

Dual Power Assignment for Network Connectivity in Wireless Sensor Networks

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Abstract—Strong connectivity has been an important feature explored in many network applications, such as sensor networks. This research focuses on a dual power assignment problem, where each sensor node has two transmission power levels. The objective is to minimize the number of wireless sensor nodes assigned to transmit messages at the high transmission power level, while the resulting sensor network is strongly connected. We propose an efficient 1.75-approximation algorithm for this challenging problem. We not only show that the approximation ratio of the proposed algorithm is tight but also demonstrate the capability of the proposed algorithm in terms of simulation experiments.

Keywords: Dual power assignment, Power level assignment, Strong connectivity, Wireless sensor network.

1. Introduction

With the advance of technology for wireless sensor nodes, sensor networks are now widely tested and deployed for different application domains. One major challenge on the deployment of a wireless sensor network is on the power consumption minimization issues. Such an observation triggered a number of studies and implementations in energy-efficient research topics, such as those for packet routing, node placement, power-level assignments, etc.

In sensor networks, connectivity is usually required to collect information sensed by nodes. In many applications, sensor nodes might even need to operate autonomously and form an ad hoc network. Sometimes nodes might even need to work together to detect some complicated events, such as those for user behaviors. Similar requirements for connectivity are often seen in the research areas for mesh networks and ad hoc networks. As a result, strong connectivity is identified in the past decade as an important topic in the literature for sensor networks, mesh networks, and ad hoc networks, e.g., [1, 3, 5, 8, 9, 11, 12]. When the available transmission power levels for each wireless sensor node are continuous in a range of reals, many researchers have proposed results for the strong connectivity of wireless sensor nodes in [1, 3, 5, 8, 9]. In particular, 2-approximation algorithms based on minimum spanning trees were proposed in [1, 9]. When wireless sensor nodes are deployed in the 2-dimensional or the 3-dimensional space, the power-level assignment problem was proved being \mathcal{NP} -hard [5, 7, 8]. Researchers also considered the power-level assignment problem for different topological constraints in [2–4, 6]. Minimization of the maximum power consumption of a node to establish a connected wireless network was explored in [9, 10].

This research is motivated by the reality in which wireless sensor nodes might only have a set of discrete power levels available for assignment. In particular, we are interested in a dual power assignment problem, in which there are two available power levels for each wireless sensor node. The objective of the dual power assignment problem is to minimize the number of nodes assigned to transmit messages at the high transmission power level, while the resulting network is strongly connected. Sholander, Frank, and Yankopolus [12] proposed a heuristic algorithm by grouping nodes

into two categories. The dual power assignment problem was proved being \mathcal{NP} -hard by Rong, Choi, and Choi [11], and a 2-approximation algorithm was given. In this paper, we propose an efficient approximation algorithm for the dual power assignment problem. The approximation ratio is proved being 1.75, and the approximation ratio is shown to be tight. The proposed algorithm is highly efficient with $O(n^2)$ time complexity, where n is the number of wireless sensor nodes in a network. The strength of the proposed algorithm is demonstrated by a series of experiments, for which significant improvement is observed, compared to the previous result in [11].

The rest of this paper is organized as follows: In Section 2, we formally define the system models under considerations and the dual power assignment problem for network connectivity. An approximation algorithm is presented in Section 3 with analysis of its properties. Section 4 reports the experimental results. Section 5 is the conclusion.

2. System Models and Problem Definition

We study a power-efficient networking problem to construct a strongly connected network for wireless sensor nodes with two transmission power levels. We are concerned with a set of n immigrated wireless sensor nodes placed in a field, where each node is specified by its location. In such a network, a packet from a sensor node may need to be delivered through several hops before reaching its final destination. Two transmission power levels are given for the sensor nodes, where the higher/lower one is denoted as *high/low* transmission power. Each transmission power level specifies its transmission range. Let r_H and r_L with $r_H > r_L$ denote the transmission ranges for the high and low transmission powers, respectively. A node is set to be high (respectively, low), if it is assigned with the high (respectively, low) transmission power level.

There is a direct link from one wireless sensor node u to another wireless sensor node v if v could receive and decode any signal from u . That is, if the distance between u and v is no more than the transmission range of the transmission power level assigned for u , v can receive and decode any signals from u . Under an assignment of transmission power levels for the sensor nodes, one routing path from a node u to another node v exists if the message delivered from u can reach v through a series of multi-hop transition. A network is said to be *strongly connected* if there exists at least one routing path from u to v for any two different sensor nodes u and v when the sensor nodes transmit messages at their assigned transmission power levels. The objective of the dual power assignment problem for network connectivity is to minimize the power consumption for n wireless sensor nodes, which is defined as the total power of the assigned transmission power levels of these nodes, while the resulting network is strongly connected. Since assigning more nodes to be high results in larger power consumption, the objective is equivalent to the minimization of the number of nodes set to be high. In this paper, we consider non-trivial cases in which the network is strongly

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connected if all nodes are set to be high (and the network is not strongly connected if some nodes are set to be low).

We formulate the dual power assignment problem for network connectivity by a graph-theoretic approach as follows. Specifically, we are given two directed graphs $G_H = (V, E_H)$ and $G_L = (V, E_L)$ on the same set V of n wireless sensor nodes, where a directed edge (u, v) from node u to node v in G_H (respectively, G_L) signifies that node v can decode the signals from node u when node u is high (respectively, low). The following properties hold for G_H and G_L :

- G_H is symmetric and strongly connected;
- G_L is symmetric but not strongly connected; and
- G_L is a proper subgraph of G_H .

A *power assignment* is represented by a subset U of V whose members are set to be high. Let $E_H(U)$ consist of the outgoing edges of G_H from the nodes in U . Let $G(U) = (V, E_H(U) \cup E_L)$. For any set S , let $|S|$ denote the cardinality of S . It is clear that setting all the nodes in U as high and the others as low results in a strongly connected network if and only if there exists a path from u to v in $G(U)$ for any two nodes u and v . Therefore, the objective of the dual power assignment problem is to derive a power assignment $U \subseteq V$ with the minimum $|U|$ such that $G(U)$ is strongly connected. The dual power assignment problem for network connectivity can be defined as a graph-theoretical problem as follows:

Dual Power Assignment Problem

Input instance: Two directed graphs $G_H = (V, E_H)$ and $G_L = (V, E_L)$ on the same vertex set V , where G_L is a proper subgraph of G_H . Both G_H and G_L are symmetric. G_H is strongly connected, whereas G_L is not.

Objective: A subset U of V such that $G(U)$ is strongly connected and $|U|$ is minimized.

In [11], Rong, et al. has shown that the Dual Power Assignment problem is \mathcal{NP} -hard and proposed a 2-approximation algorithm.

3. A 1.75-Approximation Algorithm

In this section, we present our algorithm with the approximation ratio 1.75 for the Dual Power Assignment problem. Before we proceed with further discussion, some terminologies are defined as follows. For any directed graph G , let $B(G)$ be the set of the strongly connected components in the directed graph G . Let $C(S, u)$ denote the strongly connected component of $G(S)$ that contains node u for a power assignment S . Let $\Gamma(S, u)$ consist of the nodes v in V with $(u, v) \in G_H$ and $C(S, u) \neq C(S, v)$. That is, $\Gamma(S, u)$ contains the neighbors of u in G_H that are not in the strongly connected component $C(S, u)$. A member of $B(G(S))$ is a *neighboring component* of u in $G(S)$ if it contains at least one node of $\Gamma(S, u)$. Let $\mathcal{G}(S)$ be the undirected graph on $B(G(S))$, where each strongly connected component in $B(G(S))$ is a vertex in $\mathcal{G}(S)$, and two vertices in $\mathcal{G}(S)$ are adjacent if each of them contains a node in $V - S$ such that the two nodes are adjacent in G_H . The strongly connected component of $G(S)$ represented by a vertex w in $\mathcal{G}(S)$ is denoted by $\mathcal{G}^{-1}(S, w)$.

For example, suppose that we are given 10 sensor nodes, where the corresponding directed graphs G_H and G_L are illustrated in Figures 1(a) and (b). Suppose that S is $\{v_1, v_7\}$. We know $E_H(S) = \{(v_1, v_7), (v_7, v_1), (v_1, v_2), (v_7, v_8)\}$ and $G(S)$ is $(V, E_L \cup E_H(S))$. Let w_1, w_2, w_3 , and w_4 represent the strongly connected components of $G(S)$ containing v_1, v_3, v_5 , and v_9 , respectively. $B(G(S))$ is $\{w_1, w_2, w_3, w_4\}$. $\Gamma(S, v_2)$ is $\{v_3, v_9\}$. w_2 and w_4 are neighboring components of v_2 , whereas w_3 is not. Moreover, $\Gamma(S, v_8)$ is $\{v_9\}$, w_4 is a neighboring component of v_8 , whereas w_2 and w_3 are not. $\mathcal{G}(S)$ is an undirected graph on w_1, w_2, w_3 , and w_4 , shown in Figure 1(c). Since v_2 and v_3 are adjacent in G_H , there is an undirected edge

(w_1, w_2) . The other edges are because v_4 and v_5 are adjacent (the edge (w_2, w_3)), v_6 and v_{10} are adjacent (the edge (w_3, w_4)), and v_2 and v_9 are adjacent (the edge (w_1, w_4)). Similarly, if S is an empty set, $\mathcal{G}(\emptyset)$ is an undirected graph on w'_1, w'_2, w'_3, w'_4 , and w'_5 , shown in Figure 1(d), where w'_1, w'_2, w'_3, w'_4 , and w'_5 represent the strongly connected components of $G(\emptyset)$ containing v_1, v_3, v_5, v_9 , and v_7 , respectively.

With the initialization of S as an empty set, our proposed *three-phase assignment* algorithm, referred as Algorithm TPA and summarized in Algorithm 1, inserts nodes into S incrementally in a greedy manner. Algorithm TPA has the following three phases:

- 1) First, every strongly connected component of $B(G(S))$ is unmarked. As long as there is still a node u in V that has more than one neighboring component in $G(S)$, the first phase calls $\text{select}(u, S')$, which is defined as follows, with the setting of S' as \emptyset .

The subroutine $\text{select}(u, S')$ first marks $C(S, u)$ and inserts u into S' . Then, for each neighbor v of u in $\Gamma(S, u)$, if $C(S, v)$ is unmarked, the subroutine $\text{select}(u, S')$ recursively calls $\text{select}(v, S')$.

After the recursive call returns, we insert all of the nodes in S' into S and unmark the strongly connected component of $B(G(S))$ which contains u .

- 2) The second phase repeats the following procedure until $\mathcal{G}(S)$ does not contain any cycle.

Let \bar{C} be a cycle of at least three vertices in $\mathcal{G}(S)$. Suppose that $\bar{C} = (c_0, c_1, c_2, \dots, c_{|\bar{C}|-1}, c_0)$. Let v_i be a node within the strongly connected component, represented by c_i (i.e., $\mathcal{G}^{-1}(S, c_i)$), such that the strongly connected component, represented by c_{i+1} (i.e., $\mathcal{G}^{-1}(S, c_{i+1})$), is a neighboring component of v_i in $G(S)$. The second phase then inserts these $|\bar{C}|$ nodes $v_1, v_2, \dots, v_{|\bar{C}|}$ into S .

- 3) After the above procedures, the resulting $\mathcal{G}(S)$ is a tree. Then, the third phase repeats the following procedure until there is no edge exists in $\mathcal{G}(S)$.

Let u_1 be a vertex in $\mathcal{G}(S)$ and u_2 be a neighbor of u_1 in $\mathcal{G}(S)$. For notational brevity, let $u_3 = u_1$. For $i \leq 2$, let v_i be a node of V within the strongly connected component represented by u_i (i.e., $\mathcal{G}^{-1}(S, u_i)$) such that the strongly connected component represented by u_{i+1} (i.e., $\mathcal{G}^{-1}(S, u_{i+1})$) is a neighboring component of v_i in $G(S)$. The third phase then inserts nodes v_1 and v_2 into S .

It is clear that $\mathcal{G}(S)$ has only one vertex after executing Algorithm TPA. Therefore, $G(S)$ is a strongly connected graph. Consider the input instance described by G_H and G_L in Figures 1(a) and 1(b) as an example for illustrating Algorithm TPA. Initially, $S = \emptyset$. Since v_2 has 2 neighboring components, Algorithm TPA calls $\text{select}(v_2, S')$ in the first phase, where $S' = \emptyset$ initially. v_2, v_3, v_9 , and v_8 are inserted into S' during the recursive call of $\text{select}(v_2, S')$. Then, v_2, v_3, v_9 , and v_8 are inserted into S . The first phase then terminates since no node has more than one neighboring component. After that, $v_1, v_2, v_3, v_4, v_7, v_8, v_9$, and v_{10} form a strongly connected component, and so do v_5 and v_6 . Since no cycle exists for $\mathcal{G}(S)$, the second phase inserts no node into S . In the third phase, v_4 and v_6 are inserted into S . Therefore, the number of nodes in S is 6. The optimal solution for such an input instance is 5 by assigning v_1, v_3, v_5, v_8 , and v_{10} as high.

A straightforward implementation of Algorithm TPA requires $O(|V|(|V| + |E_H|))$ time complexity. We could apply the disjoint-set data structures in our implementations so that the time complexity is $O(|V| \log |V| + |E_H|) = O(n^2)$, where n is the number of sensor

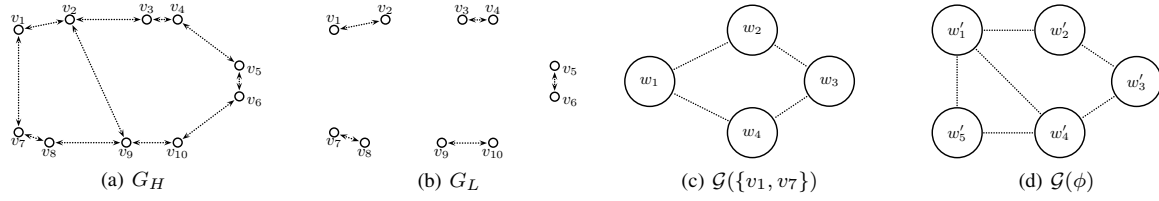


Figure 1. An example for notation explanations: (a) G_H and (b) G_L , where each dot represents a sensor node and an arrow between two nodes v_i and v_j indicates that there are two directed edges (v_i, v_j) and (v_j, v_i) in the graph. (c) $\mathcal{G}(\{v_1, v_7\})$, where w_1, w_2, w_3 , and w_4 represent the strongly connected components of $G(\{v_1, v_7\})$ containing v_1, v_3, v_5 , and v_9 , respectively. (d) $\mathcal{G}(\emptyset)$, where w'_1, w'_2, w'_3, w'_4 , and w'_5 represent the strongly connected components of $G(\emptyset)$ containing v_1, v_3, v_5, v_9 , and v_7 , respectively.

Algorithm 1 : Three Phase Assignment (TPA)

Input: (G_H, G_L) ;
Output: A subset S of V such that $G(S)$ is strongly connected;

- 1: $S \leftarrow \emptyset$;
- {first phase}
- 2: unmark every strongly connected component in $B(G(S))$;
- 3: **while** there exists a node $u \in V$ whose number of neighboring components in $B(G(S))$ is more than 1 **do**
- 4: call $\text{select}(u, S' \leftarrow \emptyset)$;
- 5: insert all of the nodes in S' into S ;
- 6: unmark the strongly connected component $C(S, u)$;
- {second phase}
- 7: **while** there exists a cycle $\bar{C} = (c_0, c_1, c_2, \dots, c_{|\bar{C}|} = c_0)$ in $\mathcal{G}(S)$ **do**
- 8: let v_i be a vertex in V , where $C(S, v_i)$ is equal to $\mathcal{G}^{-1}(S, c_i)$ and $\exists u \in \Gamma(S, v_i) \cap u \in \mathcal{G}^{-1}(S, c_{i+1})$;
- 9: insert $v_1, v_2, \dots, v_{|\bar{C}|}$ into S ;
- {third phase}
- 10: **while** there exists an edge $(u_1, u_2) \in \mathcal{G}(S)$ **do**
- 11: let v_i be a vertex in V , where $C(S, v_i)$ is equal to $\mathcal{G}^{-1}(S, u_i)$ and $\exists w \in \Gamma(S, v_i) \cap w \in \mathcal{G}^{-1}(S, u_{i+1})$, for $i \leq 2$; (remark: $u_3 = u_1$)
- 12: insert v_1 and v_2 into S ;
- 13: return S ;

Procedure: $\text{select}(u, S')$

- 1: insert u into S' and mark the strongly connected component $C(S, u)$;
- 2: **for all** $v \in \Gamma(S, u)$ **do**
- 3: **if** $C(S, v)$ is unmarked **then**
- 4: call $\text{select}(v, S')$;

nodes under considerations.

Now we show that the approximation ratio of Algorithm TPA is 1.75. For notational brevity, let S_2 denote the set S after the second phase of Algorithm TPA. For the rest of the section, let U be a power assignment of V such that $G(U)$ is strongly connected. We can derive two different lower bounds of $|U|$.

Lemma 1: $|U| \geq |B(G_L)| \geq 2$.

Proof: For each connected component in $B(G_L)$, there must be at least one node included in U . Otherwise, $G(U)$ is not strongly connected. ■

Lemma 2: Let U' be a subset of V . If no cycle exists in $\mathcal{G}(U')$ and each vertex in V has at most one neighboring component in $G(U')$, then $|U| \geq 2(|B(G(U'))| - 1)$.

Proof: We prove this lemma by contradiction with the assumption of $|U| < 2(|B(G(U'))| - 1)$. Let $\bar{G}(U', \emptyset)$ be a directed graph on $B(G(U'))$. For notational brevity, vertices are labeled with the same labels in $\bar{G}(U', \emptyset)$ and $\mathcal{G}(U')$. Note that no edge exists in $\bar{G}(U', \emptyset)$. $\bar{G}(U', U)$ is constructed as follows: For a node u in U , let v be the vertex in $\mathcal{G}(U')$, where $\mathcal{G}^{-1}(U', v)$ is equal to $C(U', u)$. Since each vertex in V has at most one neighboring component in $G(U')$, there is at most one vertex s in $\bar{G}(U', \emptyset)$ which represents the neighboring

component of u in $G(U')$. If such a vertex s exists, the directed edge (v, s) is added into $\bar{G}(U', \emptyset)$. Let $\bar{G}(U', U)$ be the resulted directed graph by considering all of the nodes in U and eliminating the duplicated directed edges in the above step.

Since $G(U)$ is strongly connected, $\bar{G}(U', U)$ is strongly connected. However, at most one directed edge will be added into $\bar{G}(U', U)$ for each element in U during the process in constructing $\bar{G}(U', U)$. Because no cycle exists in $\mathcal{G}(U')$, and G_H is strongly connected, $\mathcal{G}(U')$ is a tree. Besides, if a directed edge (u, v) is in $\bar{G}(U', U)$, then there exists a corresponding undirected edge (u, v) in $\mathcal{G}(U')$. There are $(|B(G(U'))| - 1)$ edges in $\mathcal{G}(U')$. $\bar{G}(U', U)$ is strongly connected only if there are at least $2(|B(G(U'))| - 1)$ directed edges are added during the construction of $\bar{G}(U', U)$. Thus, $|U| < 2(|B(G(U'))| - 1)$ implies that $\bar{G}(U', U)$ is not strongly connected. Therefore, $G(U)$ is not connected. A contradiction is reached. ■

S_2 is a subset of V which satisfies the properties of U' stated in Lemma 2. Combining with Lemma 1, we have

$$|U| \geq \max\{|B(G_L)|, 2(|B(G(S_2))| - 1)\}. \quad (1)$$

In the following, we prove an upper bound on the cardinality of the power assignment S derived from Algorithm TPA.

Lemma 3: $|S| \leq 1.5|B(G_L)| + 0.5|B(G(S_2))| - 2$.

Proof: For Step 4 in each iteration of the while loop in Algorithm 1, i.e., the recursive call of $\text{select}(u, \emptyset)$, let k be the number of strongly connect components marked for the recursive call. That is, k nodes in V are inserted into S' after the recursive call. These k marked strongly connected components become a connected component in $G(S)$ after inserting these k nodes into S . Therefore, these $k - 1$ strongly connected components are merged into one new strongly connected component. The number of nodes inserted into S for each of the $k - 1$ contracted strongly connected components is amortized to be $\frac{k}{k-1}$. Similarly, when a cycle \bar{C} is considered in the second phase, $|\bar{C}|$ connected components become a connected component after inserting $|\bar{C}|$ nodes into S . The number of nodes inserted into S for each of the $|\bar{C}| - 1$ contracted connected components in this iteration is amortized to be $\frac{|\bar{C}|}{|\bar{C}|-1}$. Let k_1, k_2, \dots, k_m denote the number of nodes inserted into S for Step 4 in each iteration of the while loop in the first phase of Algorithm 1. Let $\bar{C}_1, \bar{C}_2, \dots, \bar{C}_q$ denote the cycles considered in the second phase. There are $|B(G_L)| - |B(G(S_2))|$ strongly connected components contracted after the second phase of Algorithm TPA. Therefore, we have

$$\begin{aligned} |S_2| &= \sum_{i=1}^m \frac{k_i}{k_i-1} (k_i - 1) + \sum_{i=1}^q \frac{|\bar{C}_i|}{|\bar{C}_i|-1} (|\bar{C}_i| - 1) \\ &\leq 1.5(\sum_{i=1}^m (k_i - 1) + \sum_{i=1}^q (|\bar{C}_i| - 1)) \\ &= 1.5(|B(G_L)| - |B(G(S_2))|), \end{aligned}$$

because $k_i \geq 3$ and $|\bar{C}_i| \geq 3$ for every i ($\frac{k_i}{k_i-1} \leq 1.5$ and $\frac{|\bar{C}_i|}{|\bar{C}_i|-1} \leq 1.5$). Since there are only $(|B(G(S_2))| - 1)$ edges before we proceed to the third phase, the third phase inserts exactly $2(|B(G(S_2))| - 1)$

nodes into S . Therefore, we have

$$\begin{aligned} |S| &\leq 1.5(|B(G_L)| - |B(G(S_2))|) + 2(|B(G(S_2))| - 1) \\ &= 1.5|B(G_L)| + 0.5|B(G(S_2))| - 2. \end{aligned}$$

Theorem 1: Algorithm TPA is a polynomial-time 1.75-approximation algorithm for the Dual Power Assignment problem.

Proof: We prove this theorem by showing that

$$\frac{|S|}{|U|} \leq \frac{1.5|B(G_L)| + 0.5|B(G(S_2))| - 2}{\max\{|B(G_L)|, 2(|B(G(S_2))| - 1)\}} < 1.75.$$

If $|B(G_L)| \geq 2(|B(G(S_2))| - 1)$, then

$$\frac{|S|}{|U|} \leq \frac{1.5|B(G_L)| + 0.25|B(G_L)| - 1.5}{|B(G_L)|} < 1.75.$$

If $|B(G_L)| < 2(|B(G(S_2))| - 1)$, then

$$\frac{|S|}{|U|} \leq \frac{3(|B(G(S_2))| - 1) + 0.5|B(G(S_2))| - 2}{2(|B(G(S_2))| - 1)} < 1.75.$$

After the approximation ratio of our algorithm is shown, we show the tightness of the approximation bound by presenting a set of input instances. Consider the input instance shown in Figure 2, where each dot represents a sensor node. For any two nodes u and v inside a circle in Figure 2, there is a directed edge (u, v) from u to v in both G_H and G_L ; that is, the distance between u and v is no longer than r_L . In Figure 2(a), an arrow between two nodes u and v indicates that there are two directed edges (u, v) and (v, u) in G_H ; that is, the distance between u and v is longer than r_L and no longer than r_H . As shown in Figure 2, the pattern of A_1, A_2, A_3 , and A_4 repeats k times. Totally, there are $4k + 2$ circles. The optimal power assignment assigns only $4k + 2$ nodes to be high as shown in Figure 2(c), where the larger solid nodes are set to be high. Algorithm TPA returns a solution with $7k + 3$ nodes assigned to be high as shown in Figure 2(d), where the hollow nodes are included in the first phase and the larger solid nodes are included in the third phase, whereas no nodes are inserted in the second phase. Therefore, the approximation ratio of Algorithm TPA is tight for sufficiently large k .

4. Experimental Results

A. Experimental Setups and Performance Metric

Algorithm TPA is simulated extensively with comparison to the algorithm proposed in [11] (denoted as Algorithm SP), which is a 2-approximation algorithm for the Dual Power Assignment problem. We consider sensor nodes in the \mathbb{R}^2 space. Three types of distributions on sensor nodes are considered, in which the parameter setup is similar to that in [11]. For the first type of distributions of sensor nodes, both of the x -ordinate and y -ordinate of a sensor node are uniform random variables between 0 and 1000. For the second type of distributions, nodes are deployed according to a Poisson distribution by setting the mean value as 500. In addition, for the first and second types of distributions, simulations are conducted for different network sizes ($n = 30, 50, 100$). For the third type of distributions, we consider a specific application scenario, in which 300 sensor nodes are deployed homogeneously. Specifically, a 1000×1000 \mathbb{R}^2 plane is divided into rectangular regions with equal size. Each region is associated with two nodes. If (x_1, y_1) and (x_2, y_2) are the ordinates of the left-bottom and the right-top points of a region, two nodes are deployed in this region in which the x -ordinates (respectively, y -ordinates) are uniform random variables between x_1 and x_2 (respectively, y_1 and y_2). After that, the other 100 sensor

nodes are deployed by setting both x -ordinate and y -ordinate as uniform random variables between 0 and 1000.

Given a deployment of sensor nodes, we have to determine the transmission ranges r_H and r_L . In our experiments, r_H is set as the shortest transmission range associated for each node such that assigning all of the sensor nodes to high makes the wireless sensor network strongly connected. In our experiments, we simulate the two algorithms by varying the ratio of r_L to r_H from 5% to 80%.

We compare the performance of the two algorithms with an estimated lower bound, derived from Equation (1). The *ratio of relative high power nodes* of an algorithm for an input instance is defined as the ratio of the number of nodes assigned to be high in the power assignment derived from the algorithm to the estimated lower bound of the input instance. The average and maximum ratios of relative high power nodes are measured and conducted from 500 independent experiments for each parameter configuration.

B. Simulation Results

Figures 3 (a)-(d) show the performance result when nodes are deployed by uniform distributions, and the ratio of r_L to r_H varies from 0.05 to 0.8 stepped by 0.05. The average ratios of relative high power nodes when $n = 50$ and $n = 100$ are reported in Figures 3 (a) and (b), respectively. The maximum ratios of relative high power nodes when $n = 50$ and $n = 100$ are reported in Figures 3 (c) and (d), respectively. Similar results were observed when $n = 30$. As shown in Figures 3(a)-(d), for the metric of the ratios of relative high power nodes, Algorithm TPA outperforms Algorithm SP in either worst cases or average cases. Besides, the maximum ratio of relative high power nodes of Algorithm TPA is no more than 1.75, which is as the same as the analysis in Section 3. The maximum ratio of relative high power nodes of Algorithm TPA is at most 1.7, whereas that of Algorithm SP is at most 1.9. When the ratio of r_L to r_H is close to 1, both of the ratios of relative high power nodes of the two algorithms tend to approach their theoretical bounds, i.e., 1.75 for Algorithm TPA and 2 for Algorithm SP. In the worst cases, as the ratio of r_L to r_H increases, the number of nodes in a connected component of G_L becomes larger. Thus, the number of candidates of nodes to be set being high also increases. In both algorithms, once a node is set as high, one node of its neighboring components should be high. Therefore, assigning nodes in an incorrect sequence may be a poor choice compared to the estimated lower bound. As for average cases, there is a peak in Figures 3(a) and (b), i.e., when the ratio of r_L to r_H is 0.65. This is because that when the ratio of r_L to r_H is low, to make the network strongly connected, most nodes have to be assigned to be high either for power assignments derived from the two algorithms or for the corresponding estimated lower bound. Besides, when the ratio of r_L to r_H is large enough, in most cases, assigning most nodes to transmit message at the low power level could make the network strongly connected either in power assignments derived from the two algorithms or in the corresponding estimated lower bound.

Similarly, Figures 3(e)-(h) report the simulation results when nodes are deployed by Poisson distributions. The average ratios of relative high power nodes when $n = 50$ and $n = 100$ are reported in Figures 3 (e) and (f), respectively. The maximum ratios of relative high power nodes when $n = 50$ and $n = 100$ are reported in Figures 3 (g) and (h), respectively. The results in Figures 3(e)-(h) are similar to those in Figures 3(a)-(d). However, the peaks in Figure 3(g)-(h) shift to left, where the ratio of r_L to r_H is about 0.5 or 0.55. This comes from the characteristics of Poisson distributions since most nodes are not very far away.

Figure 4 shows the results for the third type of deployment distributions by varying r_L/r_H from 0.05 to 0.8 stepped by 0.05.

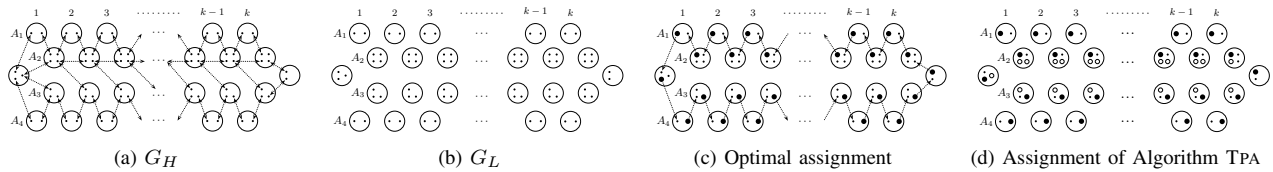


Figure 2. An input instance for a tight example: (a) G_H , (b) G_L , (c) an optimal assignment by assigning the larger solid nodes as high, and (d) an assignment derived from Algorithm TPA, where the hollow nodes are included in the first phase and the larger solid nodes are included in the third phase.

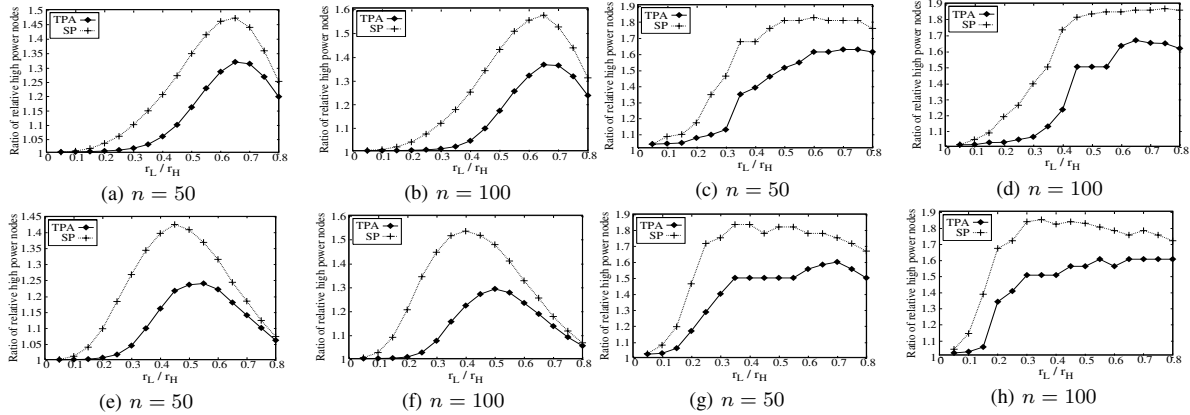


Figure 3. (a)-(b): average ratios of relative high power nodes for $n = 50, 100$ when nodes are deployed by a uniform distribution, respectively. (c)-(d): maximum ratios of relative high power nodes for $n = 50, 100$ when nodes are deployed by a uniform distribution, respectively. (e)-(f): average ratios of relative high power nodes for $n = 50, 100$ when nodes are deployed by a Poisson distribution, respectively. (g)-(h): maximum ratios of relative high power nodes for $n = 50, 100$ when nodes are deployed by a Poisson distribution, respectively.

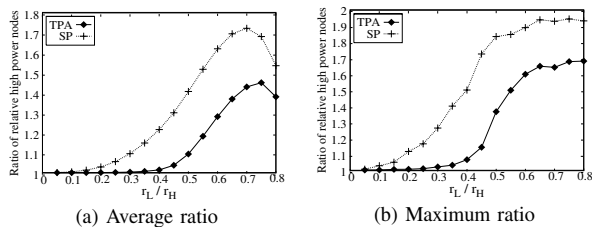


Figure 4. (a) average ratios of relative high power nodes and (b) maximum ratios of relative high power nodes for $n = 300$ when the plane is divided into 100 regions and each region has at least 2 nodes.

The average and maximum ratios of relative high power nodes are reported in Figures 4 (a) and (b), respectively. The results in Figures 4 are similar to those in Figures 3.

5. Conclusion

In this paper, we explore a power-efficient networking problem to maintain the property of strong connectivity for wireless sensor networks so that the power consumption is minimized. We consider a dual power assignment problem for a set of wireless sensor nodes, where each node has two transmission power levels. The objective is to minimize the number of nodes assigned to transmit messages in the high power level, while the resulting network is strongly connected. Given the NP-hardness of the problem, we present an efficient approximation algorithm with the approximation ratio 1.75. We not only show the tightness of the ratio but also demonstrate the capability of the proposed algorithm in terms of simulation experiments, for which very encouraging results are shown. We must point out that even though the proposed algorithm is presented for power-level assignment of sensor networks, it could also be applied to the power-level assignment for other kinds of networks.

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