Adaptive array beamforming with robust capabilities under random sensor position errors

J.-H. Lee and C.-C. Wang

Abstract: The problem of adaptive array beamforming with multiple-beam constraints in the presence of steering error caused by random sensor position errors is considered. First the statistical relationship between the random sensor position errors and the induced random phase perturbation is derived. Based on the result, a cost function consisting of terms which utilise *a posteriori* information owing to the received array data and *a priori* information owing to the probabilistic distribution of the resulting phase perturbation, respectively, is constructed. Then, an appropriate estimate of the actual phase angle vector associated with each of the desired signals can be obtained by performing a nonlinear optimisation problem. An implementation algorithm is further presented to solve iteratively the problem. Theoretical analysis regarding the convergence property of the iterative procedure is also investigated. Finally, several computer simulation examples are provided for demonstrating the effectiveness of the proposed technique.

1 Introduction

An adaptive array beamformer is a spatial filter designed for automatically preserving the desired signals while cancelling the interference and noise. The only *a priori* knowledge for a main-beam or a multiple-beam constrained beamformer is the actual direction vectors of the desired signals. A direction vector of a desired signal can be obtained from the information of the array sensor locations, signal impinging directions, and the propagation characteristics. However, the information may not be perfectly known in practice. This results in a mismatch between the presumed steering vectors and the actual direction vectors. Many reports have shown that the performance of a steered beam adaptive array beamformer is very sensitive to the mismatch [1–5]. Particularly, it has been shown in [6] that the amplitude and phase errors owing to array imperfections produce the effect of reducing interelement correlation, leading to a lowering of main lobe gain and decreased ability to discriminate against interferers.

To cure the problem of array performance degradation owing to the above mismatch, most robust techniques propose to impose additional constraints such as multiple linear constraints, derivative constraints, and norm constraints on the array weight vector [5, 7–16]. However, imposing additional constraints deteriorates the array capability in suppressing interference and noise. In contrast, the authors of [17] presented a robust approach based on the worst-case performance optimisation for curing the problem of array performance degradation owing to the signal covariance matrix with some fixed error. The authors of [18] proposed a diagonal loading approach for the beamforming problem of the desired signal with non-random steering vector error. Recently, based on the assumption that the steering vector error is an additive Gaussian random vector, two methods have been presented in [19] to find two appropriate closed-form solutions for estimating the optimal steering constraint vector. To deal with the case of steering vector errors owing to phase perturbation, the techniques of [19] resorted to an iterative algorithm to estimate the actual phase angle vector because of the resulting nonlinear programming problem. Nevertheless, the convergence of the modified techniques is not guaranteed. Moreover, all of the above mentioned techniques [5, 7–19] are developed under the situation of adaptive beamforming with main-beam constraint. In many applications, such as satellite communications [20], an antenna array must possess beamforming capability to receive more than one signal with specified gain requirements while suppressing all jammers. This purpose can be achieved effectively by using an antenna array with multiple-beam pattern [20, 21]. Recently, a technique for adaptive beamforming with capability of providing multiple-beam constraints (MBC) has been presented in [22]. In addition, the theoretical result was extended to deal with adaptive beamforming using the well-known generalised sidelobe canceller (GSC) in the presence of random array position errors [23]. A technique based on the use of an iterative Toeplitz approximation scheme in conjunction with derivative constraints was also presented to tackle the problem owing to random array position errors.

In this paper, we consider the problem of adaptive beamforming with MBC under the random errors in array sensor locations. A robust method in conjunction with an iterative procedure is presented for coping with the considered problem. We first develop the statistical relationship between the sensor position errors and the induced array phase perturbations. As a result, the considered problem can be viewed as a problem associated with the random phase angle errors. To find the optimal phase angle vector, we construct a cost function consisting of the squared norm of the projection of the steering vector on the noise subspace and a constraint related to a likelihood function associated with the random phase error vector. Minimising the

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squared norm of the projection of the steering vector on the noise subspace is equivalent to maximising the squared norm of the projection of the steering vector on the signal plus interference subspace. The constraint related to a likelihood function associated with the random phase error vector is utilised to prevent that the obtained optimal phase angle vector for each desired signal becomes one of the interference phase angle vectors. Since the resulting minimisation problem is highly nonlinear, we use a gradient method to iteratively search the solution. During the iteration process, the algorithm developed by [19] is extended to update each beam position in turn for the desired signals. It is shown that using the constraint related to a likelihood function of the random phase error vector provides the advantage of properly adjusting the step size during the gradient search procedure. The analysis regarding the investigation of the convergence property of the proposed method is also presented. Several computer simulation examples provide the illustration and comparison.

This paper is organised as follows. Section 2 formulates the problem of adaptive beamforming with MBC in the presence of random sensor position errors and derives the statistical relationship between the random sensor position errors and the resulting random phase angle errors. Then, a robust technique is presented in Section 3 for dealing with the considered problem. In Section 4, we present a theoretical analysis to provide a proof regarding the convergence property of the proposed technique. Section 5 shows several simulation examples to show the effectiveness of the proposed technique. Finally, we conclude the paper in Section 6.

2 Problem formulation

Consider a uniform linear array (ULA) with M sensors and interelement spacing equal to half of the smallest signal wavelength λ of the signals with specified gain/null arrangements. Let K narrow-band and far-field signals be impinging on the array from direction angles θ_i, $i = 1, 2, \ldots, K$, off broadside. The signal received at the mth array sensor can be expressed as

$$z_m(t) = \sum_{i=1}^{K} s_i(t) a_m(\theta_i) + n_m(t), \quad m = 1, 2, \ldots, M \quad (1)$$

where $a_m(\theta_i) = \exp(j(2\pi d_m \sin\theta_i)/\lambda)$ and $d_m = \lambda(m-1)/2$ is the distance between the mth and the first array sensors, $s_i(t)$ is the complex waveform of the ith signal, and $n_m(t)$ is the spatially white noise with mean zero and variance σ_n^2 received at the mth array sensor. In matrix form, we can write the data vector received by the ULA as follows

$$z(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \quad (2)$$

where the matrix $\boldsymbol{A} = [\boldsymbol{a}(\theta_1)\, \boldsymbol{a}(\theta_2) \cdots \boldsymbol{a}(\theta_K)]$ with the direction vector of the ith signal given by $\boldsymbol{a}(\theta_i) = [a_1(\theta_i)\ \ a_2(\theta_i) \cdots a_M(\theta_i)]^{\mathrm{T}}$, the signal source vector is $\boldsymbol{s}(t) = [s_1(t)\ \ s_2(t) \cdots s_K(t)]^{\mathrm{T}}$, and the noise vector is $\boldsymbol{n}(t) = [n_1(t)\ \ n_2(t) \cdots n_M(t)]^{\mathrm{T}}$. The superscript T denotes transpose operation. Under the assumption that $\boldsymbol{s}(t)$ and $\boldsymbol{n}(t)$ are uncorrelated, the $M \times M$ ensemble correlation matrix of $z(t)$ is Toeplitz-Hermitian and given by

$$\boldsymbol{R}_z = [R_{kl}] = [R(k-l)] = \mathrm{E}\{z(t)z(t)^{\mathrm{H}}\} = \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^{\mathrm{H}} + \sigma_n^2 \boldsymbol{I} \quad (3)$$

where the superscript H denotes the complex conjugate transpose. $\boldsymbol{S} = \mathrm{E}\{\boldsymbol{s}(t)\boldsymbol{s}(t)^{\mathrm{H}}\}$ has rank K if the K signals are uncorrelated.

Let the ULA use a weight vector $\boldsymbol{w} = [w_1\ w_2 \cdots w_M]$ for processing the received data vector $\boldsymbol{z}(t)$ to produce the array output signal $u(t) = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{z}(t)$. Assume that the selective gain/null requirements are specified by assigning a gain c_j at the direction vector $\boldsymbol{a}(\theta_p)$ for $p = 1, 2, \ldots, P$, where P denotes the number of signals with gain/null constraint. In the case without errors, the steering constraint vectors $\boldsymbol{s}_p$, $p = 1, 2, \ldots, P$, required for adaptive beamforming are set to the direction vectors, i.e., $\boldsymbol{s}_p = \boldsymbol{a}(\theta_p)$, $p = 1, 2, \ldots, P$. Then, the optimal weight vector for the adaptive array can be found from the following constrained optimisation problem [20]

$$\begin{aligned} &\text{Minimise} \quad \mathrm{E}\{|u(t)|^2\} = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_z\boldsymbol{w} \\ &\text{Subject to} \quad \boldsymbol{G}^{\mathrm{H}}\boldsymbol{w} = \boldsymbol{c} \end{aligned} \quad (4)$$

where the matrix $\boldsymbol{G} = [\boldsymbol{a}(\theta_1)\, \boldsymbol{a}(\theta_2) \cdots \boldsymbol{a}(\theta_P)]$ denotes the constraint matrix and $\boldsymbol{c} = [c_1\, c_2 \cdots c_P]^{\mathrm{T}}$ denotes the gain vector. Accordingly, the optimal weight vector is given by

$$\boldsymbol{w}_o = \boldsymbol{R}_z^{-1}\boldsymbol{G}(\boldsymbol{G}^{\mathrm{H}}\boldsymbol{R}_z^{-1}\boldsymbol{G})^{-1}\boldsymbol{c} \quad (5)$$

Substituting (5) into $\mathrm{E}\{|u(t)|^2\} = \boldsymbol{w}^{\mathrm{H}}\mathbf{R}_z\boldsymbol{w}$ yields the corresponding array output power equal to

$$\mathrm{E}\{|u(t)|^2\} = \boldsymbol{w}^{\mathrm{H}}\boldsymbol{R}_z\boldsymbol{w} = \boldsymbol{c}^{\mathrm{H}}(\boldsymbol{G}^{\mathrm{H}}\boldsymbol{R}_z^{-1}\boldsymbol{G})^{-1}\boldsymbol{c} \quad (6)$$

2.1 Random errors in sensor positions

In the presence of random sensor position errors, the steering constraint vectors $\boldsymbol{s}_i$ can not be set to the direction vectors $\boldsymbol{a}(\theta_i) = [a_1(\theta_i)\, a_2(\theta_i) \cdots a_M(\theta_i)]^{\mathrm{T}}$, $i = 1, 2, \ldots, P$, because the actual direction vectors $\boldsymbol{a}(\theta_i) = [a_1(\theta_i)\ a_2(\theta_i) \cdots a_M(\theta_i)]^{\mathrm{T}}$, $i = 1, 2, \ldots, P$, are not exactly known owing to the errors. We assume that $\hat{\boldsymbol{u}}_m = [x_m, y_m]$ is the location of the mth array sensor as shown in Fig. 1 with

$$\hat{\boldsymbol{u}}_m = \boldsymbol{u}_m + \Delta\boldsymbol{u}_m = [\lambda(m-1)/2, 0] + [\Delta x_m, \Delta y_m] \quad (7)$$

where $\boldsymbol{u}_m = [\lambda(m-1)/2,\ 0]$ and $\Delta\boldsymbol{u}_m = [\Delta x_m,\ \Delta y_m]$, Δx_m

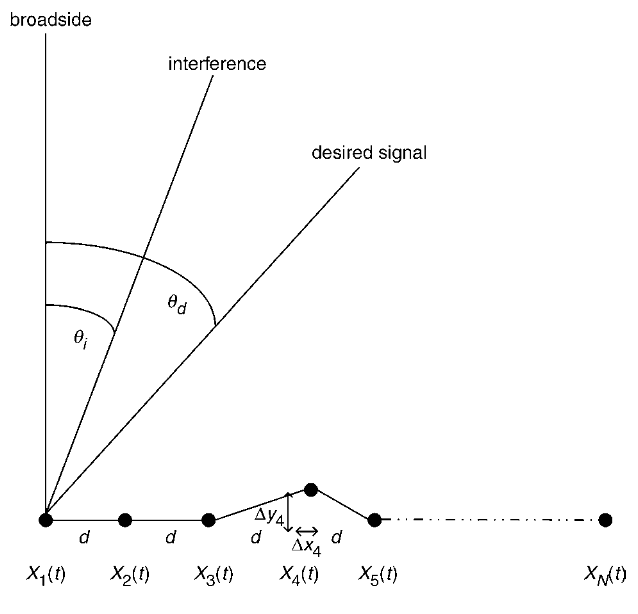

Fig. 1 *Geometrical illustration of 1-D array with sensor position errors*

and Δy_m are the associated independent Gaussian random position errors with zero mean and the same variance σ_e^2. The signal received at the mth array sensor becomes

$$z_m(t) = \sum_{i=1}^{K} s_i(t)\hat{a}_m(\theta_i) + n_m(t), \quad m = 1, 2, \ldots, M \quad (8)$$

where $\hat{a}_m(\theta_i) = \exp(j2\pi\hat{\boldsymbol{u}}_m^{\mathrm{T}}\boldsymbol{\Theta}_i/\lambda)$ and $\boldsymbol{\Theta}_i = [\sin\theta_i, \cos\theta_i]^{\mathrm{T}}$. As a result, the correlation matrix becomes

$$\begin{aligned}\boldsymbol{R}_z &= \mathrm{E}\{z(t)z(t)^{\mathrm{H}}\} \\ &= \sum_{i=1}^{K} \mathrm{E}\{|s_i(t)|^2\}\mathbf{B}_i\boldsymbol{a}(\theta_i)\boldsymbol{a}(\theta_i)^{\mathrm{H}}\mathbf{B}_i^{\mathrm{H}} + \sigma_n^2\mathbf{I} \quad (9)\end{aligned}$$

where

$$\begin{aligned}\boldsymbol{B}_i = \mathrm{diag}\{&\exp(j2\pi\Delta\boldsymbol{u}_1\boldsymbol{\Theta}_i/\lambda), \exp(j2\pi\Delta\boldsymbol{u}_2\boldsymbol{\Theta}_i/\lambda), \ldots, \\ &\exp(j2\pi\Delta\boldsymbol{u}_M\boldsymbol{\Theta}_i/\lambda)\} \quad (10)\end{aligned}$$

We note from (3) and (10) that the $\boldsymbol{B}_i$ in (10) represents the effect induced by sensor position errors. In addition, $\boldsymbol{B}_i$ depends on the source bearing θ_i and $\Delta\boldsymbol{u}_m$, $m = 1, 2, \ldots, M$. As a result, each of the $\boldsymbol{B}_i$ matrices is a function of the position error vectors $\Delta\boldsymbol{u}_m$, $m = 1, 2, \ldots, M$. The structure of the correlation matrix given by (10) reveals that the Toeplitz property possessed by (3) is destroyed by the existence of $\boldsymbol{B}_i$ and the resulting signal subspace is spanned by $\boldsymbol{B}_i\boldsymbol{a}(\theta_i)$ instead of $\boldsymbol{a}(\theta_i)$, $i = 1, 2, \ldots, K$. Consequently, the array performance will be deteriorated owing to random array position errors.

2.2 The resulting random phase angle errors

The phase observed at the mth array sensor owing to the signal with directional angle θ_i impinging on the array is given by $a_m(\theta_i) = \exp(j(2\pi d_m \sin\theta_i)/\lambda)$. The corresponding phase angle $\phi_m = (2\pi d_m \sin\theta_i)/\lambda)$ is proportional to the distance d_m between the first and the mth array sensors. Hence, the random position error $\Delta\boldsymbol{u}_m = [\Delta x_m, \Delta y_m]$ will cause a random error in ϕ_m. Substituting (7) into $\hat{a}_m(\theta_i) = \exp(j2\pi\hat{\boldsymbol{u}}_m^{\mathrm{T}}\boldsymbol{\Theta}_i/\lambda)$ and performing the necessary algebraic manipulations provides

$$\begin{aligned}\hat{a}_m(\theta_i) = \exp(j2\pi\hat{\boldsymbol{u}}_m^{\mathrm{T}}\boldsymbol{\Theta}_i/\lambda) &= \exp(j2\pi/\lambda[(m-1)d\sin\theta_i] \\ &+ j2\pi[d_{xm}\sin\theta_i + d_{ym}\cos\theta_i]) \quad (11)\end{aligned}$$

where $d_{xm} = \Delta x_m/\lambda$ and $d_{ym} = \Delta_{ym}/\lambda$. From (11), the resulting phase angle error ϕ_{em} corresponding to the random position error $\Delta\boldsymbol{u}_m = [\Delta x_m, \Delta y_m]$ is also Gaussian and is given by

$$\phi_{em} = 2\pi(d_{xm}\sin\theta_i + d_{ym}\cos\theta_i) \quad (12)$$

Since the Gaussian random position errors Δx_m and Δy_m are independent with zero mean and the same variance σ_e^2 for $m = 1, 2, \ldots, M$, the random phase angle error ϕ_{em} has zero mean and variance given by

$$\begin{aligned}\mathrm{Var}\{\phi_{em}\} &= \mathrm{Var}\{2\pi(d_{xm}\sin\theta_i + d_{ym}\cos\theta_i)\} \\ &= (2\pi)^2[\mathrm{Var}\{d_{xm}\}(\sin\theta_i)^2 + \mathrm{Var}\{d_{ym}(\cos\theta_i)^2] \\ &= (4\pi^2\sigma_e^2)/\lambda^2 \quad (13)\end{aligned}$$

Moreover, the covariance $\mathrm{E}\{\phi_{em}\,\phi_{en}\}$ between ϕ_{em} and ϕ_{en} is zero for $m \neq n$. As a result, the $M \times M$ covariance matrix $\boldsymbol{C}_i$ associated with the random phase angle error vector $\boldsymbol{\Theta}_{ei} = [\phi_{e1}, \phi_{e2}, \ldots, \phi_{eM}]^{\mathrm{T}}$ becomes a diagonal matrix given by $4\pi^2\sigma_e^2\boldsymbol{I}$, where $\boldsymbol{I}$ denotes the identity matrix.

2.3 The resulting likelihood function

Following the beamforming criterion established by (4), we consider here that the phase angle error vector for the signal with direction angle θ_p, $p = 1, 2, \ldots, P$, owing to the random position errors is given by

$$\boldsymbol{\Theta}_{ep} = \boldsymbol{\Theta}_p - \boldsymbol{\Theta}_{dp} \quad (14)$$

where $\boldsymbol{\Theta}_p$ and $\boldsymbol{\Theta}_{dp}$ denote the phase angle vectors associated with actual direction vector $\boldsymbol{a}(\theta_p)$ and the presumed direction vector $\boldsymbol{a}_d(\theta_p)$, respectively. Without loss of generality, let the mth entry of the direction vector $\boldsymbol{a}(\theta_p)$ be expressed as $a_m(\theta_p) = \exp(j\upsilon_{pm})$ and the corresponding phase angle vector be constructed as $\boldsymbol{\Theta}_p = [\upsilon_{p1}\ \upsilon_{p2} \ldots \upsilon_{pM}]^{\mathrm{T}}$. Similarly, let the mth entry of the direction vector $\boldsymbol{a}_d(\theta_p)$ be expressed as $a_{dm}(\theta_p) = \exp(j\upsilon_{dpm})$ and the corresponding phase angle vector be constructed as $\boldsymbol{\Theta}_{dp} = [\upsilon_{dp1}\ \upsilon_{dp2} \ldots \upsilon_{dpM}]^{\mathrm{T}}$. According to the discussion presented in Section 2.2, the phase angle error vector $\boldsymbol{\Theta}_{ep}$ shown by (14) is a real Gaussian random vector with mean zero and covariance matrix $\boldsymbol{C}_p$. Hence, the probability density function (PDF) for $\boldsymbol{\Theta}_{ep}$ is given by

$$PDF(\boldsymbol{\Theta}_{ep}) = [(2\pi)^{\mathrm{M}}\ \det(\boldsymbol{C}_p)]^{-1/2}\exp\{-(\boldsymbol{\Theta}_{ep}^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{\Theta}_{ep})/2\} \quad (15)$$

Accordingly, the likelihood function regarding the phase angle vector error for the signal with gain c_p can be defined as

$$\begin{aligned}LF &= \exp\{-(\boldsymbol{\Theta}_{ep}^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{\Theta}_{ep})/2\} \\ &= \exp\{-[(\boldsymbol{\Theta}_p - \boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_p - \boldsymbol{\Theta}_{dp})]/2\} \quad (16)\end{aligned}$$

To deal with the problem of array beamforming with MBC in the presence of random position errors as described above, we present a robust method in the next Section.

3 A robust method

From the property of a gain constrained array beamformer, it is well known that the output power of the beamformer will achieve its maximum when each presumed direction vector $\boldsymbol{a}_d(\theta_p)$ of the constraint matrix $\boldsymbol{G}$ coincides with the actual direction vector $\boldsymbol{a}(\theta_p)$, $p = 1, 2, \ldots, P$. Moreover, from the eigendecomposition of $\boldsymbol{R}_z$, we can express

$$\boldsymbol{R}_z = \sum_{i=1}^{M} \lambda_i\boldsymbol{e}_i\boldsymbol{e}_i^{\mathrm{H}} \quad (17)$$

where $\lambda_1 \geqq \lambda_2 \geqq \cdots \geqq \lambda_{J+P} \geqq \lambda_{J+P+1} = \cdots = \lambda_M = \sigma_n^2$, are the eigenvalues of $\boldsymbol{R}_z$ in the descending order, and $\boldsymbol{e}_i$ are the corresponding eigenvectors. J is the number of interferers. The eigenvectors associated with the minimum eigenvalue σ_n^2 are orthogonal to the direction vectors of the signals with specified gain/null constraints and interferers. Therefore, the subspaces spanned by $\boldsymbol{E}_n = \{\boldsymbol{e}_{J+P+1}, \ldots, \boldsymbol{e}_M\}$ (called the noise subspace) and $\boldsymbol{E}_s = \{\boldsymbol{e}_1, \boldsymbol{e}_2, \ldots, \boldsymbol{e}_{J+P}\}$ (called the signal plus interference subspace) are orthogonal. Consequently, we can rewrite $\boldsymbol{R}_z$ as follows

$$\boldsymbol{R}_z = \sum_{i=1}^{M} \lambda_i\boldsymbol{e}_i\boldsymbol{e}_i^{\mathrm{H}} = \boldsymbol{E}_s\boldsymbol{\Lambda}_s\boldsymbol{E}_s^{\mathrm{H}} + \boldsymbol{E}_n\boldsymbol{\Lambda}_n\boldsymbol{E}_n^{\mathrm{H}} \quad (18)$$

where $\boldsymbol{\Lambda}_s = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{J+P}\}$ and $\boldsymbol{\Lambda}_n = \sigma_n^2\boldsymbol{I}$, where $\boldsymbol{I}$ denotes the identity matrix with appropriate size.

Based on (6), (16), and (18), an appropriate cost function regarding the phase angle error for the signal with gain c_p is defined as

$$J(\boldsymbol{\Theta}_{sp}) = (\boldsymbol{s}_p)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p) - \kappa \exp\{[(\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp})]/2\} \quad (19)$$

where $\boldsymbol{s}_p = [\boldsymbol{s}_{p1}\ \boldsymbol{s}_{p2} \cdots \boldsymbol{s}_{pM}]^{\mathrm{T}} = [\exp(j\nu_{p1})\ \ \exp(j\nu_{p2}) \cdots \exp(j\nu_{pM})]^{\mathrm{T}}$, and $\boldsymbol{\Theta}_{sp} = [\nu_{p1}\ \nu_{p2} \cdots \nu_{pM}]^{\mathrm{T}}$. The first term of (19) represents the squared norm of the projection of the steering constraint vectors $\boldsymbol{s}_p$, $p = 1, 2, \ldots, P$, onto the noise subspace spanned by $\boldsymbol{E}_n$. The second term is the likelihood function related constraint. κ denotes a positive weighting parameter providing the relative weight between these terms. As a result, the optimal solution $\boldsymbol{\Theta}_{spo}$ of minimising (19) can then be used as an appropriate estimate of the actual phase angle vectors $\boldsymbol{\Theta}_p, p = 1, 2, \ldots, P$, for array beamforming. However, the cost function of (19) is a highly nonlinear function of the phase angle vectors $\boldsymbol{\Theta}_{sp}$, $p = 1, 2, \ldots, P$. Thus, a closed form solution for the optimal solution cannot exist. We resort to an iterative procedure to solve this problem as follows. First, the gradient vector of $J(\boldsymbol{\Theta}_{sp})$ is computed according to

$$\begin{aligned}\nabla_{\Phi} J(\boldsymbol{\Theta}_{sp}) = {} & [-2\,\mathrm{Re}\{j\{(\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p) \otimes \boldsymbol{s}_p^*\}\}]^{\mathrm{T}} \\ & + \kappa[\exp\{-[(\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp})]/2\} \\ & \times \boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp})] \end{aligned} \quad (20)$$

where Re{x} denotes the real part of x and $\otimes$ the Hadamard (or elementwise) product. Then, we update the phase angle vector $\boldsymbol{\Theta}_{sp}$ and the corresponding steering constraint vector $\boldsymbol{s}_p$ as follows

$$\boldsymbol{\Theta}_{sp}^{k+1} = \boldsymbol{\Theta}_{sp}^{k} - \varepsilon \nabla_{\Phi} J(\boldsymbol{\Theta}_{sp}^{k}) \quad (21)$$

$$s_{pm}^{k+1} = \exp(j\nu_{pm}^{k+1}), \quad m = 1, 2, \ldots, M \quad (22)$$

where the superscript k denotes the kth iteration and ε the preset positive step size. From (20), we note that the second term includes the factor of likelihood function related to the phase angle vector error $\boldsymbol{\Theta}_{sp} - \boldsymbol{\Theta}_{dp}$, $p = 1, 2, \ldots, P$, at the kth iteration. Hence, it would be expected that the resulting gradient approach for finding the optimal solution of $\boldsymbol{\Theta}_{sp}$ can provide a more appropriate estimate of $\boldsymbol{\Theta}_{sp}$ since the resulting step size becomes variable according to the exponential term as shown in (20). After finding the optimal estimate for the actual phase angle vector $\boldsymbol{\Theta}_p$, we substitute the optimal estimate into the cost function and repeat the above iteration process to find the optimal estimate for another actual phase angle vector $\boldsymbol{\Theta}_q$, $q = 1, 2, \ldots, P$ but $q \neq p$. From (19) and (20), we note that $\boldsymbol{\Theta}_{dp}$ stays constant throughout the proposed iteration process to provide a weighting to stop the obtained solution moving too far and, hence, the obtained solution can provide satisfactory beamforming performance.

Consider the required computational complexity. For practical implementation, we compute the sample data correlation matrix $\boldsymbol{R}_z(i)$ using i data snapshots as follows

$$\boldsymbol{R}_z(i) = \left(1 - \frac{1}{i}\right)\boldsymbol{R}_z(i-1) + \frac{1}{i}\boldsymbol{z}(i)\boldsymbol{z}(i)^{\mathrm{H}} \quad (23)$$

which is used as the estimate of the required $\boldsymbol{R}_z$, where $\boldsymbol{z}(i)$ denotes the ith data snapshot sampled from the received array data vector $\boldsymbol{z}(t)$. After computing the sample data correlation matrix $\boldsymbol{R}_z(i)$ using i data snapshots, we perform the iterative process by computing the gradient vector shown by (20) and then updating the steering constraint vectors shown by (22) to iteratively search the optimal estimate of the steering constraint vectors required for adaptive beamforming. Accordingly, the proposed robust method finds the solution after the iterative procedure is completed for every new snapshot. More than one iteration may be required. According to the ergodic property, the sample data correlation matrix $\boldsymbol{R}_z(i)$ computed by (23) gradually approaches the ensemble data correlation matrix $\boldsymbol{R}_z$ given by (3) as the number of snapshots increases. This will lead to the result that the array performance gradually approaches its steady state as expected. The required computational complexity is $O(M^2)$. Then, using a conventional approach to perform the eigendecomposition of $\boldsymbol{R}_z(i)$ provides the corresponding basis matrix $\boldsymbol{E}_n(i)$ and needs $O(M^3)$ in computational complexity. To construct the matrix $\boldsymbol{E}_n(i)\boldsymbol{E}_n(i)^{\mathrm{H}}$ requires $O(M^3)$. Therefore, the required computational complexity is about $O(M^3) + O(M^2) + M^2 + M$ in order to obtain the first term of the right-hand side of (20). Moreover, it is easy to show that the computational complexity for obtaining the second term of the right-hand side of (20) is about $4M$. As a result, the computational complexity required for computing the gradient of $J(\boldsymbol{\Phi})$ when receiving i data snapshots is about $O(M^3) + O(M^2) + M^2 + 5M$.

Next, we present an appropriate scheme for choosing the initial guess for each of the steering constraint vectors $\boldsymbol{s}_p$, $p = 1, 2, \ldots, P$, to initiate the iterative process of the proposed robust method. According to the optimal weight vector given by (5), the output of the adaptive array is approximately given by

$$u(t) = \boldsymbol{w}_o^{\mathrm{H}}\boldsymbol{z}(t) \approx \sum_{p=1}^{P} s_p(t) g_p + \boldsymbol{w}_o^{\mathrm{H}}\boldsymbol{n}(t) \quad (24)$$

based on the assumptions that $M > K$ and the interference signals are suppressed enough, where $g_p \equiv \boldsymbol{w}_o^{\mathrm{H}}\boldsymbol{a}(\theta_p)$ denotes the array gain for the pth specified signal. Equation (24) reveals that the output of the adaptive array can be used as a reference signal to find the actual phase angle vector $\boldsymbol{\Theta}_p$. Consider the cross-correlation between $\boldsymbol{z}(t)$ and $u(t)$. We have

$$\begin{aligned} E\{\boldsymbol{z}(t)u(t)^*\} &= E\{\boldsymbol{z}(t)\boldsymbol{z}(t)^{\mathrm{H}}\}\boldsymbol{w}_o = \boldsymbol{R}_z\boldsymbol{w}_o \\ &\approx \sum_{p=1}^{P} \pi_p g_p^* \boldsymbol{a}(\theta_p) + \sigma_n^2 \boldsymbol{w}_o \end{aligned} \quad (25)$$

where π_p denotes the power associated with the pth specified signal. In practice, the noise power σ_n^2 is unknown. However, it can be obtained by setting the average of the $(M - P)$ smallest eigenvalues of the autocorrelation matrix as the estimate of σ_n^2. From (25), we can therefore adopt the following vector for choosing the initial guess for each of $\boldsymbol{s}_p$, $p = 1, 2, \ldots, P$

$$\boldsymbol{v} = \boldsymbol{R}_z\boldsymbol{w}_o - \sigma_n^2\boldsymbol{w}_o \quad (26)$$

Based on (26), it is clear that the direction vector $\boldsymbol{a}(\theta_p)$ is approximately proportional to $\boldsymbol{v}$ with a proportional constant equal to $\pi_p g_p^*$ if we let the gain vector $\boldsymbol{c}$ have entries equal to zero except $c_p = 1$ in finding the optimal weight vector $\boldsymbol{w}_o$ from (5). Consequently, an appropriate initial guess $\boldsymbol{s}_p^{\mathrm{o}}$ for $\boldsymbol{s}_p$ can be set to the following normalised vector

$$\boldsymbol{s}_p^{\mathrm{o}} = (\nu_{p1})^{-1}\boldsymbol{v}_p \quad (27)$$

for $p = 1, 2, \ldots, P$, where ν_{p1} denotes the first entry of $\boldsymbol{v}_p$

and $\boldsymbol{v}_p$ the result given by (19) with the gain vector $\boldsymbol{c}$ having entries equal to zero except $c_p = 1$, the superscript 'o' represents the initial guess.

4 Convergence of the proposed method

In this Section, the convergence property of the proposed method is evaluated. For sake of simplicity, we let the vector $\boldsymbol{A}$ represent the second term of (21), i.e., $\boldsymbol{A} = -\varepsilon\nabla_\Phi J(\boldsymbol{\Theta}_{sp}^k) = [A_{p1}\, A_{p2} \cdots A_{pM}]^{\mathrm{T}}$, $p = 1, 2, \ldots, P$. Assume that $\boldsymbol{A}$ is a nonzero real vector with a norm small enough at the kth iteration. Then, $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A} > 0$, i.e.

$$\boldsymbol{A}^{\mathrm{T}}\{2\varepsilon\,\mathrm{Re}\{j\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p^k)\otimes(\boldsymbol{s}_p^k)^*\} - \varepsilon\kappa\exp\{-[(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})/2\} \times \boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})\} > 0$$

Hence,

$$\boldsymbol{A}^{\mathrm{T}}\{2\,\mathrm{Re}\{j\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p^k)\otimes(\boldsymbol{s}_p^k)^*\}\} > \kappa\exp\{-[(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})]/2\} \times \boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}-\boldsymbol{\Theta}_{dp})\} \tag{28}$$

and $\exp\{j\boldsymbol{A}\} \approx \boldsymbol{1} + j\boldsymbol{A}$, where $\exp\{j\boldsymbol{A}\} \equiv [\exp\{jA_{p1}\}\,\exp\{jA_{p2}\}\cdots\exp\{jA_{pM}\}]^{\mathrm{T}}$ and $\boldsymbol{1}$ denotes an $M\times 1$ vector with all entries equal to one. Therefore, the objective function after the $(k+1)$th iteration is given by

$$J(\boldsymbol{\Theta}_{sp}^{k+1}) = (\boldsymbol{s}_p^{k+1})^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p^{k+1}) - \kappa\exp\{-[(\boldsymbol{\Theta}_{sp}^{k+1}-\boldsymbol{\Theta}_{dp})^{\mathrm{T}} \times \boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^{k+1}-\boldsymbol{\Theta}_{dp})]/2\} \tag{29}$$

According to the above definition, (21), and (22), we have

$$\begin{aligned}\boldsymbol{s}_p^{k+1} &= \exp\{j\boldsymbol{\Theta}_{sp}^{k+1}\} = \exp\{j(\boldsymbol{\Theta}_{sp}^k + \boldsymbol{A})\}\\ &= \exp\{j\boldsymbol{A}\}\otimes\exp\{j\boldsymbol{\Theta}_{sp}^k\}\\ &\approx (\boldsymbol{1}+j\boldsymbol{A})\otimes\exp\{j\boldsymbol{\Theta}_{sp}^k\}\end{aligned} \tag{30}$$

Substituting the approximation $\exp\{j\boldsymbol{A}\}\approx\boldsymbol{1}+j\boldsymbol{A}$ and (30) into (29) and performing the necessary algebraic manipulations yields

$$\begin{aligned}J(\boldsymbol{\Theta}_{sp}^{k+1}) &\approx [(\boldsymbol{1}+j\boldsymbol{A})\otimes\exp\{j\boldsymbol{\Theta}_{sp}^k\}]^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{1}+j\boldsymbol{A})\\ &\quad\otimes\exp\{j\boldsymbol{\Theta}_{sp}^k\} - \kappa\exp\{-[(\boldsymbol{\Theta}_{sp}^k+\boldsymbol{A}-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\\ &\quad\times\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k+\boldsymbol{A}-\boldsymbol{\Theta}_{dp})]/2\} \approx (\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p^k)\\ &\quad-\kappa\,\exp\{-[(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})]/2\}\\ &\quad+(\boldsymbol{s}_p^k)^{\mathrm{H}}\otimes[-j\boldsymbol{A}]\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}(\boldsymbol{s}_p^k)+(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\\ &\quad\otimes(\boldsymbol{s}_p^k)+(\boldsymbol{s}_p^k)^{\mathrm{H}}\otimes[-j\boldsymbol{A}]\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\\ &\quad+\{(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}+\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}/2\}\\ &\quad\times\kappa\exp\{-[(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})]/2\}\\ &\approx J(\boldsymbol{\Theta}_{sp}^k)+2\,\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}\\ &\quad+(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}\kappa\,\exp\{-[(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\\ &\quad\times\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})]/2\}\end{aligned} \tag{31}$$

since the norm of $\boldsymbol{A}$ is small enough and we neglect the terms $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}$ and $(\boldsymbol{s}_p^k)^{\mathrm{H}}\otimes[-j\boldsymbol{A}]\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)$.

From (31), it is clear that we have to show

$$2\,\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}+(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}\kappa \times\exp\{-[(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})]/2\}\leqq 0 \tag{32}$$

for any k in order to ensure the convergence property of the proposed method. Based on (28), the condition of (31) can be reformulated as follows

$$2\,\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}+\boldsymbol{A}^{\mathrm{T}}2\,\mathrm{Re}\{j[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p^k]\otimes(\boldsymbol{s}_p^k)^*\}\leqq 0 \tag{33}$$

Next, we investigate the left hand side of (33) as follows

$$\begin{aligned}&2\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}+\boldsymbol{A}^{\mathrm{T}}2\,\mathrm{Re}\{[j\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p^k]\otimes(\boldsymbol{s}_p^k)^*\}\\ &=2\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)+\boldsymbol{A}^{\mathrm{T}}j[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p^k]\otimes(\boldsymbol{s}_p^k)^*\}\\ &=2\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)+[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p^k]^{\mathrm{T}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)^*\}\\ &=2\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)+\{(\boldsymbol{s}_p^k)^{\mathrm{H}}[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}]^{\mathrm{H}}[-j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}^*\}\\ &=2\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)-\{(\boldsymbol{s}_p^k)^{\mathrm{H}}[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}][j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}^*\}\\ &=2\mathrm{Re}\{2j\,\mathrm{Im}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}\}=0\end{aligned} \tag{34}$$

Hence, the result given by the left hand side of (33) is always equal to zero, i.e.

$$2\,\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}+\boldsymbol{A}^{\mathrm{T}}2\,\mathrm{Re}\{j[\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}\boldsymbol{s}_p^k]\otimes(\boldsymbol{s}_p^k)^*\}=0 \tag{35}$$

Consequently, we obtain

$$2\,\mathrm{Re}\{(\boldsymbol{s}_p^k)^{\mathrm{H}}\boldsymbol{E}_n\boldsymbol{E}_n^{\mathrm{H}}[j\boldsymbol{A}]\otimes(\boldsymbol{s}_p^k)\}+(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}\boldsymbol{A}\kappa \times\exp\{-[(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})^{\mathrm{T}}\boldsymbol{C}_p^{-1}(\boldsymbol{\Theta}_{sp}^k-\boldsymbol{\Theta}_{dp})]/2\}<0 \tag{36}$$

It follows from (31) and (36) that

$$J(\boldsymbol{\Theta}_{sp}^{k+1}) < J(\boldsymbol{\Theta}_{sp}^k) \tag{37}$$

The result shown by (37) ensures the convergence of the proposed method.

5 Computer simulation examples

In this Section, we present several simulation examples for illustration and comparison. For all simulation examples, we use a ULA with interelement spacing equal to the half of the minimum wavelength λ of the signals with specified gain/null requirements. Let the random position error for each of Δx_m and Δy_m have zero mean and variance of $0.01\lambda^2$. All simulation results presented are obtained by averaging 50 independent runs with independent noise samples for each run. The iterative procedure is completed for every new snapshot. More than one iteration may be required. The parameters ε and κ used for simulations are set to 0.1 and 0.0001, respectively. The dash curve in each of the Figures represents the simulation result obtained from a constrained beamformer with no correction for sensor position errors.

Example 1: In this example, three signal sources with signal-to-noise (SNR) equal to 5, 10, and 10 dB, respectively, are impinging on the array with size $M = 12$ from direction angles 0°, 60°, and 80°, respectively. Assume that the specified signal is the first signal with $c_1 = 1$ and the others are the jammers. Figure 2 plots the simulation

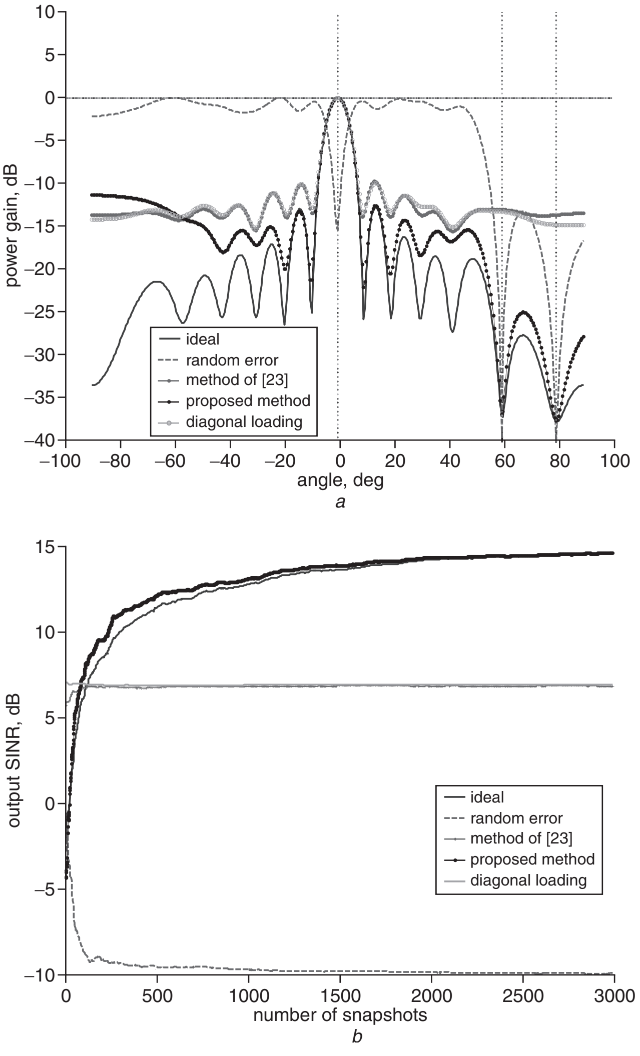

Fig. 2 *Array beam patterns and SINR against number of snapshots for example 1*

a Array beam patterns
b Output SINR against number of snapshots

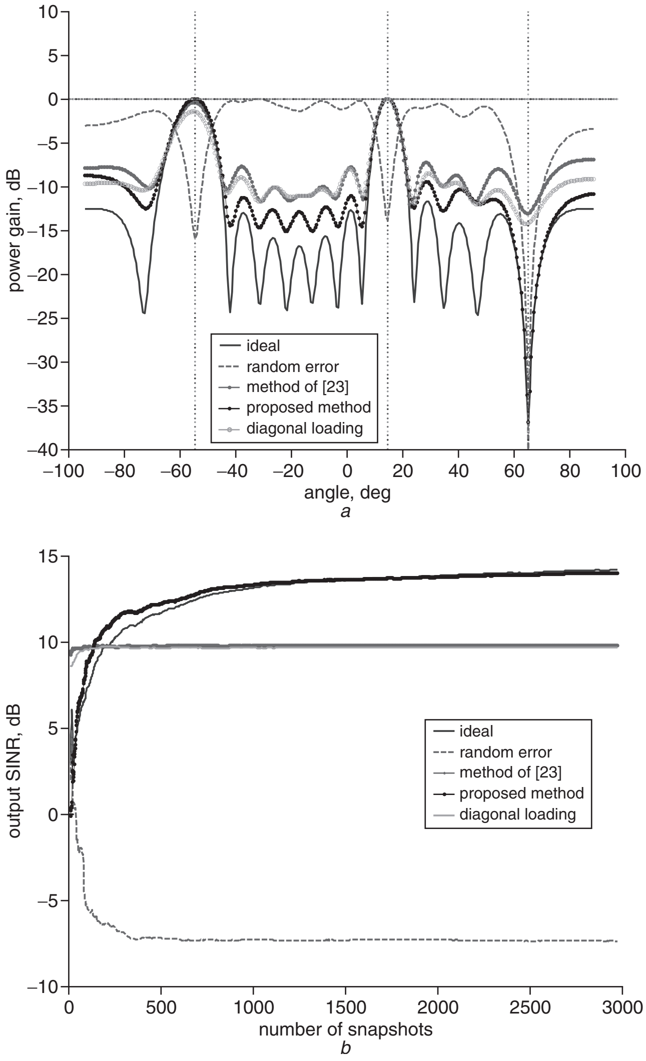

Fig. 3 *Array beam patterns and SINR against number of snapshots for example 2*

a Array beam patterns
b Output SINR against number of snapshots

results in terms of the array beam patterns and the corresponding array output signal-to-interference-plus-noise ratio (SINR) with and without utilising the proposed method. For comparison, the results of using the diagonal loading method [18] with loading factor of 2000 which is chosen optimally by experiment, the method of [23], and without random position errors are also shown. The output SINRs obtained by using 15,000 data snapshots for the results of using the proposed method after 143 iterations, the diagonal loading method, the method of [23], and the result without random position errors are 15.25, 6.90, 6.77, and 15.50 dB, respectively. We observe from the simulation results that the proposed method can cope effectively with the performance degradation owing to the random array position errors.

Example 2: Here, we consider the case of three signals with SNR equal to 3, 4, and 10 dB, respectively, are impinging on the array with size $M = 12$ from direction angles 17°, -51°, and 67°, respectively. Assume that the specified signals are the first two signals with $c_1 = c_2 = 1$ and the third one is the jammer. Figure 3 depicts the simulation results in terms of the array beampatterns and the corresponding array output SINR with and without utilising the proposed method. For comparison, the results of using the diagonal loading method [18] with loading factor of 2000 which is chosen optimally by experiment, the method of [23], and without random position errors are also shown. The output SINRs obtained by using 15,000 data snapshots for the results of using the proposed method after 223 iterations, the diagonal loading method, the method of [23], and the result without random position errors are 14.18, 9.55, 9.65, and 14.52 dB, respectively. For this case with multiple-beam constraints, we observe from the simulation results that the proposed method can effectively cure the performance degradation owing to the random array position errors.

Example 3: Here, the case of four signals with SNR equal to 5, 6, 7, and 10 dB, respectively, are impinging on the array with size $M = 15$ from direction angles 20°, -20°, -60°, and 70°, respectively. Assume that the specified signals are the first three signals with $c_1 = c_2 = c_3 = 1$ and the fourth one is the jammer. Figure 4 depicts the simulation results in terms of the array beam patterns and the corresponding array output SINR with and without utilising the proposed method. For comparison, the results of using the diagonal loading method [18] with loading factor of 2000 which is chosen optimally by experiment, the method of [23], and without random position errors are

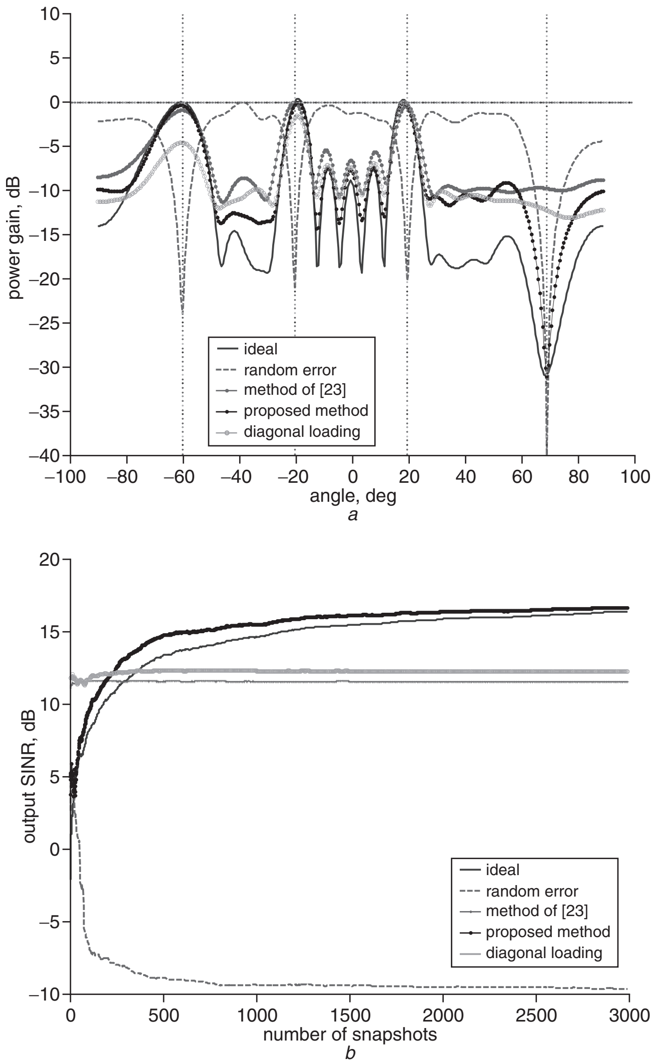

Fig. 4 *Array beam patterns and SINR against number of snapshots for example 3*

a Array beam patterns
b Output SINR against number of snapshots

also shown. The output SINRs obtained by using 15,000 data snapshots for the results of using the proposed method after 268 iterations, the diagonal loading method, the method of [23], and the result without random position errors are 17.11, 12.17, 11.50, and 17.25 dB, respectively. Again, we observe from the simulation results that the proposed method can provide very satisfactory performance in this case with multiple-beam constraints.

6 Conclusion

This paper has presented an efficient method for multiple-beam adaptive beamforming in the presence of random array position errors. We have illustrated that the performance degradation of an adaptive beamformer with multiple-beam constraints owing to random array position errors is significant. According to the proposed method, the equivalent random phase angle errors owing to random array position errors is first developed. Then, we construct a cost function consisting of the squared norm of the projection of the steering vector on the noise subspace and a constraint related to a likelihood function associated with the resulting random phase error vector. The resulting minimisation problem is highly nonlinear but can be solved through the use of an iterative procedure. In conjunction with a steepest-descent algorithm, the estimates for the phase angles of the signals with specified gain constraints can be obtained simultaneously. The corresponding constraint matrix required for finding the optimal weight vector is then constructed. The convergence property regarding the proposed method has been investigated. Several simulation examples have shown the effectiveness of the proposed method in dealing with multiple-beam adaptive beamforming under random array position errors. In general, the proposed robust method would not work with any amplitude errors owing to array imperfections. It is worth further investigating the improvement of the proposed robust method to deal with array performance degradation due to any amplitude errors.

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