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QCD Sum Rules and Couplings of the Nucleon to Axial Fields?

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We describe the method of QCD sum rules, using calculation of the couplings of the nucleon to axial fields as an illustrative example. Our interest is centered about the behavior of a system of quarks, antiquarks, and gluons under the influence of an external field, leading to a modified version of the method of QCD sum rules. In particular, the isovector axial coupling constant g_A is determined from a QCD sum rule for $(g_A - 1)$, and it is shown that, with standard values of the quark condensates, $g_A = 1.26 \pm 0.08$. A sum rule is also obtained for the "isoscalar" axial coupling constant $g_A{}^S$, which is found to be 0.13 if the isovector values of susceptibilities are used.

I. INTRODUCTION

It is believed that quantum chromodynamics (QCD), an $SU(3)$ gauge theory constructed out of the *exact* color $SU(3)$ symmetry, describes strong interactions among quarks, antiquarks, and gluons. At high energies, i.e., large Q^2 ($>> 1\ GeV^2$) or very high resolution $<r>$, the asymptotically free nature of QCD allows for perturbative treatments of physical processes involving hadrons. At low energies ($\leq 1\ GeV^2$), the nonperturbative physics dominates such that the physical ground state (vacuum) differs from the trivial ground state (vacuum) while chiral symmetry is spontaneously broken in the physical vacuum (leading to identification of lowlying pseudoscalar mesons as Goldstone bosons). Indeed, it is seen that hadrons, including baryons, mesons, and glueballs all of which are color singlet objects consisting of quarks, antiquarks, and gluons, act as the *effective* degrees of freedom in hadron physics (strong interaction physics).

It is clear that the study of hadron physics (strong interaction physics) is essential for progresses in research areas such as nuclear physics, particle physics, collapse of heavy stars, and the hadron era or nucleo-synthesis in the early universe. The nonperturbative feature of QCD defies our attempts for finding the solutions to most strong interaction problems. The lattice simulation requires an incredible amount of computing time in order to achieve a desired

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accuracy. Modeling QCD via quark models, on the other hand, always faces the critique of "giving up too soon in trying to solve QCD". Therefore, the method of QCD sum rules, as originally proposed by Shifman, Vainshtein, and Zakharov,[1] may well be the only efficient method, available to date, for extracting or analyzing consequences of QCD. In short, the method of QCD sum rules is based on the idea of finding a Q^2 region ($\approx 1\ GeV^2$) where one may incorporate nonperturbative physics, through Wilson's operator product expansion, into the perturbative QCD treatment of physical processes involving hadrons.

The purpose of this talk is to explain in a concise manner the method of QCD sum rules, using calculation of the couplings of the nucleon to axial fields as an illustrative example. Our interest is centered about the behavior of a system of quarks, antiquarks, and gluons under the influence of an external field, leading to a modified version[2-5] of the method of QCD sum rules. In particular, the isovector axial coupling constant g_A is determined from a QCD sum rule for $(g_A - 1)$, and it is shown that, with standard values of the quark condensates, $g_A = 1.26 \pm 0.08$. A sum rule is also obtained for the "isoscalar" axial coupling constant g_A, which is found to be 0.13 if the isovector values of susceptibilities are used.

II. METHOD OF QCD SUM RULES

We shall attempt to divide our presentation into several key steps (subsections), so that each major ingredient associated with the method of QCD sum rules may be elucidated to some extent.

11-l. How do quarks (or gluons) propagate in a nonperturbative physical vacuum?

As indicated earlier, the asymptotically free nature of QCD at large Q^2 allows for perturbative treatments of physical processes involving hadrons while the nonperturbative physics dominates at low Q^2 such that the physical vacuum $|0>$ differs from the trivial vacuum and chiral symmetry is spontaneously broken in the physical vacuum (leading to identification of lowlying pseudoscalar mesons as Goldstone bosons). At intermediate energies ($Q^2 \approx 1\ GeV^2$), therefore, we need to examine how propagation of quarks (or gluons) gets modified by the presence of nonzero condensates, $<\bar{q}q> \neq 0$ and $<G_{\mu\nu}{}^a G^{a\mu\nu}> \neq 0$. To this end, we consider

$$
\begin{aligned}
&< 0|T(q_i^a(x)\bar{q}_j^b(0))|0 > \\
&\qquad = iS_{ij}^{ab}(x) + < 0| : q_i^a(x)\bar{q}_j^b(0) : |0 >,
\end{aligned}
\tag{1}
$$

with a, b the color indices and i, j the indices of Dirac spinors. The quantity $iS_{ij}{}^{ab}(x)$ is the standard Green's function (or propagator) which we encounter in the perturbative field theory:

$$
\begin{aligned}
iS_{ij}^{ab}(x) &= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS_{ij}^{ab}(p) \\
&= \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} \delta^{ab} \frac{i(\hat{p}+m)_{ij}}{p^2-m^2+i\epsilon} \\
&\rightarrow \frac{i\delta^{ab}}{2\pi^2} \frac{(\gamma\cdot x)_{ij}}{x^4}, \quad \text{with } m=0 \text{ and } x_\mu \rightarrow 0.
\end{aligned}
\tag{2}
$$

Note that we adopt $\hat{a} \equiv \gamma\cdot a \equiv \gamma_\mu a^\mu$. On the other hand, we may expand the second term in Eq. (1) as a Taylor series:

$$
\begin{aligned}
&< 0| : q_i^a(x)\bar{q}_j^b(0) : |0> \\
&\quad =< 0| : q_i^a(x)\bar{q}_j^b(0) : |0> + x_\mu < 0| : (\nabla^\mu q_i^a(0))\bar{q}_j^b(0)) : |0> \\
&\quad + \frac{1}{2} x_\mu x_\nu \; < 0| : (\nabla^\mu \nabla^\nu q_i^a(0))\bar{q}_j^b(0) : |0> + \cdots,
\end{aligned}
\tag{3}
$$

with $\nabla^\mu \equiv \partial^\mu + ig_c A^{a\mu} \lambda^a/2$ the gauge-invariant derivative.

To see what is really in Eq. (3), we may write

$$< 0| : q_i^a(0)\bar{q}_j^b(0) : |0 >= K\delta^{ab}\delta_{ij}.$$

Contracting both sides by $\Sigma\delta^{ab}\delta_{ij}$, we find

$$
\begin{aligned}
&r.h.s. = 12K, \\
&l.h.s. = -\sum_{a,i} < 0| : \bar{q}_i^a(0) q_i^a(0) : |0> \equiv - < \bar{q}q > .
\end{aligned}
\tag{4}
$$

Or, we find

$$< 0| : q_i^a(0)\bar{q}_j^b(0) : |0 >= -\frac{1}{12}\delta^{ab}\delta_{ij} < \bar{q}q > . \tag{5}$$

Similar consideration may be applied to the remaining terms in Eq. (3), leading to the expression:

$$
\begin{aligned}
iS^{ab} = \; & \frac{i\delta^{ab}\hat{x}}{2\pi^2 x^4} + \frac{i}{32\pi^2 x^2} g_c \frac{\lambda^n_{ab}}{2} G^n_{\mu\nu}(\hat{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\hat{x}) \\
& - \frac{1}{12}\delta^{ab} < \bar{q}q > (1 + \frac{1}{16}x^2 m_0^2) + \cdots,
\end{aligned}
\tag{6}
$$

with

$$< 0|\bar{q}g_c\sigma \cdot Gq|0 > \equiv -m_0^2 < \bar{q}q > \tag{7}$$

To obtain gauge-invariant results, it is convenient to adopt the " fixed-point gauge " for the gluon fields:

$$x_\mu A^{a\mu}(x) = 0, \tag{8a}$$

so that

$$\begin{aligned} A_\mu(x) &= \int_0^1 d\alpha x^\nu G_{\nu\mu}(\alpha x) \\ &= \frac{1}{2}x^\nu G_{\nu\mu}(0) + \frac{1}{3}x^\nu x^\sigma D_\sigma G_{\nu\mu}(0) + \ldots \end{aligned} \tag{8b}$$

Note that, in Eq. (7), we have used $\sigma \cdot G \equiv \sigma^{\mu\nu} G_{\mu\nu}{}^a(0)\,\lambda^a/2$. Note also that, in Eq. (6), the second term refers to the perturbative contribution in which a quark emits a gluon while propagating, and $G_{\mu\nu}{}^n$ is the same as the $G_{\mu\nu}{}^n(0)$ of Eq. (8b). The four terms in Eq. (6) may be represented, respectively, by Figs. l(a), l(c), l(d), and l(e). The remaining diagrams in Fig. 1 are explained immediately below.

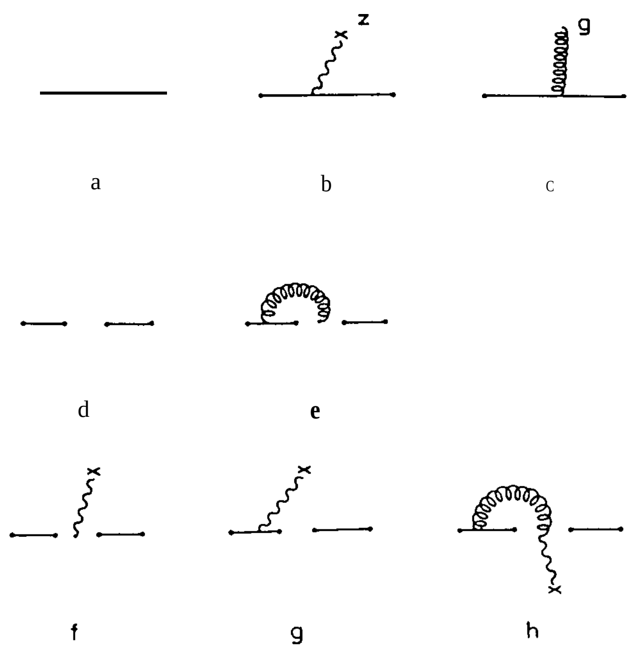

FIG. 1. Diagrams included in the quark propagator of Eq. (11).

11-2. How do quarks (or gluons) propagate in a nonperturbative physical vacuum which is under the influence of an external field, say, the axial field Z_μ?

In the presence of an external axial field,

$$Z_\mu(x) = Z^0_\mu + \frac{1}{2} Z^1_{\mu\nu} x^\nu + \cdots, \tag{9}$$

there may be induced changes on the physical vacuum so that propagation of quarks is modified. For example, we expect that

$$< 0| : \bar{q}(0)\gamma_\mu\gamma_5 q(0) : |0 > \equiv g\chi Z^0_\mu < 0| : \bar{q}(0)q(0) : |0 >, \tag{10}$$

may differ from zero, leading to a non-zero susceptibility χ. Subsequently, we may write

$$< 0| : q^a_i(0)\bar{q}^b_j(0) : |0 >_Z = a\delta^{ab}(\gamma_\mu\gamma_5)_{ij}.$$

Contracting both sides by $\Sigma\delta^{ab}(\gamma_\nu\gamma_5)_{ij}$, we find

$$a = \frac{1}{12} < 0| : \bar{q}(0)\gamma^\mu\gamma_5 q(0) : |0 > .$$

The consideration may again be applied to the remaining terms in Eq. (3), yielding results on the changes induced by the presence of the external axial field.

The final result on the quark propagator is given by

$$\begin{aligned} iS^{ab} = & \frac{\delta^{ab}}{2\pi^2 x^4}(i\hat{x} - gx\cdot Z\hat{x}\gamma_5) + \frac{i}{32\pi^2 x^2} g_c \frac{\lambda^n_{ab}}{2} G^n_{\mu\nu}(\hat{x}\sigma^{\mu\nu} + \sigma^{\mu\nu}\hat{x}) \\ & + \delta^{ab} < \bar{q}q > \{-\frac{1}{12}(1 + \frac{1}{16}x^2 m_0^2) + \frac{1}{12} g\chi\hat{Z}\gamma_5 + \frac{1}{36} g x^\alpha Z^\beta \sigma_{\alpha\beta}\gamma_5 \\ & + \frac{1}{216} g\kappa(\frac{5}{2}x^2\hat{Z} - x\,.\,Z\hat{x})\gamma_5\} + \cdots . \end{aligned} \tag{11}$$

The first three terms in Eq. (11) are the perturbative free quark propagator, and the quark propagator with a Z and a gluon, depicted in Figs. l(a-c). The next five nonperturbative terms, proportional to $<\bar{q}q>$, are the quark condensate and this same condensate in the presence of gluonic and external Z fields, depicted in the five diagrams of Fig. l(d-h). The other quantities appearing in Eq. (11) are the Z-quark coupling constant (which already enters Eq. (10); $g = g_u = -g_d$ for the isovector axial coupling g_A or $g = g_u = g_d$ for the isoscalar axial coupling $g_A{}^S$) and the condensate parameters defined by Eqs. (7),(10), and

$$< 0|\bar{q}g_c\tilde{G}_{\mu\nu}\gamma^\nu q|0 > = g\kappa Z_\mu < \bar{q}q > \tag{12}$$

Our definition of κ differs in sign from that of Ref. 4. Although the last term in the quark propagator (11) differs in sign and by a factor of 3 with that of Ref. 4, the sign is due to the difference in definition and the factor of 3 is absorbed in the definition of κ in Ref. 4. In addition to the quark and gluon condensates, one has the parameter m_0^2 and the two susceptibilities κ and χ.

II-3. Evaluation of the Correlation Function $\Pi(p)$

Consider the correlation operator, $\Pi(p)$, which is defined as[2-5]

$$\Pi(p) \equiv i\int d^4x e^{ip\cdot x} < 0|T(\eta(x)\bar{\eta}(0))|0>, \tag{13}$$

where for the nucleon current we may use a standard (but not unique) form[6]

$$\eta(x) = \epsilon^{abc}\{u^a(x)^T C\gamma_\mu u^b(x)\}\gamma^\mu\gamma^5 d^c(x), \tag{14a}$$

$$< 0|\eta(0)|N(p) > \equiv \lambda_N v_N(p), \tag{14b}$$

with C the charge conjugation operator, $a, b,$ c color indices, and $v_N(p)$ the nucleon spinor normalized such that $\bar{v}v = 2M_N$. Embedding the system in au external Z_μ field and introducing intermediate states we can express the polarization operator in the limit of a constant external field, $Z_\mu(x) = Z_\mu$, as[4,5]

$$\begin{aligned}\Pi(p) &= -|\lambda_N|^2\frac{1}{\hat{p} - M_N} \\ &-|\lambda_N|^2\frac{1}{\hat{p}-M_N}g_A\hat{Z}\gamma_5\frac{1}{\hat{p}-M_N} + \ldots,\end{aligned} \tag{15}$$

where we have adopted the on-shell definition of the nucleon axial form factor:

$$\begin{aligned}&< N(p',\lambda')|J^5_\mu(0)|N(p,\lambda) > \\ &= \bar{u}_{\lambda'}(p')\{g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\}u_\lambda(p),\end{aligned} \tag{16}$$

with $q_\mu \equiv p_\mu' - p_\mu$. The term shown in Eq. (15) corresponds to nucleon intermediate states; continuum contributions to II are shown simply by '...' in that equation. The axial coupling constant g_A in Eq. (15) is defined at $q^2 = 0$. Eq. (15) is the expression for the phenomenological form, in which $\Pi(p)$ is evaluated at the hadron level.

The correlation function $\Pi(p)$ may also be evaluated at the quark level:

$$\begin{aligned}i\Pi(p) = &\int d^4x e^{ip\cdot x} 2\epsilon^{abc}\epsilon^{a'b'c'}Tr\{iS(x)^{bb'}_u\gamma_\nu CiS(x)^{aa'T}_u C\gamma_\mu\} \\ &\cdot\gamma_5\gamma^\mu iS(x)^{cc'}_d\gamma^\nu\gamma_5.\end{aligned} \tag{17}$$

Substituting the quark propagator (11) into Eq. (17), we obtain the processes shown in Figs. 2 (a-h), which enter the sum rule calculation for the couplings of the nucleon to the axial fields.

Note that Figs. 2(b) and 2(h) may be evaluated with the aid of the identity for the gluon condensate:

$$< g_c^2 G^n_{\rho\sigma} G^m_{\alpha\beta} > = \frac{\delta^{nm}}{96} (g_{\rho\alpha} g_{\sigma\beta} - g_{\rho\beta} g_{\sigma\alpha}) < g_c^2 G^2 > . \tag{18}$$

On the other hand, Figs. 2(f) are evaluated using the relation:

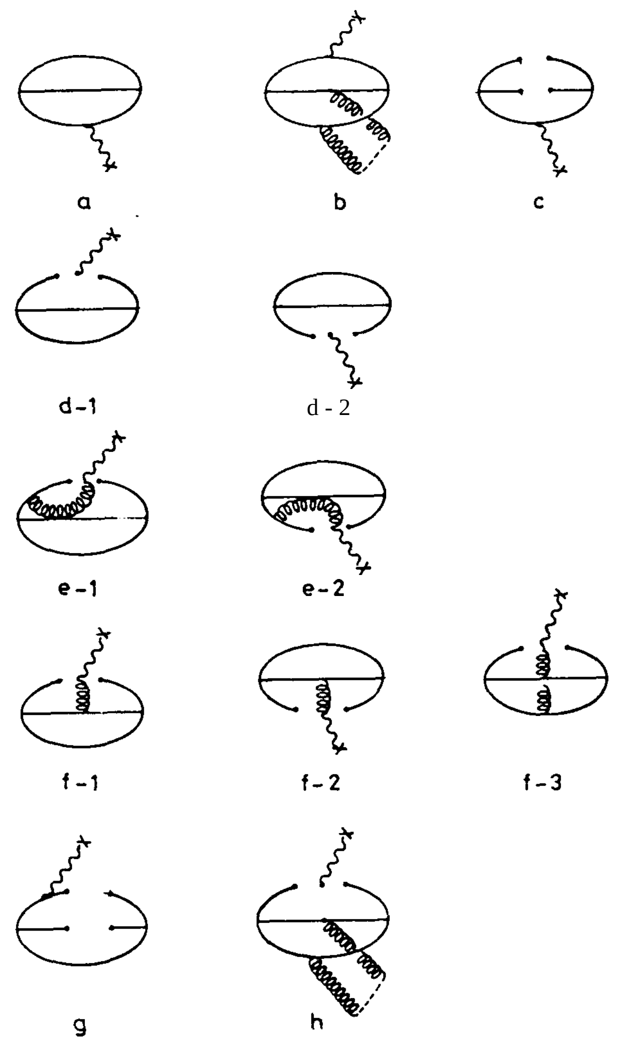

FIG. 2. Processes included in the polarization function leading to the sum rules when the coefficients of $p \cdot Z \hat{p} \gamma_5$ and $\hat{Z} \gamma_5$ are compared.

$$< q_i^a G_{\mu\nu}^m \bar{q}_j^b >_Z = \frac{1}{96}(\gamma_\alpha \sigma_{\mu\nu} + \sigma_{\mu\nu}\gamma_\alpha)\gamma_5 \frac{\lambda_{ab}^m}{2} < \bar{q}\tilde{G}^{\alpha\beta}\gamma_\beta q >, \tag{19}$$

with $\tilde{G}^{\alpha\beta} \equiv 1/2\, \varepsilon^{\alpha\beta\rho\sigma} G_{\rho\sigma}^n \lambda^n/2$. Note that, in Eqs. (18) and (19), all the field operators are evaluated at $x = 0$.

In addition to terms included in Refs. 3, 4, 7, and 8, we have added Fig. 2(h) and others so that contributions are included consistently up to dimension eight ($d = 8$).

When evaluating the polarization operator $\Pi(p)$ at the quark level and comparing it with Eq. (15), one is led to three QCD sum rules involving g_A, which[5] may not be consistent among themselves although there is indeed one sum rule which seems most appropriate for g_A. Note that Figs. 2 (a-h) enter the sum rules when the coefficients of $p \cdot Z\hat{p}\gamma_5$ and $\hat{Z}\gamma_5$ are compared. Specifically, we obtain

$$\begin{aligned} & g_d p^2 [\ln(-p^2) + \frac{5}{6}] - \frac{1}{4} g_u < g^2 G^2 > \frac{1}{p^2} - \frac{4}{3} g_d a^2 \frac{1}{p^4} \\ & - \frac{2}{3}(g_u + g_d)\chi a[\ln(-p^2) + \frac{3}{2}] - \frac{2}{3}(\frac{5}{3} g_u + g_d)\kappa a \frac{1}{p^2} + \frac{2}{3}(g_u + g_d)\kappa a \frac{1}{p^2} \\ & + \frac{8}{9} g_u a^2 \frac{1}{p^4} + \frac{1}{36} g_u \chi a < g_c^2 G^2 > \frac{1}{p^4} \\ & = 2\beta_N^2 g_A \frac{1}{(p^2 - M_N^2 + i\varepsilon)^2} + \ldots, \end{aligned} \tag{20}$$

with $a \equiv -(2\pi)^2 < \bar{q}q >$ and $\beta_N^2 \equiv (1/4)(2\pi)^4 |\lambda_N|^2$. Note that the eight terms correspond, in the alphabetic sequence, to the contributions from Figs. 2 (a-h), respectively. Eq. (20) is what we call a "QCD sum rule" for g_A.

II-4. Additional Improvements

Consider the n-th moment of the correlation function:

$$\begin{aligned} M_n(Q_0^2) &= \frac{1}{n!}(-\frac{d}{dQ^2})^n \Pi(Q^2)|_{Q^2=Q_0^2} \\ &= \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{Im\Pi(s)ds}{(s + Q_0^2)^{n+1}} \\ &\propto \frac{\gamma_N^2}{(m^2 + Q_0^2)^{n+1}} + \frac{\gamma_{N*}^2}{(m^{*2} + Q_0^2)^{n+1}} + \cdots \\ &\rightarrow \frac{\gamma_N^2}{(m^2 + Q_0^2)^{n+1}} \quad \text{as } n \rightarrow \infty. \end{aligned} \tag{21}$$

Thus, we may suppress contributions from excited states by performing the Borel transformation to both sides of Eq. (20):

$$\mathcal{B}[f(p^2)] \equiv \lim \frac{1}{n!}(-p^2)^{n+1}(\frac{d}{dp^2})^n f(p^2), \tag{22}$$

with $n \to \infty$, $-p^2 \to \infty$, and $-p^2/n = M^2$ defined for the limiting procedure.

As the next improvement, we note that the anomalous dimensions for the terms (operators) in Eq. (20) are different, indicating slightly different QCD evolution behaviors of the various terms. Specifically, some relevant anomalous dimensions are listed below:

$$\begin{aligned} &\eta(x) : +\frac{2}{9}, \quad && G^n_{\alpha\beta}G^{n\alpha\beta} : 0, \\ &\bar{q}q : +\frac{4}{9}, && \bar{q}\gamma_\mu\gamma_5 q : 0, \\ &\bar{q}\sigma_{\mu\nu}q : -\frac{4}{27}, && \bar{q}\tilde{G}_{\mu\nu}\gamma^\nu q : -\frac{32}{81}. \end{aligned} \tag{23}$$

To remedy the situation, we introduce, with A the QCD scale,

$$L \equiv \frac{\ln(\frac{M_B^2}{\Lambda^2})}{\ln(\frac{\mu^2}{\Lambda^2})}, \tag{24}$$

and insert powers of L to the various terms in the QCD sum rule in order to describe the proper QCD evolution in $\ln(M_B{}^2/\Lambda^2)$, thereby increasing the range of the validity of the derived QCD sum rule.

After carrying out the Bore1 transformation and inserting powers of L, *we* obtain a sum rule for g_A, with $g_u = -g_d = 1$:

$$\begin{aligned} &\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 - \frac{M_B^2}{18L^{68/81}}\kappa a E_0 + \frac{5}{18}a^2 L^{4/9} \\ &\qquad + \frac{1}{288L^{4/9}}\chi a < g_c^2 G^2 > \\ &\qquad = \beta_N^2(g_A + AM_B^2)\exp(-M_N^2/M_B^2), \end{aligned} \tag{25}$$

where $L = 0.621 \ln(10M_B)$, corresponding to $\Lambda_{QCD} = 0.1$ *GeV* with the Bore1 mass, M_B, in *GeV*. The most important terms on the left hand side are the first term and that proportional to a^2, corresponding to Figs. 2(a) and 2(c) and 2(g), respectively. Note that only the standard quark and gluon condensates and the susceptibilities κ and χ enter, and that the term involving the latter is numerically small.

Note that, in obtaining Eq. (25), the factors $E_0 = 1 - e^{-x}$, $E_1 = 1 - (1 + x)e^{-x}$, and $E_2 = 1 - (1 + x + x^2/2)e^{-x}$, with $x \equiv W^2/M_B{}^2 \approx (2.3\ GeV^2)/M_B{}^2$ (see Ref. 2) are used to correct the sum rule to obtain consistent $M_B{}^2$ dependence for contributions from excited states through perturbative QCD techniques.[6,7] They also serve to restrict the range of the integration and increase the weight given to the nucleon. We have thus made the usual assumption in Eq. (25). The constant A is introduced to represent the residual continuum contribution to the dispersion integral.

On the same footing, we may obtain the sum rule for $g_A{}^S$ (with $g_u = g_d = 1$)

$$-\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 + \frac{1}{6L^{4/9}} \chi a M_B^4 E_1 - \frac{M_B^2}{18L^{68/81}} \kappa a E_0$$
$$- \frac{1}{18} a^2 L^{4/9} + \frac{1}{288L^{4/9}} \chi a < g_c^2 G^2 > \tag{26}$$
$$= \beta_N^2 (g_A^S + A^S M_B^2) \exp(-M_N^2/M_B^2),$$

This is the sum rule for the "isoscalar" axial coupling constant $g_A{}^S$; it agrees with that of Ref. 8, except that their κ should be $\kappa/3$. It is assumed that the susceptibilities and W^2 are identical to those for the isovector case. This assumption can be investigated, but we may adopt it here for simplicity. The most important terms on the left hand side are the first term and that proportional to $\chi a M_B{}^4$, corresponding to Figs. 2(a) and 2 (d-l) and (d-2), respectively. Note that the susceptibility χ is very important in the sum rule for $g_A{}^S$ but only makes a small correction to g_A.

III. COUPLINGS OF THE NUCLEON TO AXIAL FIELDS

As already indicated in Refs. 3 and 5, the sum rule for g_A can be combined with that for the mass to obtain a sum rule for $g_A - 1$, which yields predictions relatively stable against reasonable variations in the Bore1 mass M_B. In particular, we make use of a Belyaev-Ioffe sum rule[6] for the determination of the nucleon mass:

$$\frac{M_B^6 E_2}{8L^{4/9}} + \frac{M_B^2}{32L^{4/9}} < g_c^2 G^2 > E_0 + \frac{1}{6} a^2 L^{4/9} - \frac{1}{24M_B^2} a^2 m_0^2$$
$$= \beta_N^2 \exp(-M_N^2/M_B^2), \tag{27}$$

Note that the first two terms in the left-hand side of the two sum rules, Eqs. (25) and (27), are equal. By subtracting Eq. (27) from Eq. (25), one obtains a sum rule for $(g_A - 1)$ involving the condensates a, m_0^2, and the susceptibilities κ and χ. These parameters have been estimated to be[1,2,6,9]

$$
\begin{aligned}
&a \approx 0.55 GeV^3, \\
&\kappa a \approx 0.140 GeV^4, \\
&\chi a \approx 0.70 GeV^2, \\
&< g_c^2 G^2 > \approx 0.47 GeV^4, \\
&m_0^2 \approx 0.8 GeV^2,
\end{aligned}
\tag{28}
$$

Because κ is less well known than the other constants, we also consider $\kappa a \approx -0.140\ GeV^4$ in order to estimate (roughly) the error of the sum rule method. The parameter β_N^2 has been determined[2] through the mass sum rule to be $\beta_N^2 \approx 0.26\ GeV^6$. In Eq. (28) we use the standard value[3] of the quark condensate. Subtracting Eq. (27) from Eq. (25), we obtain a sum rule very similar to the one obtained[3] by Belyaev and Kogan:

$$
\begin{aligned}
&\frac{1}{9} a^2 L^{4/9} + \frac{1}{24} \frac{a^2 m_0^2}{M_B^2} - \frac{1}{18} \frac{\kappa a M_B^2}{L^{68/81}} E_0 + \frac{1}{288 L^{4/9}} \chi a < g_c^2 G^2 > \\
&\quad = \beta_N^2 \{ (g_A - 1) + A M_B^2 \} \exp(-M_N^2/M_B^2).
\end{aligned}
\tag{29}
$$

This sum rule is only very weakly dependent on χa; its dominant contribution on the left-hand side is the first term; the second term is less important and the other ones are small.

Analogously, we obtain, by adding together Eqs. (25) and (26),

$$
\begin{aligned}
&\frac{M_B^2}{16 L^{4/9}} < g_c^2 G^2 > E_0 + \frac{1}{6 L^{4/9}} \chi a M_B^4 E_1 - \frac{M_B^2}{9 L^{68/81}} \kappa a E_0 \\
&\quad + \frac{2}{9} a^2 L^{4/9} + \frac{1}{144 L^{4/9} M_B^2} \chi a < g_c^2 G^2 > \\
&\quad = \beta_N^2 (g_A + g_A^S + A' M_B^2) \exp(-M_N^2/M_B^2).
\end{aligned}
\tag{30}
$$

Eqs. (29) and (30) are the main result for the axial couplings of the nucleon. It is clear from Eqs. (28) and (29) that, for $(g_A - 1)$, the quark condensate (represented by a) dominates and that the induced condensates (proportional to the susceptibilities χ and κ) are not important. This is not so for the "isoscalar" (g_A^S) sum rule, and it causes greater uncertainty in the results for this quantity.

In the numerical analysis, after moving the factor $\exp(-M_N^2/M_B^2)$ to the l.h.s., we may compare the l.h.s. to a straight-line approximation C + DM_B^2. In practice, for a given Bore1 mass M_B, we may determine the straight line which go through the points $M_B \pm \delta M_B$ (with, say, δM_B = 0.1 *GeV*) and then compare the values of the l.h.s. and r.h.s. of the sum rule at M_B. When both sides agree with the desired accuracy, the sum rule is said to hold to that accuray and it allows for extraction of the constants C and D.

We obtain solutions for the (g_A - 1) sum rule, Eq. (29), for values of the Bore1 mass $M_B \geq$ 1.8 *GeV*. It is worrisome that such large values of M_B are present in our analysis, since it is expected that M_B is of the same magnitude as the mass of the baryon of interest. I.e., one expects solutions for M_B in the range of 1 *GeV*. Larger values of M_B might appear to indicate that our g_A is distorted by coupling to baryon resonances and perhaps other states of higher energy. However, in our analysis these continuum contributions to the sum rule for (g_A - 1) are very small. Continuum contributions show up at two places for (g_A - 1): (1) the term proportional to A and (2) the deviation of E_0 from unity in Eq. (29). The former is very small, and we obtain almost identical solutions to Eq. (29) if we let $E_0 = 1$. Therefore, we conclude that the continuum contributions are small and are handled reasonably well in our calculation, even though M_B is larger than expected. The large values of M_B are still puzzling, and may be a consequence of the fact that in the sum rule for (g_A - 1) the continuum contributions of the g_A sum rule and of the mass sum rule almost cancel.

For the $g_A + g_A{}^S$ sum rule, Eq. (30), we also obtain solutions for values of the Borel mass $M_B \geq$ 1.8 *GeV*. The situation here, however, is quite different: the continuum corrections provided by E_0, E_1, and E_2 are quite important for our final results. For this reason we have examined the dependence of $g_A{}^S$ on the value of M_B. We seek solutions to Eq. (30) for g_A + $g_A{}^S$ simultaneously with the mass sum rule, Eq. (27), by fixing the value of the nucleon mass at its physical value and adjusting $\beta_N{}^2$ accordingly. We can obtain solutions of the $g_A{}^S$ sum rule for $M_B \approx$ 1.2 *GeV*, but with about a 35% increase in the value of the parameter $\beta_N{}^2$. The resulting $g_A{}^S$ is not significantly changed. In other words, the value of $g_A{}^S$ is quite insensitive to the value of M_B for $M_B \geq$ 1.2 *GeV*. Numerically, we obtain (with $M_B \geq$ 1.8 *GeV*)

$$g_A = 1.26 \pm 0.08, \tag{31a}$$

$$g_A^S = 0.13 \pm 0.08. \tag{31b}$$

A most satisfactory aspect of the result is that we obtain a value of g_A consistent with experiment with a value of the quark condensate parameter a which gives rise to the correct magnetic moments of nucleons.[2] On the other hand, the value for $g_A{}^S$, which is very sensitive to the susceptibility χ, is not very different from the EMC data." The EMC data, together with an analysis of strange baryon decays, yields[10,11]

$$g_A^S = \text{Au} + \text{Ad} = 0.28 \pm 0.08,$$

a value slightly larger than, but compatible with, ours. (Note that Au and Ad extracted from the EMC data contains contributions from antiquarks, which have been neglected in the above consideration.)

IV. SUMMARY AND OUTLOOK

To sum up, we have described the method of QCD sum rules, using calculation of the couplings of the nucleon to axial fields as an illustrative example. Our interest is centered about the behavior of a system of quarks, antiquarks, and gluons under the influence of an external field, leading to a modified version of the method of QCD sum rules. In particular, the isovector axial coupling constant g_A is determined from a QCD sum rule for $(g_A - 1)$, and it is shown that, with standard values of the quark condensates, $g_A = 1.26 \pm 0.08$. A sum rule is also obtained for the "isoscalar" axial coupling constant $g_A{}^S$ which is found to be 0.13 if the isovector values of susceptibilities are used.

It is the goal of the present authors to apply the method of QCD sum rules for the studies of weak interactions of free hadrons and for those in nuclei. Of particular current interest are the values of g_A, $g_A{}^S$, and g_P, the induced pseudoscalar coupling constant, for the free nucleon and for the nucleon embedded in a nuclear medium. In addition, work is in progress[12] for studying baryon mass differences, with emphasis on isospin symmetry breakings and flavor $SU(3)$ symmetry breakings.

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