shallowness ratio results in higher frequency for a thin shell, but the effect is insignificant for a thick shell.

References

[1]Leissa, A. W., "Vibration of Shells," NASA SP-288, 1973.

[2]Leissa, A. W., Lee, J. K., and Wang, A. J., "Vibrations of Cantilevered Shallow Cylindrical Shells of Rectangular Planform," *Journal of Sound and Vibration*, Vol. 78, No. 3, 1981, pp. 311–328.

[3]Cheung, Y. K., Li, W. Y., and Tham, L. G., "Free Vibration Analysis of Singly Curved Shell by Spline Finite Strip Method," *Journal of Sound and Vibration*, Vol. 128, No. 3, 1989, pp. 411–422.

[4]Liew, K. M., and Lim, C. W., "Vibratory Characteristics of Cantilevered Rectangular Shallow Shells of Variable Thickness," *AIAA Journal*, Vol. 32, No. 2, 1994, pp. 387–396.

[5]Lim, C. W., and Liew, K. M., "A *pb*-2 Ritz Formulation for Flexural Vibration of Shallow Cylindrical Shells of Rectangular Planform," *Journal of Sound and Vibration*, Vol. 173, No. 3, 1994, pp. 343–375.

[6]Palazotto, A. N., and Linnemann, P. E., "Vibration and Buckling Characteristics of Composite Cylindrical Panels Incorporating the Effects of a Higher Order Shear Theory," *International Journal of Solids and Structures*, Vol. 28, No. 3, 1991, pp. 341–361.

[7]Liew, K. M., and Lim, C. W., "Vibratory Characteristics of Pretwisted Cantilevered Trapezoids of Unsymmetric Laminates," *AIAA Journal*, Vol. 34, No. 5, 1996, pp. 1041–1050.

[8]Lim, C. W., and Liew, K. M., "A Higher Order Theory for Vibration of Shear Deformable Cylindrical Shallow Shells," *International Journal of Mechanical Sciences*, Vol. 37, No. 3, 1995, pp. 277–295.

Relationship of Anisotropic and Isotropic Materials for Antiplane Problems

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Introduction

THE problem of determining the stress distribution around defects has been considered by many authors over the past years. The strength of materials is influenced by the existing defects such as cracks, which can cause the stress concentration near the defects. The problem of finding the stress singularities at the apex of an isotropic elastic wedge was considered by Williams,[1] who used the eigenfunction-expansion method. Tranter[2] used the Mellin transform in conjunction with the Airy stress function representation of plane elasticity to solve for the isotropic wedge problem. Stroh[3] obtained an analytical solution to the problem of a plane crack in an anisotropic material of infinite extent. Following the approach of Stroh, Ting[4] studied the stress distribution near the composite wedge of anisotropic materials. A complex function representation of a generalized Mellin transform was employed by Bogy[5] for analyzing stress singularities in an anisotropic wedge. The antiplane problem of two dissimilar anisotropic wedges of arbitrary angles that are bonded together perfectly along a common edge has been considered recently by Ma and Hour.[6] They found the order of the stress singularity to be always real for the antiplane dissimilar anisotropic wedge problems. That is quite a different characteristic from the in-plane case in which the complex type of stress singularity might exist.

A previous analysis of the dissimilar anisotropic antiplane wedge problem by Ma[7] showed that, if an effective angle and an effective material constant are introduced for the anisotropic case, then the order of singularity for the anisotropic material can be obtained from the result of the isotropic case. The results obtained by Ma and Hour[6] have been extended to the present study for the full field, and the correspondence in the stress-displacement relationship has been established between anisotropic and isotropic materials. The reduction in the number of elastic constants considerably simplifies the description of the stress and displacement state. The Mellin transform method, originally applied to the analysis of wedge problems by Tranter,[2] is used for analyzing the antiplane problem of anisotropic material. The material symmetry is assumed to be such that the in-plane and the antiplane deformations are uncoupled. A correspondence is obtained between the anisotropic and the isotropic problem. Through such a correspondence, the relationship of the stresses and displacement for the anisotropic and the corresponding isotropic problem is established for both polar and Cartesian coordinate systems. Any solution of anisotropic material for the antiplane problem can be obtained by solving a corresponding isotropic problem.

Received Feb. 3, 1996; revision received June 18, 1996; accepted for publication June 25, 1996; also published in *AIAA Journal on Disc*, Volume 2, Number 1.

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General Solutions in Mellin-Transform Domain

Isotropic Case

The isotropic material is considered first. It is well known that the only nonvanishing displacement component w^i is along z axis for the antiplane deformation. The equilibrium equation for the nonvanishing displacement w^i is given by the following partial differential equation:

$$\frac{\partial^2 w^i}{\partial r^2} + \frac{1}{r}\frac{\partial w^i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w^i}{\partial \theta^2} = 0 \tag{1}$$

The nonvanishing stresses are

$$\tau^i_{rz} = \mu \frac{\partial w^i}{\partial r} \tag{2}$$

$$\tau^i_{\theta z} = \frac{\mu}{r}\frac{\partial w^i}{\partial \theta} \tag{3}$$

Let the Mellin transform of a function $f(r)$ be denoted by $\hat{f}(s)$:

$$\hat{f}(s) = M\{f; s\} = \int_0^\infty f(r) r^{s-1}\,\mathrm{d}r \tag{4}$$

where s is a complex transform parameter. Let $\hat{w}^i(s,\theta)$, $\hat{\tau}^i_{rz}(s,\theta)$, $\hat{\tau}^i_{\theta z}(s,\theta)$ in this order denote the Mellin transforms of $w^i(r,\theta)$, $r\tau^i_{rz}(r,\theta)$, and $r\tau^i_{\theta z}(r,\theta)$ with respect to r. Accordingly,

$$\hat{w}^i(s,\theta) = \int_0^\infty w^i(r,\theta) r^{s-1}\,\mathrm{d}r \tag{5}$$

$$\hat{\tau}^i_{rz}(s,\theta) = \int_0^\infty \tau^i_{rz}(r,\theta) r^{s}\,\mathrm{d}r \tag{6}$$

$$\hat{\tau}^i_{\theta z}(s,\theta) = \int_0^\infty \tau^i_{\theta z}(r,\theta) r^{s}\,\mathrm{d}r \tag{7}$$

Application of the inversion formula to Eqs. (5–7) gives

$$w^i(r,\theta) = \frac{1}{2\pi i}\int_{\rho - i\infty}^{\rho + i\infty} \hat{w}^i(s,\theta) r^{-s}\,\mathrm{d}s \tag{8}$$

$$\tau^i_{rz}(r,\theta) = \frac{1}{2\pi i}\int_{\rho - i\infty}^{\rho + i\infty} \hat{\tau}^i_{rz}(s,\theta) r^{-s-1}\,\mathrm{d}s \tag{9}$$

$$\tau^i_{\theta z}(r,\theta) = \frac{1}{2\pi i}\int_{\rho - i\infty}^{\rho + i\infty} \hat{\tau}^i_{\theta z}(s,\theta) r^{-s-1}\,\mathrm{d}s \tag{10}$$

Applying the Mellin transform (5) to Eq. (1) yields an ordinary differential equation for $\hat{w}^i$, the general solution of which is

$$\hat{w}^i(s,\theta) = a(s)\sin(s\theta) + b(s)\cos(s\theta) \tag{11}$$

in which the functions $a(s)$ and $b(s)$ are to be determined through the boundary conditions. The general expressions for the shear stresses in the transform domain are

$$\hat{\tau}^i_{\theta z}(s,\theta) = \mu s[a(s)\cos(s\theta) - b(s)\sin(s\theta)] \tag{12}$$

$$\hat{\tau}^i_{rz}(s,\theta) = -\mu s[a(s)\sin(s\theta) + b(s)\cos(s\theta)] \tag{13}$$

Anisotropic Case

Next, the general solution in the Mellin-transform domain for anisotropic material is constructed. The stress components are related to the displacement as follows:

$$\tau^a_{yz} = c_{44}\frac{\partial w^a}{\partial y} + c_{45}\frac{\partial w^a}{\partial x} \tag{14}$$

$$\tau^a_{xz} = c_{45}\frac{\partial w^a}{\partial y} + c_{55}\frac{\partial w^a}{\partial x} \tag{15}$$

In the absence of body forces, the corresponding displacement equation of equilibrium for a homogeneous anisotropic material is given by

$$c_{55}\frac{\partial^2 w^a}{\partial x^2} + 2c_{45}\frac{\partial^2 w^a}{\partial x\partial y} + c_{44}\frac{\partial^2 w^a}{\partial y^2} = 0 \tag{16}$$

The governing equation (16) can be solved in the complex plane $\xi = x + py$ such that

$$w^a(x,y) = 2\mathrm{Re}[U(\xi)] \tag{17}$$

where U is an arbitrary function of ξ and p is a value dependent on the material constants. Substitution of Eq. (17) into Eq. (16) yields the following characteristic equation for p:

$$c_{44}p^2 + 2c_{45}p + c_{55} = 0 \tag{18}$$

Hence,

$$p = \frac{-c_{45} \pm i\sqrt{c_{44}c_{55} - (c_{45})^2}}{c_{44}}$$

It is expedient to define

$$\phi(z) = i\sqrt{c_{44}c_{55} - (c_{45})^2}\,\frac{\mathrm{d}U}{\mathrm{d}z} \tag{19}$$

so that the shear stresses can be written simply as

$$\tau^a_{xz} = -(p\phi + \bar{p}\bar{\phi}) \tag{20}$$

$$\tau^a_{yz} = \phi + \bar{\phi} \tag{21}$$

where the overbar denotes complex conjugate. Consider the stress transformation

$$\tau^a_{\theta z} = \tau^a_{yz}\cos\theta - \tau^a_{xz}\sin\theta \tag{22}$$

$$\tau^a_{rz} = \tau^a_{yz}\sin\theta + \tau^a_{xz}\cos\theta \tag{23}$$

The solution of the problem is obtained by use of the integral transform, which is a complex analogy to the standard Mellin transform. Let $\hat{U}(s)$ be defined by

$$\hat{U}(s) = \int_0^\infty U(z)z^{s-1}\,\mathrm{d}z = (\cos\theta + p\sin\theta)^s\int_0^\infty U(z)r^{s-1}\,\mathrm{d}r \tag{24}$$

in which the path of integration is along a ray of fixed θ and s is a complex transform parameter.

The general solution of the equilibrium equation for anisotropic material in the Mellin-transform domain then can be simplified as follows:

$$\hat{w}^a(s,\theta) = \Psi^{-s}(\theta)[a(s)\sin s\phi + b(s)\cos s\phi] \tag{25}$$

$$\hat{\tau}^a_{\theta z}(s,\theta) = Cs\Psi^{-s}(\theta)[a(s)\cos s\phi - b(s)\sin s\phi] \tag{26}$$

$$\hat{\tau}^a_{rz}(s,\theta) = Cs\Psi^{-(s+2)}(\theta)[a(s)(\Theta\cos s\phi - B\sin s\phi) - b(s)(\Theta\sin s\phi + B\cos s\phi)] \tag{27}$$

where

$$\tan\phi = \frac{\sqrt{c_{44}c_{55} - (c_{45})^2}\sin\theta}{c_{44}\cos\theta - c_{45}\sin\theta} \tag{28}$$

$$\Psi(\theta) = \left[\cos^2\theta - (c_{45}/c_{44})\sin 2\theta + (c_{55}/c_{44})\sin^2\theta\right]^{\frac{1}{2}} \tag{29}$$

$$\Theta = \sin\theta\cos\theta + (c_{45}/c_{44})\cos 2\theta - (c_{55}/c_{44})\sin\theta\cos\theta \tag{30}$$

$$B = \frac{\sqrt{c_{44}c_{55} - (c_{45})^2}}{c_{44}}$$

Relationship Between Anisotropic and Isotropic Problems

For convenience, let the coordinate system associated with the anisotropic problem be denoted by r and θ (or x and y) and the isotropic problem be denoted by R and ϕ(or X and Y) in the following context. If comparison is made of the general solution of $\hat{w}^a$, $\hat{\tau}^a_{\theta z}$, and $\hat{\tau}^a_{rz}$ in Eqs. (25), (26), and (27), respectively, for anisotropic material and in Eqs. (11), (12), and (13), respectively, for the isotropic material, then a very simple relationship for the anisotropic problem and the corresponding isotropic problem in the polar coordinate can be established as follows:

$$w^a(r,\theta) = (\mu/C)w^i(R,\phi) \tag{31}$$

$$\tau^a_{\theta z}(r,\theta) = \Psi(\theta)\tau^i_{\theta z}(R,\phi) \tag{32}$$

$$\tau^a_{rz}(r,\theta) = [1/\Psi(\theta)]\left[\Theta\tau^i_{\theta z}(R,\phi) + B\tau^i_{rz}(R,\phi)\right] \tag{33}$$

where

$$R = \Psi(\theta)r \tag{34}$$

$$\phi = \tan^{-1}\left[\frac{\sqrt{c_{44}c_{55} - (c_{45})^2}\sin\theta}{c_{44}\cos\theta - c_{45}\sin\theta}\right] \tag{35}$$

The relationships between the solutions for the displacement and shear stresses of an anisotropic problem and that for the corresponding isotropic problem are expressed in Eqs. (31–33). The solution for an anisotropic problem can be obtained by replacing the angle θ by ϕ and r by R in the isotropic solution. The geometric changes of the boundary for the associated isotropic problem can be constructed from Eqs. (34) and (35). Hence, a material point in the anisotropic

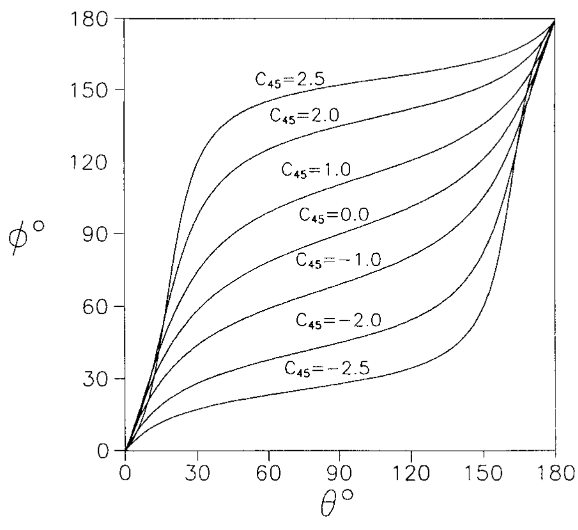

Fig. 1 Numerical results of ϕ as a function of θ for $c_{44} = 1$, $c_{55} = 8$ and different values of c_{45}.

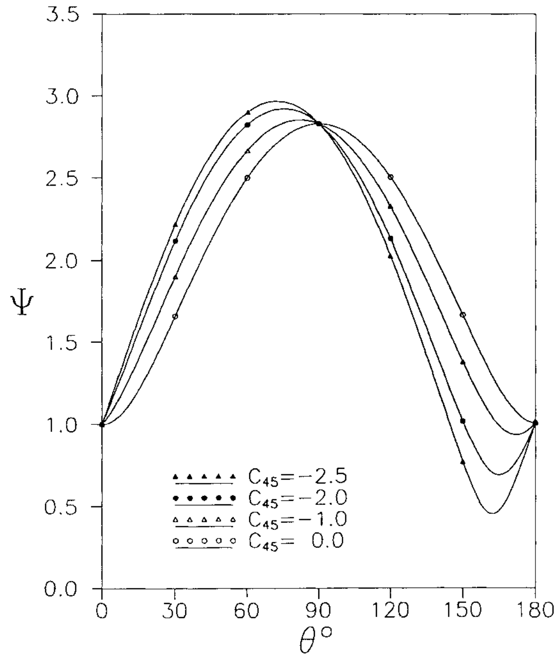

Fig. 2 Numerical results of Ψ as a function of θ for $c_{44} = 1, c_{55} = 8$ and different values of c_{45}.

problem located at (r, θ) is then to be changed to (R, ϕ) in the corresponding isotropic problem. The functions $\Psi(\theta)$ and $\phi(\theta)$ for different material constants are plotted in Figs. 1 and 2. It is shown that if $\theta = 0$ deg (or $\theta = 180$ deg), we have $\Psi = 1$ and $\phi = 0$ deg (or $\phi = 180$ deg). The material points at $\theta = 0$ (or $\theta = 180$ deg) therefore do not change when transforming to the corresponding isotropic problem. From the relationship of the transformation from the anisotropic to the isotropic problem in a polar coordinate system as shown in Eqs. (34) and (35), this relationship in the Cartesian coordinate system also can be obtained here as follows:

$$X = x - (c_{45}/c_{44})y \tag{36}$$

$$Y = (C/c_{44})y \tag{37}$$

where $x - y$ and $X - Y$ are the respective Cartesian coordinate systems in anisotropic and isotropic problems. It has some interesting features in the Cartesian coordinate transformation. For a straight line (x_1, y_0), (x_2, y_0) parallel to the x axis will still be a straight line (X_1, Y_0), (X_2, Y_0) parallel to the X axis and with the same length after transformation, i.e., $X_2 - X_1 = x_2 - x_1$. For a vertical line (x_0, y_1), (x_0, y_2) parallel to the y axis will be a straight line (X_0, Y_1), (X_1, Y_2) but with a rotation angle γ with respect to the Y axis, i.e., $\tan\gamma = -(X_1 - X_0)/(Y_2 - Y_1) = c_{45}/C$. Because $C > 0$, hence c_{45} will control the character of the rotation. There will be no rotation if $c_{45} = 0$ and will rotate counterclockwise for $c_{45} > 0$, clockwise for $c_{45} < 0$. Hence, a unit vector that is in the y direction still will be in the Y direction, but a unit vector in the x direction $\boldsymbol{i}$ will rotate to a new direction after the transformation to the corresponding isotropic problem

$$\cos\gamma\boldsymbol{i} + \sin\gamma\boldsymbol{j} = \frac{1}{\sqrt{c_{44}c_{45}}}(C\boldsymbol{i} + c_{45}\boldsymbol{j}) \tag{38}$$

A boundary point is next considered whose unit normal is denoted as $\boldsymbol{n} = n_x\boldsymbol{i} + n_y\boldsymbol{j}$ for an anisotropic problem; after transforming to the corresponding isotropic problem, the unit normal vector for the same point will change to

$$\boldsymbol{N} = \frac{1}{|N|}\left\{\frac{Cn_x}{\sqrt{c_{44}c_{55}}}\boldsymbol{i} + \left[\frac{c_{45}n_x}{\sqrt{c_{44}c_{55}}} + n_y\right]\boldsymbol{j}\right\} \tag{39}$$

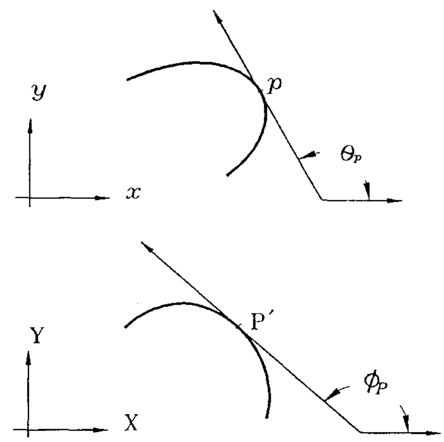

Fig. 3 Configuration of the boundary for the anisotropic problem and the corresponding isotropic problem.

where

$$|N| = \left\{1 + \frac{2c_{45}}{\sqrt{c_{44}c_{55}}}n_x n_y\right\}^{\frac{1}{2}}$$

If $\tau_n^a(p)$ represents the traction on the boundary point p of the anisotropic problem as shown in Fig. 3, the corresponding point of the isotropic problem is denoted as P. The traction at P is then given by

$$\tau_N^i(P) = (1/\Psi^*)\tau_n^a(p) \tag{40}$$

where

$$\Psi^*(x, y) = \left[\cos^2\theta_p - \frac{c_{45}}{c_{44}}\sin 2\theta_p + \frac{c_{55}}{c_{44}}\sin^2\theta_p\right]^{\frac{1}{2}}$$

$$= \left[\frac{1 - 2(c_{45}/c_{44})m + (c_{55}/c_{44})m^2}{1 + m^2}\right]^{\frac{1}{2}} \tag{41}$$

in which θ_p is the angle between the tangent line to the boundary point and the x axis, and $m = \mathrm{d}y/\mathrm{d}x$ is the slope. The ratio change of the traction in an anisotropic problem and the corresponding isotropic problem in the $X - Y$ coordinate system also can be expressed as follows:

$$\Psi^*(X, Y) = \left\{\frac{1 + 2(c_{45}/C)M + \left[\left(c_{44}^2 + c_{45}^2\right)/C^2\right]M^2}{1 + M^2}\right\}^{-\frac{1}{2}} \tag{42}$$

where

$$M = \frac{\mathrm{d}Y}{\mathrm{d}X}$$

The relationship presented in Eq. (40) is not only valid for the boundary point but also can be used for the interior point. Hence, we have

$$\tau_{xz}^a n_x + \tau_{yz}^a n_y = \Psi^*\left(\tau_{XZ}^i N_X + \tau_{YZ}^i N_Y\right) \tag{43}$$

If $(n_x, n_y) = (1, 0)$, we have

$$N_X = \frac{C}{\sqrt{c_{44}c_{55}}} \tag{44}$$

$$N_Y = \frac{c_{45}}{\sqrt{c_{44}c_{55}}} \tag{45}$$

$$\Psi^* = \sqrt{c_{55}/c_{44}} \tag{46}$$

If $(n_x, n_y) = (0, 1)$, we have

$$N_X = 0 \tag{47}$$

$$N_Y = 1 \tag{48}$$

$$\Psi^* = 1 \tag{49}$$

Substitute these two special cases (44–49) into Eq. (43). The solutions for anisotropic material can be obtained from the corresponding isotropic problem in Cartesian coordinate as follows:

$$w^a(x, y) = (\mu/C)w^i(X, Y) \tag{50}$$

$$\tau^a_{yz}(x, y) = \tau^i_{YZ}(X, Y) \tag{51}$$

$$\tau^a_{xz}(x, y) = (1/c_{44})\left[C\tau^i_{XZ}(X, Y) + c_{45}\tau^i_{YZ}(X, Y)\right] \tag{52}$$

The anisotropic antiplane problems can be converted into the one involving isotropic materials by properly changing the geometry and the tractions on the boundary. The procedure may be indicated as follows:

1) The geometry of the original anisotropic problem is changed to the corresponding isotropic problem by using Eqs. (34) and (35) in a polar coordinate system or using Eqs. (36) and (37) in a Cartesian coordinate system.

2) The boundary tractions are changed by Eq. (40). Note that the total force produced by the traction for the anisotropic problem is the same as that for the corresponding isotropic problem after transformation.

3) Solve the associated isotropic problem.

4) The relationship shown in Eqs. (31–33) for polar coordinates or Eqs. (50–52) for Cartesian coordinates is used to obtain the displacement and shear stresses for the anisotropic problem.

Conclusions

Solving an isotropic problem has always been easier than the corresponding anisotropic problem in both the analytical analysis and the numerical investigation. The antiplane problem of anisotropic materials is investigated in detail. In analytical studies, solutions of antiplane problems have served two distinct purposes. First, they could be used for shedding some light on the qualitative behavior of the solutions for somewhat more difficult inplane problems. Second, they could have practical applications in their own right in situations such as relationship for the full-field solutions of stress and displacement between the anisotropic materials and the corresponding isotropic problems have been established. Through such a correspondence, solutions for anisotropic problems could be expressed in terms of those for isotropic problems. With this correspondence at hand, investigating the complicated antiplane anisotropic problem has become very convenient. Attention needs, therefore, only be focused on the problem of isotropic materials.

Acknowledgment

The financial support by the National Science Council, Republic of China, through Grant NSC 79-0401-E-002-37 to National Taiwan University is gratefully acknowledged.

References

[1]Williams, M. L., "Stress Singularities Resulting from Various Boundary Conditions in Angular Corners of Plates in Extension," *Journal of Applied Mechanics*, Vol. 19, No. 4, 1952, pp. 526–528.

[2]Tranter, C. J., "Use of the Mellin Transform in Finding the Stress Distribution in an Infinite Wedge," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 1, No. 2, 1948, pp. 125–130.

[3]Stroh, A. N., "Dislocations and Cracks in Anisotropic Elasticity," *Philosophical Magazine*, Vol. 3, 1958, pp. 625–646.

[4]Ting, T. C. T., "Explicit Solution and Invariance of the Singularities at an Interface Crack in Anisotropic Composites," *International Journal of Solids and Structures*, Vol. 22, No. 9, 1986, pp. 965–983.

[5]Bogy, D. B., "Plane Solution for Anisotropic Elastic Wedges under Normal and Shear Loading," *Journal of Applied Mechanics*, Vol. 39, No. 4, 1972, pp. 1103–1109.

[6]Ma, C. C., and Hour, B. L., "Analysis of Dissimilar Anisotropic Wedges Subjected to Antiplane Shear Deformation," *International Journal of Solids and Structures*, Vol. 25, No. 11, 1989, pp. 1295–1309.

[7]Ma, C. C., "Antiplane Problems of Monoclinic Material," *Journal of Engineering Mechanics*, Vol. 118, No. 9, 1992, pp. 1765–1782.

Inclusion of Transverse Shear Deformation in Optimum Design of Aircraft Wing Panels

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Introduction

THE effects of shear deformation plate theory (SDPT) on the minimum mass design of composite prismatic plate assemblies of practical aircraft wing panel proportions are investigated using VICONOPT.[1] Critical buckling loads for single plates calculated using SDPT are lower than those calculated using classical plate theory (CPT), the difference often being significant for thick laminates. For plate assemblies,[2] material properties and the relative thicknesses cause greater SDPT vs CPT differences for local buckling modes than for overall ones. The differences are particularly large for sandwich panels with low density foam cores,[3] but the present Note addresses thick laminate cases only.

In the optimum designs presented, the panel mass is minimized subject to buckling, ply strain, and geometric constraints. Design variables include plate breadths and individual ply thicknesses. The results compare SDPT and CPT and use VIPASA[4] or VICON[5] buckling analysis as appropriate, both of which require the calculation, at every longitudinal half-wavelength λ and at every iteration of the Wittrick–Williams algorithm,[4] of the transcendental stiffness matrix $\boldsymbol{k}$ for a flat plate carrying in-plane longitudinal (N_x), transverse (N_y), and shear (N_{xy}) loads per unit length of their edges. The CPT used assumes a balanced, symmetric laminate and the absence of through thickness shear strains, so that normals to the centroidal plane remain so after deformation. The SDPT used is first order,[3] i.e., normals remain straight but not normal after deformation.

Nondimensional parameters β and μ represent the degradations in critical buckling load and optimum mass, respectively, because SDPT is used instead of CPT. Thus, if SDPT is used to analyze a CPT design, the panel fails to carry the design load P by a percentage β, i.e., it carries only $P(1 - \beta/100)$. SDPT redesign raises the critical buckling load to P but increases the mass M by a percentage μ, i.e., to $M(1 + \mu/100)$.

Scope of Study

Figure 1 shows a plan view of a benchmark 45 deg swept composite wing skin recently studied by the Group for Aeronautical Research and Technology in Europe (GARTEUR) Working Group on Structural Optimization.[6] The panel P8 shown is the most heavily loaded panel and was designed by VICONOPT to satisfy buckling, strength, and geometric conditions for three load cases. Figure 2 gives the geometric and layup details for panel P8, the ply properties being $E_{11} = 125$ GPa, $E_{22} = 8.8$ GPa, $G_{12} = G_{13} = 5.3$ GPa, $G_{23} = 0.5\ G_{13} = 2.65$ GPa, $\upsilon_{12} = 0.35$, and density $= 1620$ kg/m^3.

Received Feb. 24, 1996; accepted for publication July 11, 1996; also published in *AIAA Journal on Disc*, Volume 2, Number 1.

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