# Characteristics of the Domain Wall Structure and Domain Wall Damping 

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#### Abstract

The coordinate and transverse field dependence of the domain wall (DW) magnetization structure across the wall proper and the corresponding dependence of the effective DW width of both the polar and azimuthal angle distribution has been investigated. The effective width of the azimuthal angle distribution is found to vary with the orientation and strength of the transverse field and is much broader than the conventional wall width due to the polar angle distribution. On dynamics, the analysis of the DW motion in the presence of the transverse field normal to the anisotropy axis taking into account the nonconservation of the magnetization modulus has uncovered two types of the magnetization damping parameters. One is that of the Landau-Lifshitz damping constant, $\lambda_{F M R}$, which characterizes the homogeneous magnetization manifested in the ferromagnetic resonance line width. The other, $\lambda_{\mu}$, is associated with the inhomogeneous magnetization such as the DW which gives rise to an extra viscous damping. Numerically, $\lambda_{\mu}$ can be of the same order or several factors greater than $\lambda_{F M R}$. Our formulation establishes the existence of two distinctly different damping processes associated with a ferromagnet sample which the Landau-Lifshitz-Gilbert equation fails inherently..


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## I. Introduction

The structure of $180 \hat{1} \mathrm{DW}$ in a uniaxial ferromagnet under the influence of an external field normal to the anisotropy axis (the transverse field) has already been analyzed in a number of papers [1-6]. As a rule, one describes the structure of the one-dimensional DW based on the assumption that the azimuthal angle of the magnetization does not depend on the coordinate normal to the DW plane [2]. The only successful attempt which has taken into account the coordinate dependence of the azimuthal angle of magnetization was given in Ref. [7]. The latter treated the case of the one-dimensional DW moving at a low velocity.

Recently it was shown that a new one-dimensional solution to the system of equations for the polar and azimuthal angles of the magnetization $M=M\{\sin \vartheta \cos \varphi, \sin \vartheta \sin \varphi, \cos \vartheta\}$ which describes the distribution of magnetic moment inside the DW can be obtained in the case when the transverse field is arbitrarily oriented with respect to the DW plane [6]. This
new solution for the magnetization azimuthal angle $\varphi$ is a function of both the transverse field and the coordinate normal to the DW plane. The solution yields a lower value of the DW energy in comparison with the one in which $\varphi$ is assumed to maintain constant across the DW [6].

Our purpose is to analyze the coordinate dependence of the solution of both the polar and azimuthal angle of the magnetization across the one-dimensional DW. The transverse field dependence of the Bloch to Ní eel wall transition, and the DW energy variation versus the transverse field normal to the anisotropy axis arbitrarily oriented with respect to the DW plane will also be expounded.

## II. Method of solution

Consider a uniaxial ferromagnet with the anisotropy axis oriented along the $z$ axis of the coordinate frame. For convenience the DW is assumed to be pinned throughout the investigation and the wall plane is chosen to coincide with the $x-z$ plane. The orientation of the external field normal to the anisotropy axis is described by the azimuthal angle $\psi_{H}$ of the field with respect to the x axis $\mathrm{H}=H\left(\cos \psi_{H}, \sin \psi_{H}, 0\right)$. In the frames of the uniaxial ferromagnet model we shall consider the so called uncharged domain walls. The last means that the demagnetization part of the DW energy density can be written as follows [2]

$$
\begin{equation*}
\mathrm{w}_{\mathrm{m}}-2 \pi M^{2} \Delta_{B}(\sin 13 \sin \varphi-\mu)^{2} \tag{1}
\end{equation*}
$$

Here $4 \pi M$ is the saturation magnetization; $\Delta_{B}=\sqrt{A / K}$ is the Bloch wall width parameter; $K$ is the uniaxial anisotropy constant, $A$ is the exchange stiffness constant, the parameter $\mu$ in (1) is determined by the value of the normal component of the magnetization which exists inside domains in the presence of the transverse field, that is, $\mu=\sin \vartheta_{0} \sin \psi_{H}$, where $\vartheta_{0}$ is the magnetization polar angle inside the domain.

The dimensionless energy density for the DW in a uniaxial ferromagnet may be written as follows [1,2]:

$$
\begin{align*}
\mathrm{w}_{D W}= & \frac{w_{D W}}{\sqrt{A K}}=\int d \xi\left\{\vartheta^{\prime 2}+\varphi^{\prime 2} \sin ^{2} \vartheta+\sin ^{2} \vartheta+\right.  \tag{2}\\
& \left.\varepsilon(\sin \vartheta \sin \varphi-\mu)^{2}-2 h \sin \vartheta \cos \left(\varphi-\psi_{H}\right)\right\}
\end{align*}
$$

where $\xi=y / \Delta_{B} ; \vartheta^{\prime} \equiv d \vartheta / d \xi, \varphi^{\prime} \equiv d \varphi / d \xi ; \varepsilon=Q^{-1} ;$ the material quality factor $Q=$ $H_{K} / 4 \pi M\left(H_{K}=2 K / M\right) ; \mathrm{h}=H / H_{K}$ is the reduced transverse field.

The system of equations from which the distribution of the magnetization inside the DW may be obtained has the form [1-3]

$$
\begin{align*}
& \vartheta^{\prime \prime}-\sin \vartheta \cos \vartheta\left(1+\varphi^{\prime 2}+\varepsilon \sin ^{2} \varphi\right)+\varepsilon \mu \cos \vartheta \sin \varphi+\sin \vartheta_{0} \cos \vartheta \cos \left(\varphi-\psi_{H}\right)=0 \\
& \left(\sin ^{2} \vartheta \varphi^{\prime}\right)^{\prime}-\varepsilon(\sin \vartheta \sin \varphi-\mu) \sin \vartheta \cos \varphi-\sin \vartheta_{0} \sin \vartheta \sin \left(\varphi-\psi_{H}\right)=0 \tag{4}
\end{align*}
$$

The polar angle of the magnetization inside domain is determined by the relation $\sin \vartheta_{0}=\mathrm{h}$, a condition which has been properly taken care of in writing down Eqs. (3) and (4). The boundary conditions for the solutions describing the variation of the angles inside the DW have been chosen as follows

$$
\begin{align*}
& \vartheta(\xi)=\vartheta\left(\xi ; h, \psi_{H}\right)=\left\{\begin{array}{cc}
\vartheta_{0}, & \xi \rightarrow-\infty \\
\pi-\vartheta_{0}, & \xi \rightarrow+\infty
\end{array} ; \vartheta^{\prime}\left(\xi ; h, \psi_{H}\right) \rightarrow 0, \text { at } \xi \rightarrow \pm \infty\right.  \tag{5}\\
& \varphi(\xi)=\varphi\left(\xi ; h, \psi_{H}\right)=\psi_{H}, \quad \xi \rightarrow \pm \infty ; \quad \varphi^{\prime}\left(\xi ; h, \psi_{H}\right) \rightarrow 0, \text { at } \xi \rightarrow \pm \infty
\end{align*}
$$

The system of Eqs. (3) and (4) along with the boundary conditions (5) have been solved numerically using the adaptive Runge-Kutta-Fehlberg method [9]. The interval of variation of the coordinate $\xi$ was chosen to be equal to $-15 \leq \xi \leq 15$. The step of the $\xi$ variation was adjusted automatically. We have obtained the set of dependences for $\vartheta(\xi)$ and $\varphi(\xi)$ for several orientation values of the azimuthal angle of the external field $\psi_{H}=30^{\circ}, 35^{\circ}, 40^{\circ}, 45^{\circ}, 50^{\circ}, 60^{\circ}, 70^{\circ}, 80^{\circ}$, and $90^{\hat{1}}$ and for several values of the reduced transverse field $\mathbf{h}=0.01,0.05,0.1,0,2$, and 0.25 . The characteristic value of the material quality factor Q was chosen to be equal be $\mathrm{Q}=2.0$ for easy convenience of making comparison with the results obtained in Ref. [6,7] where a slightly different method of numerical solution of the system of Eqs. (3) and (4) has been used.

## III. The domain wall structure

The dependence of the azimuthal angle of magnetization upon the coordinate normal to the DW, $\xi$, has the following form $: \varphi\left(\xi ; \mathrm{h}, \psi_{H}\right)=\psi_{H}-\psi\left(\xi ; \mathrm{h}, \psi_{H}\right)$, where $\psi\left(\xi ; \mathrm{h}, \psi_{H}\right) \rightarrow$ 0 at $\xi \rightarrow \pm \infty$. Sucha form of dependence may be understood as follows. The external magnetic field applied at an angle $\psi_{H}$ with respect to the DW plane produces a deviation in the orientation of the magnetization inside the DW which in turn leads to an increase of the demagnetization energy. The type of solution for the azimuthal angle $\varphi\left(\xi ; \mathrm{h}, \psi_{H}\right)=$ $\psi_{H}-\psi\left(\xi ; h, \psi_{H}\right)$ provides, first, the fulfillment of the condition that the DW is uncharged and, second, it reduces the increase of the DW energy. This is the reason why the DW energy with the coordinate dependent azimuthal angle is always lower than when the azimuthal angle is held constant at $\varphi=\psi_{H}[6]$. The dependence of $\varphi\left(\xi ; \mathrm{h}, \psi_{H}\right)$ for the case of $\psi_{H}=30$ " for several values of transverse field $h$ is given in Fig. 1. Clearly, the maximum deviation of the magnetization inside the DW from the orientation of the transverse field decreases with increasing transverse field. The dependences of $\varphi(\xi)$ for other orientations of the transverse field $\psi_{H}$ are similar to the ones given in Fig. 1. The only difference is that at $\psi_{H}=90 \hat{\imath}$ the function $\psi\left(\xi ; \mathrm{h}, \psi_{H}\right)$ becomes identically equal to zero at $\mathrm{h}=0.225$ [6]. This is a reflection of the fact that a phase transition from the quasi-Bloch wall to the Nkel one has taken place at the value of the reduced field $\mathrm{h}=0.225$ when the latter is oriented along the normal to the DW $[2,6]$.

Using the definition of the effective DW width of the distribution of the magnetization angle inside the DW as given in Ref. [10], we analyzed the transverse field dependences of the effective DW width of the distributions of both the polar and azimuthal angles of the magnetization for our solutions to the system of Eqs. (3) and (4). The effective DW width (in unit of $\Delta_{B}$ ) of the distribution of the azimuthal angle of the magnetization, A, $\left.h, \psi_{H}\right)$, was found to decrease with increasing transverse field in the region of the reduced fields $0<\mathrm{h} \leq 0.4 \doteq 0.5$ and then it increases, tending to infinity as $\mathrm{h} \rightarrow 1$ when the sample becomes homogeneously remagnetized. The transverse field dependence of the effective DW width of the azimuthal angle of magnetization distribution, $\Delta_{\varphi}\left(h, \psi_{H}\right)$, for several orientations of the transverse field are given in Fig. 2. Note that the curve for $\psi_{H}=90 \hat{i}$ is


FIG. 1. Dependence of the azimuthal angle of magnetization $\varphi\left(\xi ; h, \psi_{H}\right)$ upon the coordinate $\xi$ normal to the DW plane for $\psi_{H}=\pi / 4$ and for severa of the transverse field $h$.


FIG. 2. The transverse field dependence of the effective DW width of the azimuthal angle of the magnetizav a 1 ution distribution $A,(h)$ for several orientations of the transverse field $\psi_{H}=45^{\circ}, 60^{\circ}$, and $90^{\circ}$.
truncated at $h=0.225$ as remarked above.
Traditionally, the effective DW width is defined in term of the polar angle of magnetization distribution $\Delta_{\vartheta}(h)$. It is of interest to compare the behavior of $\Delta_{\vartheta}(h)$ versus that of $\Delta_{\varphi}\left(h, \psi_{H}\right)$. Analysis of $\Delta_{\vartheta}(h)$ distributions inside the DW for different orientations of the transverse field showed that this distribution is insensitive to the variation in the transverse field orientation [7]. Furthermore, the numerical solutions of the system of Eqs. (3) and (4) for the polar magnetization angle do not deviate much from the dependence of

$$
\sin \vartheta(\xi)=\sin \vartheta_{0}+\frac{\cos ^{2} \vartheta_{0}}{\cosh u+\sin \vartheta_{0}}
$$

Note that the above expression represents the analytical solution of the system obtained under the approximation $\varphi=\psi_{H}$ inside the DW [2]. In this expression $\mathrm{u}=\xi \cos \vartheta_{0} \sqrt{1+\varepsilon \sin ^{2} \psi_{H}}$. According to Ref. [10], the effective DW width $\Delta_{\vartheta}(h)$ of the polar angle of the magnetization distribution can be expressed as

$$
\begin{equation*}
\Delta_{\vartheta}(h)=\pi \Delta_{B}\left[\cos \vartheta_{0} \sqrt{1+\varepsilon \sin ^{2} \psi_{H}}\right]^{-1} \tag{6}
\end{equation*}
$$

It is easy to see from (6) that $\Delta_{\vartheta}(h)$ increases monotonically with increasing transverse field and tends to infinity as $\mathbf{h}$ approaches unity. Figure 3 shows comparison of the transverse field dependence of the numerically obtained effective width $\Delta_{\vartheta}(h)$ versus that obtained directly from (6). Clearly, the difference between the two sets of curves are practically negligible except when the values of the transverse field is small.


FIG. 3. Comparison of the dependence of the effective widths of the distribution of the polar angles, $\Delta_{\vartheta}(h)$, on the transverse field $h$ at $\psi_{H}=45^{\circ}$. Solid line: numerical result. obtained from solving the system of Eqs. (3) and (4); dashed line: analytical result obtained directly from (6).


FIG. 4. The variation of the azimuthal angle of magnetization at the mid-point of the DW, $\psi(0)$, for several values of the transverse field orientations $\psi_{H}$. A Bloch wall makes an abrupt phase transition to a Ní eel wall at $h=0.225$ when $\psi_{H}=90^{\circ}$.

From the transveres field dependence shown in Figs. 2, and 3, it is clear that the effective DW width due to the azimuthal angle distribution $A$,(h) is significantly different from that of polar angle $\Delta_{\vartheta}(h)$, and that the absolute value of the effective width $A,(h)$ is always greater than that of the polar angle.

Armed with the fomulation described above it becomes relatively easy to investigate the Bloch-to-Ní eel wall transition as a function of $\left(\mathrm{h}, \psi_{H}\right)$. Figure 4 shows the variation of the azimuthal angle at the mid-point of the DW, $\psi(o)$, for several values of the transverse field orientations $\psi_{H}$. Clearly, a Bloch wall will make an abrupt phase transition to the Ní eel wall at some value of $h(h=0.225)$ provided if the transverse field is oriented normal to the anisotropy axis with $\psi_{H}=90^{\circ}$.

## IV. On dynamics

The motion of the magnetization vector in spin waves and domain wall dynamics is largely characterized by the relaxation processes, hence by the corresponding relaxation (damping) constant. However, the functional dependence of the relaxation constants for either the spin waves or domain wall (DW) dynamics is completely unclear up to the present time. Two empirical ways of measuring the relaxation constants (in garnets) are known. First is by means of measurements of the ferromagnetic resonance (FMR) line width to
obtain $\lambda_{F M R}$. Secondly, one may deduce from linear mobility of the domain wall motion to obtain $\lambda_{\mu}$. It has long been an intriguing and well known fact that the numerical value of the damping constant measured by FMR could be substantially different from the one obtained from the DW mobility measurements. The ratio between these two constants $\lambda_{\mu}$ and $\lambda_{F M R}$ may range from $<10^{-1}$ to $>10^{2}$ (e.g. in pure YIG) [3,12]. Another open question is that the presence of a transverse field may increase the DW mobility substantially [12] while the FMR line width is practically insensitive to it. All of these questions have not neen properly addressed up to the present date.

More often than not the Landau-Lifshits-Gilbert (L-L-G) equation is used to analyze the relaxation constant related to both the FMR line width and DW mobolity. However, the L-L-G equation is known to conserve the value of magnetization modulus. On the other hand, it was shown [2] that the presence of the static DW may cause the change in the magnetization modulus. It follows that the L-L-G equation is inherently incapable of delineating the distinction between these two constants since two distinctly different relaxation processes or mechanisms were involved.

Recently, a generalized dynamic equation taking into account the nonconservation of magnetization modulus has been introduced into the Landau-Lifshits (L-L) Eq. [13] to study the domain wall dynamics [14]. Numerical analysis based on this generalized L-L equation indicates that the two constants $\lambda_{\mu}$ and $\lambda_{F M R}$, that is the relaxation constant related to the DW mobility and the original L-L damping constant, are two distinctly different constants [15]. Furthermore, $\lambda_{\mu}$ is shown manifestly field dependent.

Hereunder we report the use of this generalized L-L equation to derive the new Slonzewski-like equation for the DW dynamics [3] and to deduce from it a field-dependent linear DW mobility and the important relation between the two constants $\lambda_{\mu}$ and $\lambda_{F M R}$. Based on this relation the value of longitudinal susceptibility $\chi_{\|}$compatible with spin wave theory can also be suscessfully estimated.

## V. The analysis

The generalized L-L equation for magnetization dynamics can be expressed as [13]

$$
\dot{M}=-\gamma[M \times F]+\gamma M \hat{\Lambda} F-\gamma M \lambda_{e} \nabla^{2} F,
$$

where the tensor $\hat{\Lambda}$ is determined by crystal symmetry: $\hat{\Lambda}=\lambda_{F M R} \delta_{i j}$ for a cubic crystal, $\mathrm{A}=\lambda_{F M R}\left[\delta_{i j}-n_{i} n_{j}\right][\mathbf{n}$ is the unit vector along the anisotropy axis] for a uniaxial one; $\lambda_{e}$ is the spatial dispersion of relaxation constant; $\mathrm{F}=\delta \varpi / \delta \mathrm{M}$. (In the present context the L-L damping constant $\lambda_{T}$ is being identified as equal to $\lambda_{F M R}$ ). Eenergy density w of a uniaxial ferromagnet with a Bloch wall is

$$
a=\frac{2 A}{M_{s}^{2}}(\nabla M)^{2}+\frac{\left(M^{2}-M_{s}^{2}\right)^{2}}{8 \chi_{\|} M_{s}^{2}}+2 \pi Q M^{2}\left\{\Delta_{B}^{2} \vartheta^{\prime}+\left(1+\varepsilon \sin ^{2} \varphi\right) \sin \vartheta\right\}
$$

Linear mobility of the DW is frequently used to characterize magnetic thin films [3] and is defined by the relation

$$
\begin{equation*}
\mu_{D W}\left(H_{t}\right)=\lim _{H_{\|} \rightarrow 0} \frac{d V\left(H_{\|}, H_{t}\right)}{d H_{\|}} \tag{7}
\end{equation*}
$$

In steady-state the DW velocity can be expressed as $[15,16]$

$$
\begin{equation*}
V\left(H_{\|}, H_{t}\right)=\frac{\gamma \Delta_{B} H_{\|} \sqrt{r\left(\theta_{0}\right)}}{\lambda_{F M R} a(\theta, \psi) R\left(\theta_{0}, \psi\right)} \tag{8}
\end{equation*}
$$

which is obtained by means of the consideration of the dissipative function

$$
Q=\frac{\gamma M_{s}}{2} \lambda_{F M R} \int d \mathbf{r} F_{M}^{2}
$$

where $F_{M}$ is the longitudinal component of effective field. Detail of the derivation can be found in Ref. [16].

Substituting Eq. (8) into Eq. (7) yields

$$
\begin{equation*}
\mu_{D W}\left(H_{t}\right)=\frac{\gamma \Delta_{B} \quad v_{*}^{2}}{\lambda_{F M R} \cos \theta_{0}\left[v_{*}^{2} r\left(\theta_{0}\right)+4 R_{10}\left(\theta_{0}\right)\right]}, \tag{9}
\end{equation*}
$$

where $v_{*}=\lambda_{F M R} / 4 \pi Q \chi_{\|}$and

$$
\begin{align*}
R_{10}\left(\theta_{0}\right)= & \frac{2}{\cos ^{2} \theta_{0}}\left\{\frac{2}{3} \cos \theta_{0}-\sin \theta_{0}\left[\frac{17}{3} \sin \theta_{0} \cos \theta_{0}\right.\right. \\
& \left.\left.+\frac{1}{2}\left(\pi-2 \theta_{0}\right)\left(2+\cos 2 \theta_{0}\right)+6 \sin \theta_{0} \ln \left|\tan \frac{\theta_{0}}{2}\right|\right]\right\} \tag{10}
\end{align*}
$$

in the absence of the transverse field, it becomes

$$
\begin{equation*}
\mu_{D W}\left(H_{t}=0\right)=\frac{\gamma \Delta_{B}}{\lambda_{F M R}\left[1+\frac{16}{3 v_{*}^{2}}\right]} \tag{11}
\end{equation*}
$$

Expression (11) is compatible with that obtained in the original Landau-Lifshitzí s paper in the case when the modulus of the magnetization was considered to be conserved. The transverse-field dependent relaxation parameter $\lambda_{\mu}\left(H_{t}\right)$ deduced from the DW mobility data in the linear regime becomes

$$
\begin{equation*}
\lambda_{\mu}\left(H_{t}\right)=\lambda_{F M R} \cos \theta_{0}\left[r\left(\theta_{0}\right)+\frac{4}{v_{*}^{2}} R_{10}\left(\theta_{0}\right)\right] . \tag{12}
\end{equation*}
$$

In the absence of $H_{t}$ we have

$$
\begin{equation*}
\lambda_{\mu}\left(H_{t}=0\right)=\lambda_{F M F}\left[1+\frac{16\left(4 \pi Q \chi_{\|}\right)^{2}}{3 \lambda_{F M R}^{2}}\right] . \tag{13}
\end{equation*}
$$

Note that the $v_{*}$ term in Eq. (12) is due to the nonconservation of magnetization modulus. It is to be noted that, to our knowledge, Eq. (13) is first such relation by which $\lambda_{\mu}$ and $\lambda_{F M R}$ are related. In the absence of transverse field and for the materials with large value of $v_{*}$, i.e. $v_{*}^{2} \gg 16 / 3$, the two damping constants may be treated as identical.

## VI. Discussions

Let us first survey the relevant experimental data available. The longitudinal susceptibility $\chi_{\|}$data for typical iron garnets are rather skimpy. One theoretical estimate based on spin-wave theory $[18,19]$, gives, $\chi_{\| \|}=3 \sim 7 . \not \subset 10^{-4}$. Two sets of experimental data $[20,21]$ (see also Table 2 in Ref. [13]) for $\left(\mathrm{Eu}_{0.65} \mathrm{Ga}_{1.2}\right)$ YIG are available. They are $\left(\lambda_{\mu}, \lambda_{F M R}, Q\right)=(0.14,0.023,3.3)$ and $(0.13,0.025,2.1)[21]$. Substituting these data into Eq. (13) yields $\chi_{\|}=5.4 \times 10^{-4}$ and $\chi_{\|}=8.2 \times 10^{-4}$ respectively. These numerical values are fairly close to the ones estimated by spin-wave theory. Data obtained in the presence of transverse field $H_{t}=1000 \mathrm{e}$ gives: $\left(\lambda_{\mu}, \lambda_{F M R}, Q\right)=(0.022,0.03,8.9)$ [20]. On the other hand we may use Eq. (13) to estimate the DW mobility damping. For example let $\chi_{\|} \simeq 5 \times 10^{-4}$ we have $\lambda_{\mu}=0.031$ which is comparable to the data.

Since the values of $\chi_{\| \mid}$deduced from Eq. (13) is consistent with that estimated by spinwave theory, this put our equations on a sound basis to be further applied to iron garnet films. In Fig. 5 we show the variation of the transverse-field dependent DW mobility damping constants. The material parameters used in the calculation are taken directly from Ref. $[20,21]$. Solid (dashed) curve corresponds to the sample with material parameters $\left[\chi_{\|}, \lambda_{F M R}, Q, H_{K}(0 \mathrm{e})\right]=\left[5.4 \times 10^{-4}, 0.023,3.3,517\right]\left(\left[8.2 \times 10^{-4}, 0.025,2.1,337\right]\right),\left(H_{K}=\right.$ $2 K / M_{s}$ is anisotropy field, $K$ : anisotropy constant, $M_{s}$ : saturation of magnetization). We see from Fig. 5 that: (1) in the absence of transverse field, $\lambda_{\mu}$ is always larger than $\lambda_{F M R}$; (2) if transverse field $H_{t}$ is large enough, one finds that $\lambda_{\mu}<\lambda_{F M R}$; (3) when $H_{t}$ approaches to $H_{K}, \lambda_{\mu}$ becomes vanishingly small. The latter situation is understandable since the DW broaden rapidly into homogeneous magnetization.


FIG. 5. Variation of the calculated DW damping constant $\lambda_{\mu}$ versus the transverse field. The inset $\lambda_{F M R}$ values are taken from Ref. [11,12].

For a sample homogeneously magnetized, one sees that precessional motion of the spins inside such a sample may suffer from ferromagnetic relaxation characterized by a constant $\lambda_{F M R}$. For an inhomogeneous sample with domain structures one sees the damping characterized not only by $\lambda_{F M R}$ but also the drag force due to inhomogeneous magnetization and nonconservation of magnetization modulus characterized by $\lambda_{\mu}$ which hinders the DW from moving any faster. Consequently, the experimental value in the absence of a transverse field observed for the DW mobility damping $\lambda_{\mu}$ is larger than $\lambda_{F M R}$. Almost all empirical data exhibit this common characteristic feature.

A planar DW may exhibit a fairly high DW mobility in the presence of a transverse field. For example, the damping constant observed for $\lambda_{F M R}$ and $\lambda_{\mu}$ in EuGaYIG sample is nearly identical [12]. Our analysis shows that this is the case as exemplified in Fig. 5 in which $\lambda_{\mu} \sim \lambda_{F M R}$ for the solid (dashed) curve when the transverse field is $H_{t} \sim 185 \mathrm{Oe}$ ( $H_{t} \sim 115 \mathrm{Oe}$ ). Since the mechanism responsible for the DW damping differs distinctly from that for $\lambda_{F M R}$ it appears that there is no basis for anyone to believe that a sample may exhibit $\lambda_{\mu} \sim \lambda_{F M R}$. It turns out that there is no ambiguity answer to this question. As described by Eq. (12) and by the fact that a DW may broaden into a homogeneous magnetization in the presence of a strong transverse field, as remarked above [3,22], cases for $\lambda_{\mu} \sim \lambda_{F M R}$ and that for $\lambda_{\mu}<\lambda_{F M R}$ may happen and have actually been observed as compiled in table 2 in Ref. [12].

When the strength of transverse field approaches that of anisotropy field $H_{t} \rightarrow H_{K}$ the magnetization inside the DW structure will be forced to rotate toward the direction of the field. As a consequence, the DW becomes only faintly detectable. The DW mobility damping becomes vanishingly small and meaningless.

The formulation described above is based on the assumption that quality factor Q of the sample concerned being $\mathrm{Q}>1$. We are however greatly encouraged to note that Eq. (13) works properly for pure YIG [13]. For example, according to Table III in Ref. [12], for pure YIG, it gives $\lambda_{\mu} M_{s} / \gamma=0.52 \times 10^{-7}$ and $\lambda_{F M R} M_{s} / \gamma=0.006 \times 10^{-7} \mathrm{Oe}^{2} \cdot \mathrm{sec} \cdot \mathrm{rad}^{-1}$. If the material parameters of for pure YIG $7=1.8 \times 10^{7} \mathrm{sec}^{-1} \mathrm{Oe}^{-1}, 4 \pi M_{s}=1750 \mathrm{G}$, $\Delta_{B} \simeq 8 \times 10^{-6} \mathrm{~cm}$, and $\mathrm{A}=4.15 \times 10^{-7} \mathrm{erg} / \mathrm{cm}[13]$ etc., are used, we then obtain the quality factor of YIG being $\mathrm{Q}=0.0532$. Substituting the above parameters into Eq. (13), one obtains $\chi_{\|} \simeq 4.65 \times 10^{-4}$ which is well within the bound estimated using spin-wave theory. Consistency of our calculation with the spin-wave theory indeed throws strong support to our formulation, in particular, in support of our claim that the disagreement between $\lambda_{\mu}$ and $\lambda_{F M R}$ is due to nonconservation of magnetization modulus which in turn is induced by the inhomogeneous magnetization structure of the sample.

## VII. Conclusions

New characteristics of the coordinate and transverse field dependence of the domain wall (DW) magnetization structure and the corresponding dependence of the effective DW width of both the polar and azimuthal angles of magnetization distribution has been obtained. The effective width due to the azimuthal angle distribution is found to vary sensitively with the orientation and strength of the transverse field and is much broader than the conventional wall width due to the polar angle distribution.

Analysis of the DW dynamic equation based on the new generalized Landau Lifshitz

Gilbert equation taking into account the nonconsevation of magnetization modulus has led us to two types of the magnetization damping parameters. One is ascribed to the homogeneous magnetization manifested in the ferromagnetic resonance line width characterized by the Landau-Lifshitz damping constant $\lambda_{F M R}$. The other is associated with the inhomogeneous magnetization localized within the DW proper characterized by the DW mobility damping constant $\lambda_{\mu}$. Our formulation points out that the two constants are distinctly different but interrelated. In the absence of transverse field $\lambda_{\mu}$ is always greater than $\lambda_{F M R}$. In the presence of transverse field, however, the ratio between the two constants $\lambda_{\mu}$ and $\lambda_{F M R}$ varies, ranging from $<10^{-1}$ to $>10^{2}$ as has been observed. Our formulation also allows us to estimate correctly the longitudinal susceptibility. The calculated results are consistent with the available data found in garnet samples.

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References
[1] G. A. Jones and B.K. Middelton, Int. J. Magnetism, 6, 1 (1974).
[ 2 ] A. Hubert, Theorie der domännenwände in Geordneten M edien (Sringer, Berlin, 1974) (in German).
[3] A. Molozemoff and J. Slonczewski, Magnetic Domain Walls in Bubble Materials (Acad. Press., N. Y., 1979).
[4 ] Chai Tak Teh, H. L. Huang, and V. L. Sobolev, J. Appl. Phys. 75, 7003 (1994).
[5 ] Yu A. Dimashko, Sov. Phys.-Solid State, 27, 1274 (1985).
[6] Chai Tak Teh, V. L. Sobolev, and H. L. Huang, J. Magn. Magn. Mats. 145, 382 (1995).
[7] J. Morkowski, H. Dotsch, P. E. Wigen, and R. Y. Yeh, J. Magn. Magn. Mats. 25, 39 (1981).
[ 8] Chai Tak Teh, H. L. Huang, and V. L. Sobolev, J. Magn. Magn. Mats (1995), in press.
[9] J. Morkowski, H. Dotsch, P. E. Wigen, and R. Y. Yeh, J. Magn. Magn. Mats. 25, 39 (1981).
[10] D. Kincaid and W. Cheney, Numerical analysis (Brooks/Cole Publ. Co., California, 1990).
[11] B. A. Liley, Phil. Mag. 41, 792 (1950).
[12] F. H. de Leeuw, R. van der Doel, U. Enz, Rep. Prog. Phys. 43, 690 (1980).
[13] V. G. Barí Yakhtar, Sov. Phys. JETP, 60, 863 (1984); also V. G. Barí Yakhtar, V. A. Brodovoi, B. A. Ivanov, I. V. Krutsenko, and K. A. Safaryan, Phys. Solid State, 32, 502 (1990).
[14] S. C. Chen, V. L. Sobolev, H. L. Huang, J. Magn. Magn. Mats. (1997), accepted.
[15] H. L. Huang, S. C. Chen, and V. L. Sobolev, J. Appl. Phys. (1997) accepted.
[16] Shoan Chung Chen, Ph.D. dissertation, National Tsing Hua University, HsinChu, Taiwan, ROC (1996).
[17] B. A. Ivanov and K. A. Safaryan, Sov. Phys. Solid State, 32, 2034 (1990).
[18] T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
[19] B. E. Argyle, S. H. Charap, and E. W. Pugh, Phys. Rev. 132, 2051 (1963).
[20] J. C. DeLuca and A. P. Malozemoff, AIP Conf. Proc. 34, 151 (1976).
[21] J. C. DeLuca, A. P. Malozemoff, J. L. Su, and E. B. Moore, J. Appl. Phys. 48, 1701 (1977).
[22] C. T. Teh, H. L. Huang, and V. L. Sobolev, J. Appl. Phys. 79, 6060 (1996).

