

# The Analysis of Relation between Q-factor and Phase Noise by Using Substrate-integrated Waveguide Cavity Oscillators

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**Abstract**—The relation between Q-factor and phase noise is verified in this paper. According to the phase noise model proposed by D.B. Leeson in 1966, it estimates about a 6 dB reduction in phase noise as Q-value of resonator doubled. By fabricating three substrate-integrated waveguide cavity oscillators on different substrates with different Q-values, we could obtain the same tendencies between measurements and computed values from the proposed model and thus verified the relation between Q-factor and phase noise.

## I. INTRODUCTION

An oscillator is one of basic yet essential components in the communication system. There are several factors used to judge the performance of a design and the phase noise is one of the most important indices. The phase noise represents how good the frequency selectivity of an oscillator is. An oscillator with lower phase noise means a design with purer output signals. According to the phase noise model proposed by D.B. Leeson in 1966 [1] and following researches [2]-[3], we know that the low noise performance of oscillators is primarily dominated by high Q-value (quality factor) of resonators. In a traditional hybrid oscillator design, it is common to use high-Q components such as the dielectric resonators or metal cavities to achieve the low noise objectives. However,

these components of large, 3D structures and manufactured by complex processes make them repulsive in today's small size, low cost and high integrity requirements. A new structure of substrate-integrated waveguide (SIW) proposed by K. Wu [4] et al in 2001 provides a great scheme to solve above issues. With planar structure and high-Q characteristics of SIW cavity resonator, as shown in Fig.1, it can offer pretty good performance of phase noise in oscillator designs and with low-cost and high-yield print circuit board (PCB) or low-temperature co-fired ceramic (LTCC) processes.

Although the Leeson's model has been explored for years, to our knowledge, there is no direct verification of the relation between Q-factor and phase noise from measurements. Basing on Leeson's phase noise model, we know there is about a 6 dB reduction in phase noise as Q-value of resonator doubled. Since the Q-factor depends on the cavity substrate loss and matching conditions, we could design oscillators on different substrates with different Q-values of SIW cavity resonators. By using these designed oscillators in the same parallel feedback structure, we could find out the similar inclinations between measurements and computed values from Leeson's model, and also validated the relation of Q-factor and phase noise presented by Leeson directly from experiments.

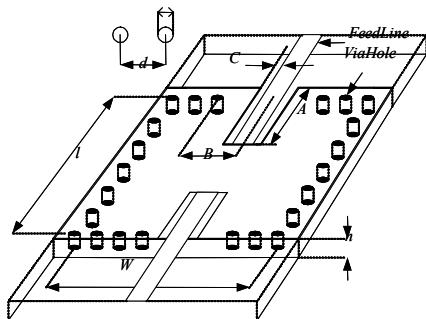


Fig.1 The structure of substrate-integrated waveguide cavity.

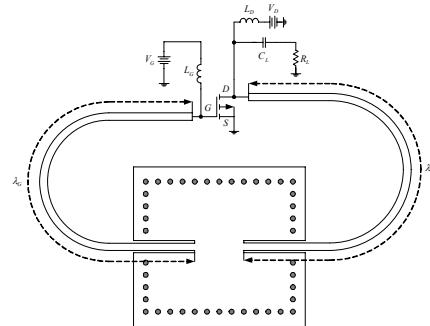


Fig.2 The structure of parallel feedback type SIW cavity oscillator.

## II. OSCILLATOR DESIGN

The design of SIW cavity oscillator is based on two prominent papers [5]-[6] and the structure is shown in Fig.2. We choose the oscillation frequency first. Since the propagation mode in SIW cavity is similar to traditional rectangular waveguide [4], we can determine the resonant frequency by

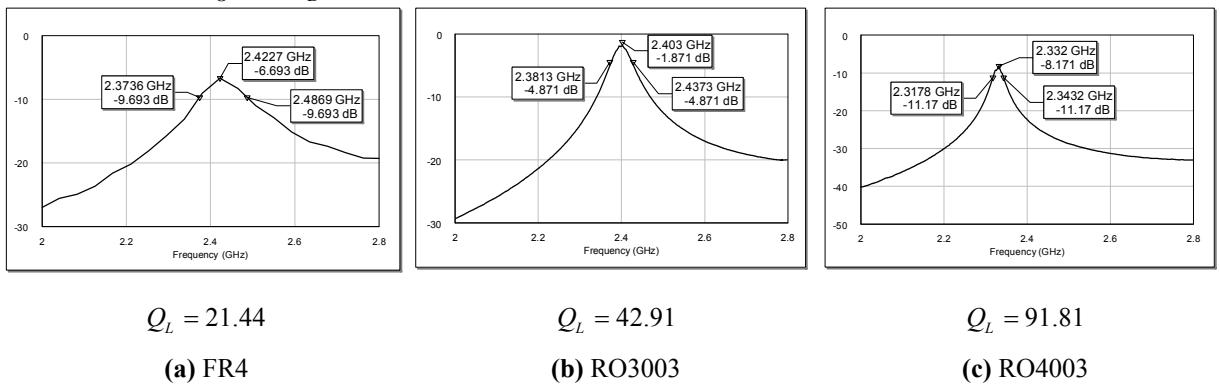
$$f_{R(\text{TE}_{m0n})} = \left( c_0 / 2\sqrt{\epsilon_r} \right) \sqrt{(m/W_{\text{eff}})^2 + (n/L_{\text{eff}})^2} ,$$

where the effective length and width are  $L_{\text{eff}} = L - D^2/(0.95 \cdot d)$ ,  $W_{\text{eff}} = W - D^2/(0.95 \cdot d)$ . The resonant frequencies of three designs are set to be 2.4 GHz. Secondly, we design the SIW cavity to our desired Q-value. Although the unloaded Q-factor of resonator is dominated by the substrate, we could vary the feeding length “A”, via surrounding distance “B” and feeding gap width “C” to vary the loaded Q-factor of cavity resonator, as shown in Fig.1. We let Q-values be increased progressively in a multiple step to get different phase noise levels. The substrates used in this paper are FR4, RO3003, and RO4003. After the characteristics of cavity resonators have been decided, we can choose the DC bias of active circuit. The purpose of amplifier is to compensate the loss of cavity resonator with positive feedback amplification. We set  $V_{\text{DS}} = 2$  V and  $I_{\text{D}} = 9$  mA to meet the gain condition of Barkhausen criterion in all three designs. The oscillators were designed using packaged pHEMT Agilent 54143 with  $G_{\text{max}}$  of 19 dB at 2.4 GHz and  $f_{\text{max}}$  is estimated to be 6 GHz. Finally, we change the electrical lengths of feedings,  $\lambda_G$  and  $\lambda_D$ , to meet the phase condition of oscillation to complete the design. Additionally, as shown in Fig.2, the DC bias inductances,  $L_G$  and  $L_D$ , are replaced by one

pole bandstop filter and surface mounted (SMD) capacitors are used for DC blocking.

## III. MEASUREMENT RESULTS AND DISCUSSIONS

Three different resonators are designed to give different progressive Q-values. Fig.3 are measured  $S_{21}$  of three SIW cavities. The Q-factors are calculated according to the transmission type measurement [7] of  $Q_L = 21.44$  on FR4,  $Q_L = 42.91$  on RO3003 and  $Q_L = 91.81$  on RO4003. The measured phase noises and photographs of fabricated oscillators are shown in Fig.4. The comparison of measured results and computed values by Leeson’s model are listed in Table I. First, let’s compare the measured phase noise with the value predicted by Leeson’s model, the difference is only about 1~2 dB among different substrates for 100 kHz offset. Secondly, let’s check the relation between the phase noise and loaded Q of resonators. The loaded Q of resonators in RO3003 is about double the Q-value of FR4’s oscillator. This gives about 5 dB reduction in measured phase noise. The loaded Q of RO4003 is about double the Q-value of RO3003, which give about 6 dB reduction in measured phase noise. For the values predicted by Leeson’s model, there are 6 dB reduction from FR4 to RO3003 and 7 dB reduction from RO3003 to RO4003. Thus, the estimated phase noise by the Leeson’s model is proved to be valid from these experiments. For the relation of Q-factor and phase noise, there is less than 1 dB difference between measurements and predicted values and the tendencies are very similar, as shown in Fig.6. Therefore, we verified the relation with about 6 dB reduction in phase noise as a multiple increased on Q-value of oscillator.



$$Q_L = 21.44$$

(a) FR4

$$Q_L = 42.91$$

(b) RO3003

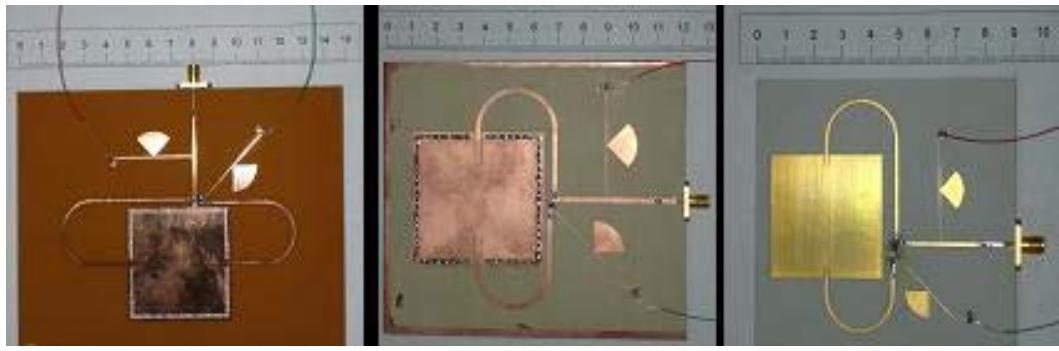
$$Q_L = 91.81$$

(c) RO4003

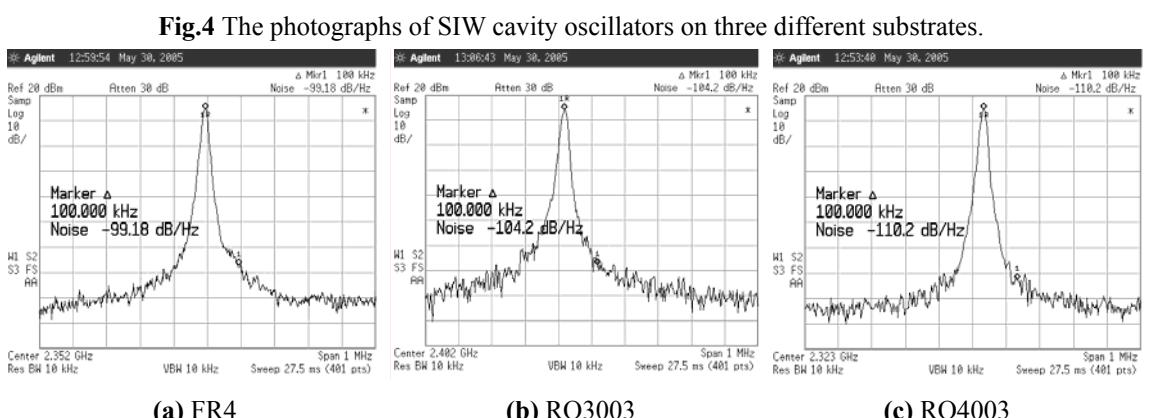
**Fig.3** The measured  $S_{21}$  and Q-factors of SIW cavities on three different substrates.

**Table I** The phase noise comparison between measurements and the Leeson's model

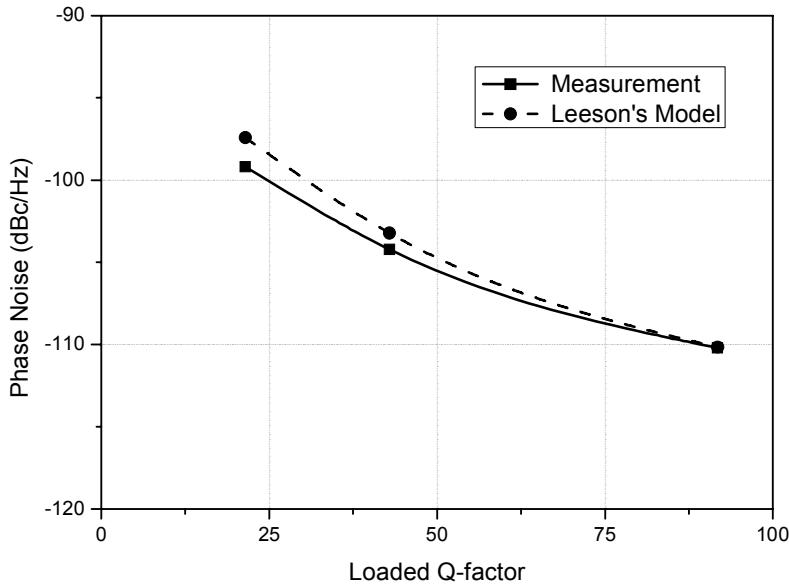
	FR4	RO3003	RO4003
<b>Measurement</b>			
$Q_L$	21.44	42.91	91.81
Phase Noise	-99.18 dBc/Hz @ 100 kHz	-104.2 dBc/Hz @ 100 kHz	-110.2 dBc/Hz @ 100 kHz
Phase noise comparison	RO3003 vs. FR4	RO4003 vs. RO3003	5.0 dB @ 100 kHz
D.B. Leeson's Phase Noise Model	$L(\Delta f) = 10 \log \left\{ \frac{KTF}{2P_{av}} \left[ \frac{f_c}{\Delta f^3} \left( \frac{f_0}{2Q_L} \right)^2 + \frac{1}{\Delta f^2} \left( \frac{f_0}{2Q_L} \right)^2 + \frac{f_c}{\Delta f} + 1 \right] \right\}$		
where the room temperature is 298 °K , $F_{min} \square 0.4$ dB , $f_c = 300$ KHz .			
Phase Noise	-97.42 dBc/Hz @ 100 kHz	-103.22 dBc/Hz @ 100 kHz	-110.15 dBc/Hz @ 100 kHz
Phase noise comparison	FR4 vs. RO3003	RO3003 vs. RO4003	5.8 dB @ 100 kHz
	6.9 dB @ 100 kHz		



**Fig.4** The photographs of SIW cavity oscillators on three different substrates.



**Fig.5** The measured phase noise of SIW cavity oscillators on three different substrates.



**Fig.6** Predicted and measured phase noise at 100 kHz offset under different Q-valued oscillators.

#### IV. CONCLUSION

The SIW cavity oscillators are designed and fabricated on three different substrates. By using these cavity oscillators with different Q-values, we conduct the experiment on effect of Q-factor to phase noise. We successfully prove the Leeson's phase noise model which predicts a 6 dB phase noise reduction as the Q-value of oscillator doubled.

#### ACKNOWLEDGEMENT

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