

# Arbitrary Waveform Coded Excitation Using Bipolar Square Wave Pulsers

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*Abstract* - This paper presents a new coded excitation scheme that efficiently synthesizes codes for arbitrary waveforms using a bipolar square wave pulser. The key idea of the proposed method is the conversion of a nonbinary code with good compression performance into a binary code by code translation and code tuning. Tukey-windowed chirps were converted into binary Tukey-windowed chirps that were compared with pseudochirps over the same spectral band. Experimental results show that the use of binary Tukey-windowed chirps can reduce the code duration by 20% or the peak sidelobe level by 6 dB compared to the commonly used pseudochirps.

## I. INTRODUCTION

In this paper, coded excitation is treated as an approach to improving the signal-to-noise ratio (SNR) under the condition of a fixed peak acoustic power [1]. In a coded excitation system, a wide transmit bandwidth is maintained while the transmit pulse length is increased. Thus, the axial resolution can be preserved at the receiver with pulse compression.

The performance of pulse compression is generally characterized by the mainlobe width (related to the axial resolution), the sidelobe level (related to the dynamic range and contrast resolution), and the SNR improvement. Given a code, once the filter length is fixed, the more stringent the constraints imposed on the compression results, the lower the output SNR is. Therefore, the code must be properly designed to meet the system requirements.

Achieving optimal pulse compression performance requires the ability to generate an arbitrary transmit waveform in order to realize the desired spectral characteristics. However, an arbitrary waveform generator is expensive. Here we propose a method that preserves the low-cost advantage of a bipolar pulser while achieving compression performance similar to that of an arbitrary waveform generator. The key idea of the

proposed method is the conversion of a nonbinary code into a binary code. It is shown that good compression performance can be achieved by using the converted binary code instead of the original code.

## II. CODE CONVERSION

The goal of code conversion (including code translation and code tuning) is to convert a nonbinary code into a binary code that exhibits similar compression performance.

### A. Code Translation

Let  $x(t)$  be the desired band-limited continuous time waveform for coded excitation. We first choose a sampling frequency  $f_s$  and discretize  $x(t)$  into  $x(n)$ , then send the sequence  $x(n)$  into a first-order one-bit sigma-delta modulator [2] ( $t$  denotes the time, and the letters  $n$  and  $k$  within parentheses denote discrete time indices) and get the output signal  $y(n)$  of the modulator.

### B. Code Tuning

The algorithm for tuning an  $N$ -bit code  $y(n)$ ,  $0 \leq n \leq N-1$ , into a new code  $y'(n)$  is illustrated using the flow diagram shown in Fig. 1. We define

$$d[y(n)] = \|[y(t; f_s) - x(t)] \otimes h_1(t)\|, \quad (1)$$

where  $h_1(t)$  is the impulse response of the transducer,  $\|\cdot\|$  denotes the  $L^2$  norm; then  $d[y(n)]$  is a measure of the similarity between  $y(t; f_s)$  and  $x(t)$ . If  $y(n)$  is tuned into a code  $y_{\text{temp},1}(n)$  such that  $d[y_{\text{temp},1}(n)] < d[y(n)]$ ,  $y_{\text{temp},1}(n)$  is considered better than  $y(n)$  and will be the current candidate for the final code  $y'(n)$ . If a code  $y_{\text{temp},2}(n)$  generated by tuning  $y_{\text{temp},1}(n)$  satisfies  $d[y_{\text{temp},2}(n)] < d[y_{\text{temp},1}(n)]$ ,  $y_{\text{temp},2}(n)$  will become the new candidate for  $y'(n)$ . The process continues until the candidate code cannot be further updated. In this algorithm, any new candidate for  $y'(n)$  is different from the current candidate in at most  $N_2$  ( $N_2 = 16$  in this paper) consecutive bits, and the start index of the  $N_2$  bits is shifted sequentially and iteratively until the similarity to  $x(n)$  cannot be further improved. The actual

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transmitted signal is

$$y'(t; f_s) = \sum_n y'(n+1)\Pi(f_s t - n), \quad (2)$$

where

$$\Pi(t) = \begin{cases} 1 & \text{if } 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}. \quad (3)$$

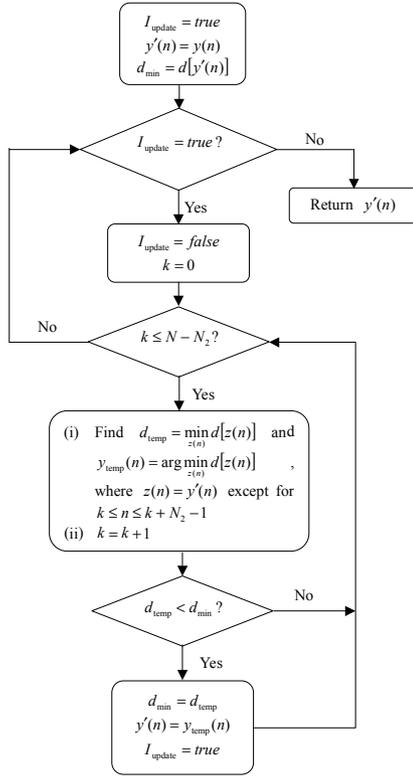


Fig. 1. Flow diagram for code tuning.

### III. COMPRESSION FILTER DESIGN

In this study, pulse compression is realized at baseband. Assume that the digitized echo signal is

$$y'_{\text{rf}}(n) = [y'(t; f_s) \otimes h_t(t) \otimes h_t(t)]_{t=n/f_{s,\text{ADC}}}, \quad (4)$$

where  $f_{s,\text{ADC}}$  is the sampling rate of the analog-to-digital converter (ADC) at the receiver. The  $y'_{\text{rf}}(n)$  is demodulated into a baseband signal

$$y'_{\text{bb}}(n) = h_k(n) \otimes [y'_{\text{rf}}(n) \cdot \exp(-j2\pi f_0 n / f_{s,\text{ADC}})], \quad (5)$$

where  $f_0$  is the center frequency of  $y'(t; f_s)$ , and  $h_k(n)$  is a Kaiser-windowed low-pass filter [3], i.e.,

$$h_k(n) = \frac{I_0 \left| \beta \left( 1 - \left[ \frac{(n - M/2)}{(M/2)} \right]^2 \right)^{1/2} \right|}{I_0(\beta)} \cdot \frac{\sin 2\pi f_c (n - M/2)}{\pi(n - M/2)}, \quad 0 \leq n \leq M, \quad (6)$$

where  $I_0(\cdot)$  represents the zeroth-order modified Bessel function of the first kind,  $f_c$  is the  $-6$  dB cutoff frequency,  $(M+1)$  is the filter length, and  $\beta$  is a shape parameter. The  $y'_{\text{bb}}[n]$  is then  $D$  times downsampled to obtain

$$y'_d(n) = y'_{\text{bb}}(Dn). \quad (7)$$

Given a compression filter  $h_{d,c}(n)$ , the compressed signal is

$$y'_{d,c}(n) = y'_d(n) \otimes h_{d,c}(n). \quad (8)$$

Let  $n_d(n)$  and  $n_{d,c}(n)$  denote the noise after downsampling and compression, respectively, and denote the autocorrelation function of a random process  $s(n)$  as  $R_s(k)$ , then [4]

$$R_{n_{d,c}}(k) = R_{n_d}(k) \otimes h_{d,c}(k) \otimes h_{d,c}^*(-k) \quad (9)$$

and the ensemble-averaged noise power after pulse compression is  $R_{n_{d,c}}(0)$ . If the peak of the compressed signal is normalized to unity by scaling the filter coefficients, the inverse of  $R_{n_{d,c}}(0)$  is the output SNR.

Let  $m$  denote the index of the peak position of  $y'_{d,c}(n)$ ,  $I_{\text{sl}}$  denote the index set of the sidelobes,  $s$  specify the predetermined allowable PSL, and  $s_{\text{dB}} = 20 \log s$ . The goal is to find a compression filter  $h_o(n)$  resulting in the minimal  $R_{n_{d,c}}(0)$  under the constraints of  $y'_{d,c}(m) = 1$ , and  $|\text{Re}\{y'_{d,c}(n)\}|, |\text{Im}\{y'_{d,c}(n)\}| \leq s/\sqrt{2}$  for  $n \in I_{\text{sl}}$ , where  $\text{Re}\{\cdot\}$  and  $\text{Im}\{\cdot\}$  denote the real part and the imaginary part, respectively. This is a quadratic programming problem with a convex feasible set [5]. If the constraints can be satisfied using a filter length of  $N_f$ , the optimal compression filter is unique and can be found.

### IV. EXPERIMENTAL RESULTS

In this section, all codes operate at a sampling frequency (or bit rate) of 40 MHz. The mainlobe is defined as the central 9 points of the compressed signal, with the rest of the signal being defined as the sidelobe region. The method introduced in Section II is applied and compared with the pseudochirp approach in [1].

Fig. 2 shows the experimental setup. A transducer with a diameter of 25.4 mm and a focal length of 71.1 mm (V304, Panametrics, Waltham, MA) was used to transmit and receive the ultrasonic signal. The image target was a nylon wire with a diameter of 0.2 mm placed 68.7 mm from the transducer. The pulse-echo signal from the wire and its spectrum obtained by using a pulser/receiver (PR5800, Panametrics) to transmit

and receive are shown in Figs. 3(a) and (b), respectively. A waveform generator (DAC200, Signatec, Corona, CA) was used to generate the coded signal at a sampling rate of 200 MHz (i.e., there are five samples per bit) with an amplitude resolution of 12 bits. The pulser/receiver was used to provide a 60 dB gain to the echo signal. The amplified echo was then digitized by an ADC (PCI-9820, ADLINK, Taipei, Taiwan) at a sampling rate of 60 MHz (i.e.,  $f_{s,ADC} = 60$  MHz) with an amplitude resolution of 14 bits. The transmit and receive timing was controlled by a function/arbitrary waveform generator (33120A, Agilent, Palo Alto, CA), which generated a 1-kHz trigger signal for the waveform generator and the ADC.

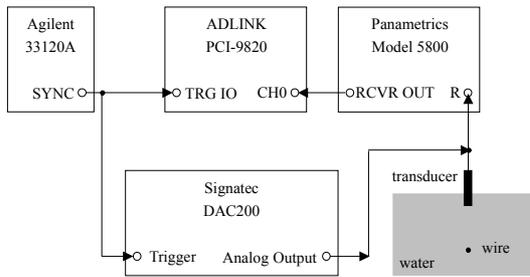


Fig. 2. Block diagram of the experimental setup

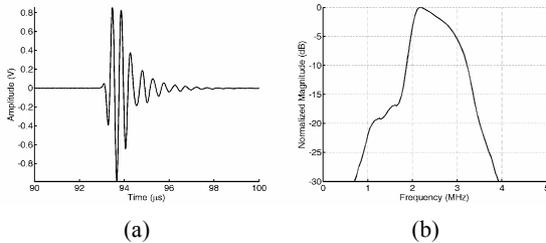


Fig. 3. (a) The pulse-echo signal and (b) the spectrum.

A dummy experiment was conducted to estimate  $R_n(k)$ . The parameters were  $M = 54$ ,  $\beta = 3.7$ ,  $f_c = 0.0375$ , and  $D = 15$  (i.e.,  $f_{s,d} = 4$  MHz). An experiment was also conducted using a one-cycle square wave with a duration of  $0.4 \mu\text{s}$  and an amplitude of  $\pi/4$  V as the transmitted signal. The SNR at baseband was 14.8 dB, and this value was used as a reference.

A chirp  $x_{cp}(n)$  with a duration of  $T$  and a sampling frequency of  $f_s$  can be defined as

$$x_{cp}(n) = \sin \left\{ 2\pi \left[ \left( f_0 - \frac{\Delta f}{2} \right) \frac{n}{f_s} + \frac{\alpha}{2} \left( \frac{n}{f_s} \right)^2 \right] \right\}, \quad (10)$$

$0 \leq n \leq N-1$ , where  $f_0$  is the carrier frequency,  $\Delta f$  is the bandwidth,  $\alpha = \Delta f / T$ ,  $N = f_s T$ , and its corresponding pseudochirp  $z_{pc}(n)$  is defined

by

$$z_{pc}(n) = \begin{cases} 1 & \text{if } x_{cp}(n) \geq 0 \\ -1 & \text{otherwise} \end{cases}. \quad (11)$$

The following pseudochirp signal was used to evaluate its performance in SNR enhancement:

$$z_{pc}(t, f_s) = \sum_n z_{pc}(n) \Pi(f_s t - n). \quad (12)$$

The transmitted signal levels were  $\pm \pi/4$  V, and the other parameters were  $f_0 = 2.5$  MHz, and  $\Delta f = 1.5$  MHz. Fig. 4(a) shows the downsampled baseband signal  $z_{pc,d}(n)$  for  $T = 12 \mu\text{s}$  (dashed line) and the compressed signal  $z_{pc,d,c}(n)$  corresponding to  $N_f = 64$  and  $s_{dB} = -40$  dB (solid line). The optimal improved SNR, defined as the optimal output SNR minus 14.8 dB (i.e., the SNR reference), is 16.8 dB. Fig. 4(b) is a plot of the optimal improved SNR versus  $N_f$  corresponding to  $s_{dB} = -40$  dB and  $T = 12 \mu\text{s}$ . The optimal SNR increases with  $N_f$ .

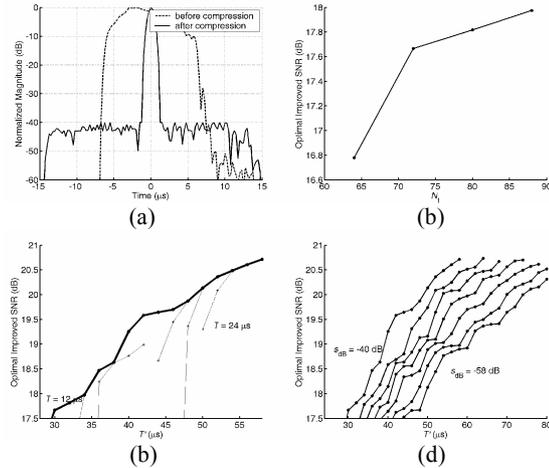


Fig. 4

Define  $T' = (T + N_f / f_{s,d})$  as the total temporal duration of the compressed signal. The performance of a code is determined by the optimal SNR, assuming  $T'$  and  $s_{dB}$  are fixed. Plots of the optimal improved SNR versus  $T'$  corresponding to  $s_{dB} = -40$  dB and  $T = 12 \mu\text{s}$  to  $24 \mu\text{s}$  with a step of  $2 \mu\text{s}$  are shown in Fig. 4(c) for the pseudochirps. At a given  $T'$ , the maximum of the optimal improved SNRs corresponding to various  $T$  values are found. A curve connecting such points is also shown in Fig. 4(c) as a thick solid line. This curve is called the characteristic curve of the pseudochirp at  $s_{dB} = -40$  dB. In this format, a code with a (vertically) higher curve outperforms a code with a lower curve. Seven characteristic curves of pseudochirps corresponding to  $s_{dB} = -40$  dB to  $-58$  dB with a step of  $-3$  dB are shown in Fig. 4(d). These curves were subsequently used to evaluate the

performance of the binary Tukey-windowed chirps.

Tukey-windowed chirps [6], defined as

$$x_{Tc}(t; T) = \sin \left\{ 2\pi \left[ \left( f_0 - \frac{\Delta f}{2} \right) t + \frac{\alpha}{2} t^2 \right] \right\} \quad (13)$$

$$\cdot w_T(t; r), 0 \leq t \leq T,$$

where  $w_T(t; r)$  is a Tukey window with a taper ratio of  $r$ , were converted into binary Tukey-windowed chirps [denoted by  $y'_{bTc}(n; T)$ ] using the method presented in Section II. All Tukey-windowed chirps had  $f_0 = 2.5$  MHz,  $\Delta f = 1.625$  MHz, and a taper ratio of 0.15. With these settings, the  $-12$  dB bandwidths of a Tukey-windowed chirp and a pseudo-chirp with  $f_0 = 2.5$  MHz and  $\Delta f = 1.5$  MHz are the same. To make the transmitted peak acoustic power associated with the binary Tukey-windowed chirp the same as that associated with the pseudo-chirp, the transmitted signal levels were  $\pm 1$  V because the magnitude of the fundamental frequency of a square wave with an amplitude of  $\pi/4$  is 1.

Figs. 5(a)–(d) are plots of the optimal improved SNR versus  $T'$  for binary Tukey-windowed chirps corresponding to  $s_{dB} = -40$  dB to  $-58$  dB with a step of  $-6$  dB, respectively. In each figure panel, the curves corresponding to three binary Tukey-windowed chirps are shown as thick solid lines from top to bottom:  $y'_{bTc}(n; 24 \mu s)$ ,  $y'_{bTc}(n; 20 \mu s)$ , and  $y'_{bTc}(n; 16 \mu s)$ . Each panel of the figure includes the characteristic curves of pseudo-chirps for comparison. The characteristic curve corresponding to the same PSL is shown as a thick dashed line, and the others are shown as thin solid lines. Fig. 5 shows that using a binary Tukey-windowed chirp improves the SNR by up to 2 dB relative to using a pseudo-chirp given an  $s_{dB}$  and a  $T'$  since, given an  $s_{dB}$ , the turning-point positions (which represent more efficient code/filter combinations) in the curves for binary Tukey-windowed chirps are generally 1–2 dB higher than the characteristic curves of pseudo-chirps. Taking a 1-dB SNR improvement as an example, this means that using a binary Tukey-windowed chirp instead of a pseudo-chirp results in a  $(1 - 10^{-1/10}) \cdot 100\% = 20.6\%$  reduction in code duration and dead zone because the SNR improvement is approximately proportional to the code duration. Moreover, comparing the turning-point positions in the curves for binary Tukey-windowed chirps with respect to the characteristic curves of pseudo-chirps shows that using a binary Tukey-windowed chirp in general results in a PSL that is 6 dB lower than that when

using a pseudo-chirp.

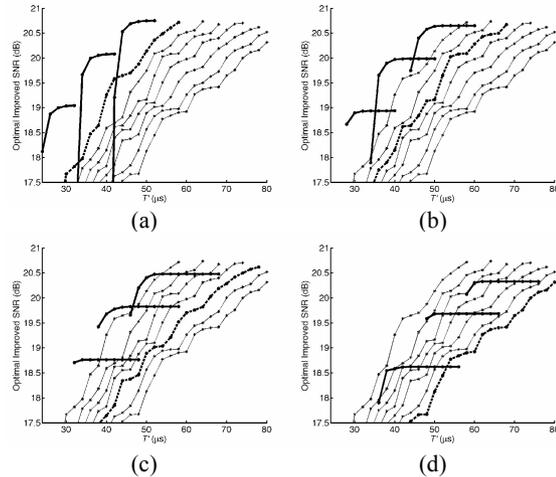


Fig. 5

## V. CONCLUSIONS

In this paper, a nonbinary code is converted into a binary code by code translation and code tuning such that the desired waveform can be transmitted using a bipolar pulser. Although the converted binary code is only an approximation of the original code after convolving with the impulse response of the transducer, the proposed method is successful at improving the SNR. With our method, the low-cost advantage of bipolar pulsers is preserved while the compression performance of the coded excitation system is enhanced in medical ultrasound. The proposed method can be used to effectively realize arbitrary waveform coded excitation with bipolar pulsers.

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