nu}_\varphi[\eta(0)].$$

It follows from the assumption (2) that $\rho : [0,\varphi^*) \to [0,\infty)$ is a smooth strictly increasing bijection. Thus the inverse function $\varphi = \varphi(\rho)$ is well-defined. Since $\rho(\varphi)$ has a physical meaning as the density of particles, we would rather use ρ to parametrize the family of measures and we write

$$\nu_\rho = \overline{\nu}_{\varphi(\rho)},\ \rho \geq 0.$$

From this convention we get

$$\varphi(\rho) = \nu_\rho[\gamma(\eta(0))],\ \rho \geq 0.$$

Now we are ready to state the hypotheses on the process.

(A1) $P(\cdot,\cdot)$ is a mean zero irreducible translation invariant transition probability on $\mathbb{Z}$ with finite range. That is, there exists a transition probability $p(\cdot)$ on $\mathbb{Z}$ such that

$$P(x,y) = P(0, y-x) = p(y-x),$$

and for some $N_0 \in \mathbb{N}$,

$$p(x) = 0 \text{ if } |x| > N_0 \text{ and } \sum_{x\in\mathbb{Z}} xp(x) = 0.$$

(A2) γ is a nonnegative function such that $0 = \gamma(0) < \gamma(x)$ for every $x \in \mathbb{N}$. Moreover, there exist two positive constants $c_1, c_2 > 0$ and $N_1 \in \mathbb{N}$ such that

$$c_1 \le \gamma(x+1) - \gamma(x) \le c_2, \quad x \ge N_1.$$

The existence of a Markov process with generator given in (1) satisfying (A1) and some condition a little weaker than (A2) was proved by Andjel ([4]).

Now fix a positive time $T > 0$ and a positive constant density $\rho > 0$. Denote by P_N^ρ the probability measure on the path space $D([0,T], \mathcal{X})$ corresponding to the Markov process with generator $N^2 L$ and initial measure ν_ρ, where L is given in (1). Denote by E_N^ρ the expectation with respect to the probability P_N^ρ.

Denote by σ^2 the variance of the probability distribution $p(\cdot) : \sigma^2 = Var(p(\cdot)) = \sum_x x^2 p(x) > 0$. Let $C(\mathbb{R})\,(C_K(\mathbb{R}))$ be the space of continuous (with compact support) functions on $\mathbb{R}$ with sup norm topology. Let $\mathcal{M}_+$ be the space of positive Radon measures on $\mathbb{R}$ equipped with the weak* topology induced by $C_K(\mathbb{R})$ by the duality

$$\langle \mu, H \rangle = \int H \, d\mu$$

for $H \in C_K(\mathbb{R})$, $\mu \in \mathcal{M}_+$.

Given any path $\{\eta_t, t \in [0,T]\}$ the empirical measure $\mu_t^N \in D([0,T], \mathcal{M}_+)$ is defined as

$$\mu_t^N = \frac{1}{N} \sum_{x\in\mathbb{Z}} \eta_t(x) \delta_{x/N},$$

where δ_u stands for the Dirac measure on u. Denote by Q_N^ρ the probability measure on $D([0,T], \mathcal{M}_+)$ corresponding to the Markov process $\mu_\cdot^N$ induced by P_N^ρ.

Denote by $C_\rho(\mathbb{R})$ the space of nonnegative continuous functions $\gamma : \mathbb{R} \to \mathbb{R}_+$ such that $\gamma - \rho$ has compact support. Let $\mathcal{A} = \mathcal{A}_\rho$ be the space of all paths μ in $D([0,T], \mathcal{M}_+)$ such that $m_t = d\mu_t / d\lambda$ solves the PDE

$$\begin{aligned} &\partial_t m = (\sigma/2)\Delta\varphi(m) - \partial_x(\varphi(m)\partial_x H), \\ &m(0,\cdot) = \gamma(\cdot), \end{aligned}$$

for some $\gamma \in C_\rho(\mathbb{R})$ and some $H \in C_K^{1,3}([0,T] \times \mathbb{R})$. In [5], Benois et al. established the following large deviations estimate for the empirical density of the process introduced above.

Proposition 1 *For every closed subset $C \subset D([0,T], \mathcal{M}_+)$ and every open subset $O \subset D([0,T], \mathcal{M}_+)$*

$$\begin{aligned} \limsup_{N\to\infty} \frac{1}{N} \log Q_N^\rho(C) &\le -\inf_{\mu\in C} I_\rho(\mu), \\ \liminf_{N\to\infty} \frac{1}{N} \log Q_N^\rho(O) &\ge -\inf_{\mu\in O\cap\mathcal{A}} I_\rho(\mu). \end{aligned}$$

Here the definition of the rate function $I_\rho(\cdot)$ is omitted.

A conventional approach which links the occupation time and the time average of the density field has been proposed by Landim ([6]) in order to study the occupation time large deviations. Following this approach Fong-Min Wu established the next super-exponential estimate, which is the main contribution in his thesis.

Lemma 2 *For every $\delta > 0, t \in [0,T]$, we have*

$$\lim_{\varepsilon\to 0}\limsup_{N\to\infty}\frac{1}{N}\log P_N^\rho\left[\left|\int_0^t V_{N,\varepsilon}(\eta_s)\,ds\right|\geq\delta\right]=-\infty,$$

where

$$V_{N,\varepsilon}(\eta)=\eta(0)-\frac{1}{2\varepsilon N+1}\sum_{|x|\leq\varepsilon N}\eta(x).$$

We skip the proof, which replies on the applications of Feynman-Kac formula, assumption (A2), an integration by parts formula, a truncation argument, and Logarithmic-Sobolev inequality for the zero range process ([7]).

By applying the contraction principle and the large deviation results in [5], we derive the lower bound of the occupation time large deviations. Because the rate function I_ρ is not convex, we are unable to yield the upper bound estimate.

To state the lower bound result, we introduce the rate function for the occupation time large deviations. For $\alpha \geq 0$, define

$$\begin{aligned}\tilde{\psi}_t(\alpha) &= \inf_{\substack{\mu;\, t^{-1}\int_0^t \mu(s,0)ds=\alpha\\ \mu\in\mathcal{A}}} I_\rho(\mu),\\ \psi_t(\alpha) &= \lim_{\varepsilon\to 0}\inf_{\substack{\beta\geq 0\\ |\beta-\alpha|<\varepsilon}}\tilde{\psi}_t(\beta).\end{aligned}$$

Theorem 3 *For every open subset $O \subset \mathbb{R}_+$, we have*

$$\liminf_{N\to\infty}\frac{1}{N}\log p_N^\rho\left(\frac{1}{t}\int_0^t \eta_s(0)\,ds\in O\right)\geq -\inf_{\alpha\in O}\psi_t(\alpha).$$

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