CHINESE JOURNAL OF PHYSICS VOL. 41 , NO. 6 DECEMBER 2003

CP Violation in $B \to PP$ Decays in the Standard Model with SU(3) Symmetry

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(Received March 26, 2003)

In this paper we study CP violation in the $B \to PP$ decays in the Standard Model using SU(3) flavor symmetry. With SU(3) symmetry, only seven hadronic parameters are needed to describe the $B \to PP$ decays in the SM, when annihilation contributions are neglected. The relevant hadronic parameters can be determined using known experimental data from $B \to \pi\pi$ and $B \to K\pi$. We predict branching ratios and CP asymmetries for some of the unmeasured $B \to PP$ decays. Some of the CP asymmetries can be large and measured at B factories. The effects of the annihilation contributions can also be studied using the present experimental data. Inclusion of annihilation contributions introduces six more hadronic parameters. We find that the annihilation contributions are in general small, but can have significant effects on CP asymmetries and some $B_s \to PP$ decays.

PACS numbers: 11.30.-j, 11.30.Hv, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

In this paper we study rare charmless hadronic $B \to PP$ decays using SU(3) flavor symmetry in the Standard Model (SM). Here P is one of the SU(3) octet pseudoscalar mesons. The SU(3) analysis for rare charmless B decays has been studied by many groups, and they have obtained several interesting results, such as relations between the different decay branching ratios, and ways to constrain and/or to determine the phase γ [1–6]. SU(3) symmetry for the $B \to PP$ decays is expected to be a good approximation, because the energies released in these decays is larger than the hadronization scale. A test of SU(3) symmetry has been shown to be possible, by using the relations predicted and also by using some $B \to PP$ and $B_s \to PP$ decays in an electroweak model independent way [4]. Here we will take SU(3) symmetry as our working hypothesis. We will also study how SU(3) breaking effects may affect the results.

Recently it has been shown that, if enough $B \to PP$ decay branching ratios can be measured in the framework of SU(3) symmetry, the associated hadronic parameters and their CP conserving final state interaction (FSI) phases can be systematically studied [7]. The CP violating phase γ in the KM matrix can also be constrained. From a comparison of the phase γ constrained this way with other constraints, the consistency of the SM can be checked. Once the hadronic parameters are determined, CP asymmetries in these decays can be predicted. In previous numerical analyses, due to limited experimental data samples, annihilation contributions were neglected. Recently more data have become available, allowing an analysis with annihilation contributions. Here we will carry out an analysis, using the most recent data on rare charmless $B \to PP$ decays, to determine the hadronic parameters, including annihilation contributions, and to predict several other decay branching ratios and CP asymmetries in $B \to PP$ decays. We found that the annihilation contributions can have large effects on CP violation.

We start with a few comments on the determination of the CP violating phase γ using information from ϵ_K in $K^0 - \bar{K}^0$ mixing, $B - \bar{B}$ mixing, $|V_{ub}/V_{cb}|$, and $\sin(2\beta)$. Very stringent constraints on the CP violating phase γ [7–9] can be obtained by using experimental information on various KM matrix elements [7, 9]. Some of the most stringent constraints come from the CP violating parameter ϵ_K, $|V_{ub}/V_{cb}|$, and Δm_B. The recently measured $\sin(2\beta)$ also provides important information. Although $B_s - \bar{B}_s$ mixing has not been measured, one can still use information on the upper bound on Δm_{B_s} to constrain the phase. One of the methods to include $B_s - \bar{B}_s$ mixing information, in a global χ^2 fit of γ, is to use the amplitude method [10]. Using the input numerical values of the parameters in these processes as in Ref. [7, 9], and the new averaged value of $\sin(2\beta) = 0.78 \pm 0.08$ [11–14], we obtain the best fit value of γ to be 59°. The 68% C.L. and 95% C.L. allowed regions, in the Wolfenstein parameters ρ and η plane, are shown for this case in Figure 1.

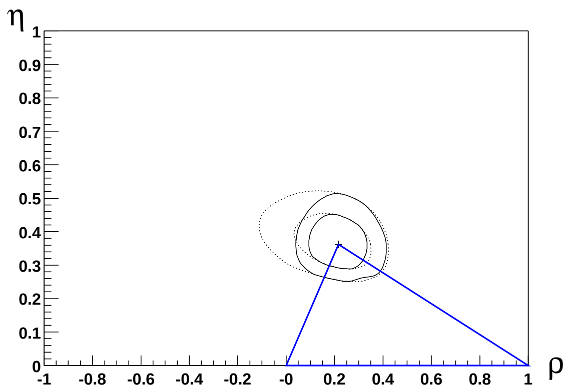

FIG. 1: The solid lines are for the fit with Δm_{B_s}, and the dashed lines are for the fit without Δm_{B_s}. The two regions from smaller to larger correspond to the the 68% C.L. and the 95% C.L. allowed regions, respectively.

The usage of an upper bound on Δm_{B_s} to constrain γ is not without controversy, because the result depends on how the bound is included. To avoid uncertainties due to this, we propose to fit the value for γ without the use of the Δm_{B_s} bound, and use the other information to predict the allowed range of the Δm_{B_s}. Carrying out a χ^2 analysis, we find that the best values with 68% C.L. errors for γ and Δm_{B_s}, and their 95% C.L. ranges are given by

$$\begin{aligned} \gamma &= 59^{+26^\circ}_{-16^\circ}, \quad 39^\circ \sim 94^\circ, \ 95\% \text{ C.L. range;} \\ \Delta m_{B_s} &= 17.9^{+4.4}_{-3.9}, \quad 10.8 < \Delta m_{B_s} < 26.1, \ 95\% \text{ C.L. range.} \end{aligned} \tag{1}$$

In Figure 1 we also show the allowed region (the dashed lines) in the ρ and η plane. Since the experiments constrain Δm_{B_s} to be larger than 14.9 ps^{-1} [15] at the 95% C.L., the range for Δm_{B_s} should be taken to be 14.9 $\sim$ 26.1 ps^{-1} at the 95% C.L. This prediction is consistent with the prediction of $\Delta m_{B_s} = 29^{+16}_{-11}$ ps^{-1} [16], from a measurement of $\Delta\Gamma_s$ and a lattice calculation of $\Delta\Gamma_s/\Delta m_{B_s}$. The predicted range of Δm_{B_s} can be measured at future hadron colliders, such as LHCb, HERAb and BTeV. This will provide an important test for the SM.

The value of γ obtained above will serve as a reference value for comparison when we study the $B \to PP$ decays. We will make use of the values obtained in two ways. We will first study the consistency of the value obtained here and the one to be obtained from $B \to PP$ decays. The other way is to fix γ at its best fit value, determined above, and to use the experimental data on the $B \to PP$ decays to fix the hadronic parameters using SU(3) symmetry and to predict other unmeasured branching ratios and CP asymmetries.

In Section II, we will briefly review the SU(3) parameterizations for the $B \to PP$ decay amplitudes in the SM and study the consistency of γ, by comparing the constraint discussed earlier and that from the $B \to PP$ decays. In Section III we study the SU(3) hadronic parameters, the branching ratios, and CP asymmetries for the $B \to PP$ decays. In Section IV, we study the effects of the annihilation amplitudes on the $B \to PP$ decays. In Section V, we discuss some of the implications from our studies and draw conclusions.

II. SU(3) HADRONIC PARAMETERS AND THE PHASE γ

In the SM the decay amplitudes for $B \to PP$ can be written as

$$A(B \to PP) =< PP|H^q_{eff}|B >= \frac{G_F}{\sqrt{2}}[V_{ub}V^*_{uq}T(q) + V_{tb}V^*_{tq}P(q)], \tag{2}$$

where $B = (B_u, B_d, B_s) = (B^-, \bar{B}^0, \bar{B}^0_s)$. $T(q)$ contains contributions from the *tree* operators as well as *penguin* operators due to charm and up quark loop corrections to the matrix elements, while $P(q)$ contains contributions purely from *penguins* due to top and charm quarks in loops.

SU(3) flavor symmetry can relate different $B \to PP$ decays. Therefore, knowing some of the branching ratios, other branching ratios and associated CP violating rate asymmetries can be predicted. As far as the SU(3) structure is concerned, the effective Hamiltonian contains $\bar{3}$, 6, and $\overline{15}$ which define three types of SU(3) invariant amplitudes. We use the notations in Ref. [7]. In Table II, we list the $B \to PP$ decays in terms of the SU(3) invariant amplitudes.

In general there are both tree and penguin amplitudes $C^{T,P}_{\bar{3},6,\overline{15}}$, $A^{T,P}_{\bar{3},6,\overline{15}}$. C_6 and A_6 always appear as $C_6 - A_6$, and we take this combination to be C_6. The amplitudes A_i are referred as annihilation amplitudes. In total there are 10 complex hadronic parameters (20 real parameters with one of them to be an overall unphysical phase). However simplification can be made because of the following relations in the SM:

TABLE I: SU(3) decay amplitudes for $B \to PP$ decays.

$\Delta S = 0$	$\Delta S = -1$
$T^{B_u}_{\pi^-\pi^0}(d) = \frac{8}{\sqrt{2}} C^T_{\overline{15}}$,	$T^{B_u}_{\pi^-\bar{K}^0}(s) = C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}}$,
$T^{B_u}_{\pi^-\eta_8}(d) = \frac{2}{\sqrt{6}}(C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} + 3C^T_{\overline{15}})$,	$T^{B_u}_{\pi^0 K^-}(s) = \frac{1}{\sqrt{2}}(C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} + 7C^T_{\overline{15}})$,
$T^{B_u}_{K^- K^0}(d) = C^T_{\bar{3}} - C^T_6 + 3A^T_{\overline{15}} - C^T_{\overline{15}}$,	$T^{B_u}_{\eta_8 K^-}(s) = \frac{1}{\sqrt{6}}(-C^T_{\bar{3}} + C^T_6 - 3A^T_{\overline{15}} + 9C^T_{\overline{15}})$,
$T^{B_d}_{\pi^+\pi^-}(d) = 2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}}$,	$T^{B_d}_{\pi^+ K^-}(s) = C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} + 3C^T_{\overline{15}}$,
$T^{B_d}_{\pi^0\pi^0}(d) = \frac{1}{\sqrt{2}}(2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} - 5C^T_{\overline{15}})$,	$T^{B_d}_{\pi^0\bar{K}^0}(s) = -\frac{1}{\sqrt{2}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}})$,
$T^{B_d}_{K^- K^+}(d) = 2(A^T_{\bar{3}} + A^T_{\overline{15}})$,	$T^{B_d}_{\eta_8\bar{K}^0}(s) = -\frac{1}{\sqrt{6}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}})$,
$T^{B_d}_{\bar{K}^0 K^0}(d) = 2A^T_{\bar{3}} + C^T_{\bar{3}} - C^T_6 - 3A^T_{\overline{15}} - C^T_{\overline{15}}$,	$T^{B_s}_{\pi^+\pi^-}(s) = 2(A^T_{\bar{3}} + A^T_{\overline{15}})$,
$T^{B_d}_{\pi^0\eta_8}(d) = \frac{1}{\sqrt{3}}(-C^T_{\bar{3}} + C^T_6 + 5A^T_{\overline{15}} + C^T_{\overline{15}})$,	$T^{B_s}_{\pi^0\pi^0}(s) = \sqrt{2}(A^T_{\bar{3}} + A^T_{\overline{15}})$,
$T^{B_d}_{\eta_8\eta_8}(d) = \frac{1}{\sqrt{2}}(2A^T_{\bar{3}} + \frac{1}{3}C^T_{\bar{3}} - C^T_6 - A^T_{\overline{15}} + C^T_{\overline{15}})$,	$T^{B_s}_{K^+K^-}(s) = 2A^T_{\bar{3}} + C^T_{\bar{3}} + C^T_6 + A^T_{\overline{15}} + 3C^T_{\overline{15}}$,
$T^{B_s}_{K^+\pi^-}(d) = C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} + 3C^T_{\overline{15}}$,	$T^{B_s}_{K^0\bar{K}^0}(s) = 2A^T_{\bar{3}} + C^T_{\bar{3}} - C^T_6 - 3A^T_{\overline{15}} - C^T_{\overline{15}}$,
$T^{B_s}_{K^0\pi^0}(d) = -\frac{1}{\sqrt{2}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}})$,	$T^{B_s}_{\pi^0\eta_8}(s) = \frac{2}{\sqrt{3}}(C^T_6 + 2A^T_{\overline{15}} - 2C^T_{\overline{15}})$,
$T^{B_s}_{K^0\eta_8}(d) = -\frac{1}{\sqrt{6}}(C^T_{\bar{3}} + C^T_6 - A^T_{\overline{15}} - 5C^T_{\overline{15}})$,	$T^{B_s}_{\eta_8\eta_8}(s) = \sqrt{2}(A^T_{\bar{3}} + \frac{2}{3}C^T_{\bar{3}} - A^T_{\overline{15}} - 2C^T_{\overline{15}})$.

$$\begin{aligned} C^P_6 &= -\frac{3}{2}\frac{c^{tc}_9 - c^{tc}_{10}}{c_1 - c_2 - 3(c^{uc}_9 - c^{uc}_{10})/2} C^T_6 \approx -0.013 C^T_6 \ , \\ C^P_{\overline{15}}(A^P_{\overline{15}}) &= -\frac{3}{2}\frac{c^{tc}_9 + c^{tc}_{10}}{c_1 + c_2 - 3(c^{uc}_9 + c^{uc}_{10})/2} C^T_{\overline{15}}(A^T_{\overline{15}}) \approx +0.015 C^T_{\overline{15}}(A^T_{\overline{15}}). \end{aligned} \tag{3}$$

Here we have used the Wilson coefficients obtained in Ref. [17]. We would like to comment that, in principle, the above relations are renormalization scheme and scale dependent. At leading order in the QCD correction, the above relations are renormalization scheme independent, but scale dependent [7]. Higher order corrections introduce a dependence on the renormalization schemes. We have checked with different renormalization schemes and find that numerically the changes are less than 15% for the different schemes. Although the changes are not sizable, there is a scheme dependence. The total decay amplitudes are not renormalization scheme dependent, therefore the hadronic matrix elements determined depend on the renormalization scheme used to determine the ratios, $(c_9 \pm c_{10})/(c_1 \pm c_2)$. One should consistently use the same scheme. In obtaining the above relations, we have also neglected small contributions from $c_{7,8}$ which cause less than 1% deviations.

With the above relations, there are less independent parameters; we choose them to be $C^{T,P}_{\bar{3}}(A^{T,P}_{\bar{3}})$, C^T_6, and $C^T_{\overline{15}}(A^T_{\overline{15}})$. Using the fact that an overall phase can be removed without loss of generality, we will set $C^P_{\bar{3}}$ to be real; there are in fact only 13 real independent parameters for $B \to PP$ in the SM,

$$C^P_{\bar{3}},\ C^T_{\bar{3}} e^{i\delta_{\bar{3}}},\ C^T_6 e^{i\delta_6},\ C^T_{\overline{15}} e^{i\delta_{\overline{15}}}\ A^T_{\bar{3}} e^{i\delta_{A^T_{\bar{3}}}},\ A^P_{\bar{3}} e^{i\delta_{A^P_{\bar{3}}}},\ A^T_{\overline{15}} e^{i\delta_{A^T_{\overline{15}}}}.$$

Further, the amplitudes A_i correspond to the annihilation contributions and are expected to be small. In this section, we neglect these amplitudes. In this case there are only 7 independent hadronic parameters

$$C^P_{\bar{3}},\ C^T_{\bar{3}} e^{i\delta_{\bar{3}}},\ C^T_{6} e^{i\delta_{6}},\ C^T_{\overline{15}} e^{i\delta_{\overline{15}}}. \tag{4}$$

The phases in the above are defined in such a way that all $C_i^{T,P}$ are real positive numbers. We will discuss how the annihilation contributions affect the decays in Section IV.

SU(3) may not be an exact symmetry for $B \to PP$. The amplitudes C_i for $B \to \pi\pi$ and $B \to K\pi$ will be different if SU(3) is broken. At present it is not possible to have a full calculation of the breaking effects. To have some idea about the size of the SU(3) breaking effects, we work with the factorization estimate. To leading order, the relation between the amplitudes for the $B \to \pi\pi$ decays $C_i(\pi\pi)$ and the amplitudes for $B \to K\pi$ decays $C_i(K\pi)$ can be parameterized as $C_i(K\pi) = rC_i(\pi\pi)$, where r is approximately given by $r \approx f_K/f_\pi = 1.22$. Here we have assumed that the SU(3) breaking effects in f_i and $F_0^{B\to i}$ are similar in magnitude, that is, $f_K/f_\pi \approx F_0^{B\to K}/F_0^{B\to\pi}$. Using the above to represent the SU(3) breaking effect, we can obtain another set of fitting results. Compared with $B \to K\pi$, there is also an SU(3) breaking effect in $B_s \to K\pi$, proportional to $F^{B_s\to K}/F^{B\to K}$ or $F^{B_s\to\pi}/F^{B\to\pi}$. We will take them to be approximately 1.

There are different ways to determine the hadronic parameters C_i and δ_i. A consistent and systematic way of carrying out such an analysis is to perform a χ^2 analysis, taking into account all experimental data on $B \to PP$. We will use this method to obtain the hadronic parameters and also the CP violating phase γ.

In Table II we list the presently available experimental data on $B \to PP$ decays. In general the errors for the experimental data in Table II are correlated. Due to the lack of knowledge of the error correlation from experiments, in our analysis, for simplicity, we take them to be uncorrelated and assume that the errors obey a Gaussian distribution, taking the larger one between σ_+ and σ_- to be on the conservative side. When combining different measurements, we take the weighted average. For the data which is only presented as upper bounds, we assume them to obey a Gaussian distribution and take the error σ accordingly. We would like to emphasize that, without the full knowledge of the error correlations, the results obtained should be taken as indications. A combination of the method developed here with detailed information on the error correlations can provide more reliable results.

Besides the hadronic parameters discussed previously, there are also the KM parameters involved in the analysis. Since we are primarily concerned with the hadronic parameters and the CP violating phase γ, we will carry out our χ^2 analysis with the KM matrix elements $V_{us} = \lambda$, $V_{cb} = A\lambda^2$, and $V_{ub} = |V_{ub}|exp(-i\gamma)$ fixed by [8] $\lambda = 0.2196$, $A = 0.835$, and $|V_{ub}| = 0.09|V_{cb}|$. We also take the phase γ to be a free parameter, to be determined in this section, to check the consistency of the SM. The total parameters to be determined are, therefore, the 7 hadronic parameters in Eq. (4) and the phase γ.

In Figure 2 we show χ^2 as a function of the phase γ. From the figure we see that, for the case with exact SU(3) symmetry and γ between $20^\circ \sim 160^\circ$, χ^2 is reasonably small and allowed at the one sigma level. Although there are minimal points in the curves, they

TABLE II: The branching ratios for $B \to PP$ in units of 10^{-6}.

Branching ratio and CP asymmetries	Cleo [18]	Belle [19]	Babar [20]	Averaged Value
$Br(B_u \to \pi^-\pi^0)$	$5.6^{+2.6}_{-2.3} \pm 1.7$	$7.0 \pm 2.2 \pm 0.8$	$5.5^{+1.1}_{-0.9} \pm 0.6$	5.8 ± 1.0
$Br(B_u \to K^-K^0)$	< 5.1(90%C.L.)	< 3.8(90%C.L.)	< 1.3(90%C.L.)	0 ± 0.8
$Br(B_d \to \pi^+\pi^-)$	$4.3^{+1.6}_{-1.4} \pm 0.5$	$5.1 \pm 1.1 \pm 0.4$	$5.4 \pm 0.7 \pm 0.4$	5.2 ± 0.6
$Br(B_d \to \pi^0\pi^0)$	$2.2^{+1.7+0.7}_{-1.3-0.7}$	$3.2 \pm 1.5 \pm 0.7$	$1.6^{+0.7+0.6}_{-0.6-0.3}$	2.0 ± 0.7
$Br(B_d \to K^-K^+)$	< 1.9(90%C.L.)	< 0.5(90%C.L.)	< 1.1(90%C.L.)	0 ± 0.4
$Br(B_d \to \bar{K}^0K^0)$	$1.8^{+1.8}_{-1.2} \pm 1.8$	< 4.1	< 7.3	1.8 ± 2.5
$Br(B_u \to \pi^-\bar{K}^0)$	$18.2^{+4.6}_{-4.0} \pm 1.6$	$18.8 \pm 3.0 \pm 1.5$	$17.5^{+1.8}_{-1.7} \pm 1.3$	17.9 ± 1.7
$Br(B_u \to \pi^0K^-)$	$11.6^{+3.0+1.4}_{-2.7-1.3}$	$12.5 \pm 2.4 \pm 1.2$	$12.8^{+1.2}_{-1.1} \pm 1.0$	12.6 ± 1.3
$Br(B_d \to \pi^+K^-)$	$17.2^{+2.5}_{-2.4} \pm 1.2$	$21.8 \pm 1.8 \pm 1.5$	$17.8 \pm 1.1 \pm 0.8$	18.6 ± 1.1
$Br(B_d \to \pi^0\bar{K}^0)$	$14.6^{+5.9+2.4}_{-5.1-3.3}$	$7.7 \pm 3.2 \pm 1.6$	$10.4 \pm 1.5 \pm 0.8$	10.2 ± 1.5
$A_{CP}(B_u \to \pi^-\pi^0)$		$0.31 \pm 0.31 \pm 0.05$	$-0.03^{+0.18}_{-0.17} \pm 0.02$	0.05 ± 0.16
$A_{CP}(B_d \to \pi^+\pi^-)$		$0.94^{+0.25}_{-0.31} \pm 0.09$	$-0.02 \pm 0.29 \pm 0.07$	0.44 ± 0.22
$A_{CP}(B_u \to \pi^-\bar{K}^0)$	0.18 ± 0.24	$0.46 \pm 0.15 \pm 0.02$	$-0.17 \pm 0.10 \pm 0.02$	0.04 ± 0.08
$A_{CP}(B_u \to \pi^0K^-)$	-0.29 ± 0.23	$-0.04 \pm 0.19 \pm 0.03$	$-0.09 \pm 0.09 \pm 0.01$	-0.1 ± 0.08
$A_{CP}(B_d \to \pi^+K^-)$	-0.04 ± 0.16	$-0.06 \pm 0.08 \pm 0.01$	$-0.05 \pm 0.06 \pm 0.01$	-0.09 ± 0.04
$A_{CP}(B_d \to \bar{K}^0\pi^0)$			$0.03 \pm 0.36 \pm 0.09$	0.03 ± 0.37

are not deep enough to single out one point with a high confidence level. γ around 60° is certainly allowed. There is no inconsistency between the allowed range of γ obtained here and that in the previous section. For the case with broken SU(3) symmetry, the region with γ around 110° is not favored. But γ around 60° is still allowed at the 90% C.L.. These results indicate that the fitting in this analysis is not sensitive to the phase γ. With improved data and full knowledge of error correlations, one may improve the situation.

III. BRANCHING RATIOS AND CP ASYMMETRIES FOR $B \to PP$

In the previous section we have seen that the CP violating phase γ, determined using data from $B \to PP$ and from ϵ_K, $B - \bar{B}$ mixing, $|V_{ub}/V_{cb}|$, and $\sin(2\beta)$, is not in conflict. One may want to combine these two to predict a combined best fit value for γ. At present the fit from the first section for γ has a much better error range. The combined fit will give a value for γ similar to the one in the previous section [7]. From now on we will use the best fit value of $\gamma = 59^\circ$ from the first section as a known value to study the hadronic parameters in more detail for the $B \to PP$ decays.

The best fit values for the hadronic parameters are given in Table III. The magnitudes of the C_i are of the same order of magnitude as the factorization predictions [7], and

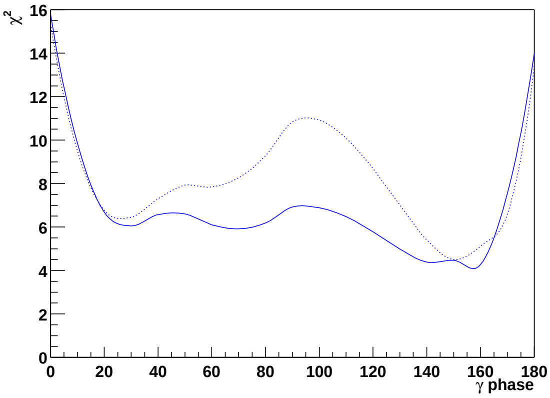

FIG. 2: χ^2 vs. γ phase without annihilation terms. The solid line is for the case with exact SU(3) symmetry, and the dotted line is for the case with the SU(3) breaking described in the text.

TABLE III: The best fit values and their 1σ errors for the hadronic parameters, using all data in Table II, with annihilation terms set to be zero and $\gamma = 59°$.

Invariant Parameters	SU(3) exact		SU(3) break		QCD Factorization
	central value	error range	central value	error range	
$C_{\bar{3}}^P$	0.139	0.003	0.114	0.003	0.08
$C_{\bar{3}}^T$	0.227	0.114	0.229	0.076	0.35
C_6^T	0.146	0.103	0.136	0.084	0.20
$C_{\overline{15}}^T$	0.154	0.012	0.156	0.011	0.15
$\delta_{\bar{3}}$	55.73^0	31.40^0	47.67^0	22.76^0	3.3^0
δ_6	89.72^0	38.78^0	83.81^0	28.28^0	7.9^0
$\delta_{\overline{15}}$	13.42^0	15.24^0	10.48^0	13.05^0	-2.8^0

the QCD improved factorization estimate of the relevant parameters are listed in the last column in Table III. The CP conserving phases δ_i, which can not be reliably calculated in the factorization approximation, can be determined from the χ^2 analysis performed here. We see from Table III that the CP conserving phases, obtained from fitting the data, can be much larger than that estimated from the QCD improved factorization.

Using the above determined hadronic parameters, one can easily obtain the branching ratios and CP asymmetries for $B \to PP$. We use the following definition for the CP violating rate asymmetry,

$$A_{CP} = \frac{\Gamma(B_i \to PP) - \Gamma(\bar{B}_i \to \bar{P}\bar{P})}{\Gamma(B_i \to PP) + \Gamma(\bar{B}_i \to \bar{P}\bar{P})}. \quad (5)$$

In general P can be any one of the SU(3) pseudoscalar octet mesons: π, K, and η_8. Here we will limit our study to $P = \pi, K$ to avoid complications associated with η_1 and η_8 mixing. In this case there are a total of 16 decay modes. Among them the decay amplitudes for $B_d \to K^- K^+$, $B_s \to \pi^- \pi^+, and \pi^0 \pi^0$ only receive annihilation contributions. Since we have neglected annihilation contributions they would have vanishing branching ratios. At present none of them have been measured experimentally. The present bound on $B_d \to K^- K^+$ is consistent with this prediction.

In Tables IV and V we show the results for the branching ratios and CP asymmetries for the other 13 decays. We see that the best fit values of the branching ratios for the ones having experimental measurements are similar and agree with each other within error bars. We also predict the branching ratios for the $B_s \to K^+\pi^-, K^0\pi^0, K^-K^+$, and $K^0\bar{K}^0$ decays. These decay modes are predicted to be large and can be measured at hadron colliders, such as CDF, HERAb, and LHCb. The SM and SU(3) flavor symmetry can be tested.

SU(3) symmetry predicts some of the CP asymmetries to be equal. From Table I we obtain

$$\begin{aligned}
&A_{CP}(B_d \to \bar{K}^0 K^0) = A_{CP}(B_u \to K^- K^0),\\
&A_{CP}(B_d \to \pi^+ \pi^-) = A_{CP}(B_s \to K^+ \pi^-),\\
&A_{CP}(B_d \to \pi^0 \pi^0) = A_{CP}(B_s \to K^0 \pi^0),\\
&A_{CP}(B_d \to \pi^+ K^-) = A_{CP}(B_s \to K^+ K^-),\\
&A_{CP}(B_u \to \pi^- \bar{K}^0) = A_{CP}(B_s \to K^0 \bar{K}^0).
\end{aligned} \tag{6}$$

When SU(3) is broken, in general, these relations may no longer hold. However, in the special pattern of the SU(3) breaking we are dealing with, the above relations still hold. Experimental measurements of CP asymmetries for these modes can provide an important test for SU(3) flavor symmetry.

In the SU(3) limit there are also some relations between the rate differences defined as $\Delta(B_i \to PP) = \Gamma(B_i \to PP) - \Gamma(\bar{B}_i \to \bar{P}\bar{P})$ between the $\Delta S = 0$ and $\Delta S = -1$ modes, due to a unique feature of the SM in the KM matrix element that [21] $Im(V_{ub}V_{ud}^* V_{tb}^* V_{td}) = -Im(V_{ub}V_{us}^* V_{tb}^* V_{ts})$. We find [4]

$$\begin{aligned}
&\Delta(B_d \to \pi^+ \pi^-) = -\Delta(B_d \to \pi^+ K^-),\\
&\Delta(B_d \to \pi^0 \pi^0) = -\Delta(B_d \to \pi^0 \bar{K}^0),\\
&\Delta(B_d \to \bar{K}^0 K^0) = -\Delta(B_u \to \pi^- \bar{K}^0).
\end{aligned} \tag{7}$$

These rate difference relations can also provide important information.

The best fit values for A_{CP} can be large, with several of them reaching more than 10%, such as the asymmetries for $B_d \to \pi^+\pi^-, \pi^0\pi^0$, $K^+\pi^-$, and $B_s \to K^+\pi^-, K^0\pi^0$, K^+K^-. $B_d \to \pi^+\pi^-$ provides the best chance to measure CP asymmetry. The fact that the size of A_{CP} for these modes are large can be easily understood from the following. Using the above relations, one would obtain

$$A_{CP}(B_d \to \pi^+\pi^-) = A_{CP}(B_s \to K^+\pi^-) = -A_{CP}(B_d \to \pi^+K^-)\frac{Br(B_d \to \pi^+K^-)}{Br(B_d \to \pi^+\pi^-)},$$
$$A_{CP}(B_d \to \pi^0\pi^0) = A_{CP}(B_s \to K^0\pi^0) = -A_{CP}(B_d \to \pi^0\bar{K}^0)\frac{Br(B_d \to \pi^0\bar{K}^0)}{Br(B_d \to \pi^0\pi^0)}. \quad (8)$$

In all the above cases the ratio of the branching ratios are larger than one; a small A_{CP} of the decay modes on the right hand side can induce a large A_{CP} for the decay modes on the left hand side. The situation with the SU(3) breaking case is also similar. We note that the averaged central experimental values of $A_{cp}(B_d \to \pi^+\pi^-)$ and $A_{cp}(B_d \to \pi^+K^-)$ are consistent with the above predictions.

The cases for $B_u \to \pi^-\bar{K}^0$, $B_d \to K^0\bar{K}^0$ and $B_s \to K^0\bar{K}^0$, $B_u \to K^-K^0$ are particularly interesting. In the factorization approximation, the tree amplitudes for these modes are almost zero. In terms of the SU(3) amplitudes, that implies $\Delta C = C^T_{\bar{3}} - C^T_{6} - C^T_{\overline{15}}$ is close to zero. CP asymmetries are predicted to be very small. That ΔC is small, however, does not follow from SU(3) symmetry. The rescattering effect may make it significantly deviate from zero. Indeed, from Table II, one sees that the experimental central values of $\mathrm{Br}(B_d \to K^0\bar{K}^0)$ and $\mathrm{Br}(B_u \to K^-K^0)$ indicate that ΔC is small. One, however, should also notice that the errors associated do not rule out the possibility of large deviations. We should seek the information more carefully for our fitting.

From Table III we can see that the best fit value, $\Delta C = -0.023 + 0.005i$, for the case with exact $SU(3)$ ($\Delta C = -0.014 + 0.005i$, for the case with broken $SU(3)$), is small. But within errors it can be non-zero. Translating this into CP violating rate asymmetries for $B_u \to \bar{K}^0\pi^-$ and $B_s \to K^0\bar{K}^0$, we see that the best fit value is small, but non-zero asymmetries at the level of a few percent cannot be ruled out. This can lead to large asymmetries for $B_u \to K^-K^0$ and $B_d \to K^0\bar{K}^0$ within the error bars, as can be seen from the Table V experiments.

We note that the CP asymmetry for $B_u \to \pi^-\pi^0$ is zero in Table V, resulting from SU(3) (or isospin) symmetry. In principle, it should have a small asymmetry, due to the different short distance strong and electroweak penguins, but it is negligibly small and has been neglected.

At present no CP asymmetry in $B \to PP$ has been measured. To see how sensitive the bounds on CP asymmetries in Table II affect the analysis, we carried out an analysis using mostly branching ratio information. If we do not use any CP violating data, we find that the branching ratios are not affected very much. However, in this case there is a degeneracy in identifying particle and anti-particle branching ratios. This implies that one can only determine the size of the asymmetries but not the signs. To determine the sign, one should use at least one CP asymmetry data point to lift the degeneracy. For this purpose we select one CP asymmetry data point, the asymmetry for $B_d \to \pi^+K^-$, for which all experimental measurements have similar central values although there is still a large error bar to establish the measurement. We list the results in Tables VI, VII and VIII.

TABLE IV: The prediction of the branching ratio without annihilation terms and γ=59°.

Branching ratio	SU(3) Exact		SU(3) break	
	central value	Error(Max , Min)	central value	Error(Max , Min)
$B_u \to \pi^-\pi^0$	6.2	(7.1 , 5.2)	6.4	(7.3 , 5.4)
$B_u \to K^-K^0$	0.7	(1.0 , 0.4)	1.0	(1.4 , 0.9)
$B_d \to \pi^+\pi^-$	5.1	(5.6 , 4.5)	4.9	(5.5 , 4.4)
$B_d \to \pi^0\pi^0$	1.7	(2.3 , 1.1)	1.5	(2.1 , 1.0)
$B_d \to \bar{K}^0K^0$	0.6	(0.9 , 0.3)	1.0	(1.3 , 0.9)
$B_u \to \pi^-\bar{K}^0$	19.4	(20.6, 18.3)	19.3	(20.3, 18.3)
$B_u \to \pi^0K^-$	10.7	(11.3, 10.1)	11.0	(11.5, 10.6)
$B_d \to \pi^+K^-$	19.1	(20.0, 18.7)	19.0	(19.3, 18.4)
$B_d \to \pi^0\bar{K}^0$	8.8	(9.2 , 8.3)	8.6	(8.9 , 8.2)
$B_s \to K^+\pi^-$	4.8	(5.3 , 4.3)	6.9	(7.7 , 6.2)
$B_s \to K^0\pi^0$	1.6	(2.2 , 1.0)	2.1	(2.9 , 1.4)
$B_s \to K^+K^-$	17.9	(18.8, 17.6)	27.6	(29.0, 26.1)
$B_s \to K^0\bar{K}^0$	17.1	(18.1, 16.1)	26.2	(27.3, 24.8)

TABLE V: The prediction of the CP asymmetry without annihilation terms and γ=59°.

Asymmetry	SU(3) Exact		SU(3) break	
	Central value	Error (Max,Min)	Central value	Error (Max,Min)
$B_u \to \pi^-\pi^0$	0	(0 , 0)	0	(0 , 0)
$B_d \to \pi^+\pi^-$	0.41	(0.53 , 0.29)	0.31	(0.40 , 0.21)
$B_u \to \pi^-\bar{K}^0$	-0.0013	(0.04 , -0.05)	-0.0018	(0.03 , -0.04)
$B_u \to \pi^0K^-$	-0.06	(0.01 , -0.14)	-0.06	(0.01 , -0.14)
$B_d \to \pi^+K^-$	-0.11	(-0.08 , -0.14)	-0.12	(-0.08 , -0.16)
$B_u \to K^-K^0$	0.04	(0.9 , -0.89)	0.05	(0.79 , -0.75)
$B_d \to \pi^0\pi^0$	0.16	(0.43 , -0.5)	0.15	(0.37 , -0.31)
$B_d \to \bar{K}^0K^0$	0.04	(0.9 , -0.89)	0.05	(0.79 , -0.75)
$B_d \to \pi^0\bar{K}^0$	-0.03	(0.07 , -0.10)	-0.04	(0.07 , -0.12)
$B_s \to K^+\pi^-$	0.41	(0.53 , 0.29)	0.31	(0.40 , 0.21)
$B_s \to K^0\pi^0$	0.16	(0.43 , -0.5)	0.15	(0.37 , -0.31)
$B_s \to K^+K^-$	-0.11	(-0.08 , -0.14)	-0.12	(-0.08 , -0.16)
$B_s \to K^0\bar{K}^0$	-0.0013	(0.04 , -0.05)	-0.0018	(0.03 , -0.04)

TABLE VI: The best fit values and their error ranges for the hadronic parameters without annihilation terms and $\gamma = 59°$, using data on branching ratios and CP asymmetry on $B_d \to K^+\pi^-$.

Invariant Parameters	SU(3) exact		SU(3) break		QCD Factorization
	central value	error range	central value	error range	
$C_{\bar{3}}^P$	0.139	0.003	0.114	0.003	0.08
$C_{\bar{3}}^T$	0.253	0.124	0.263	0.074	0.35
C_6^T	0.196	0.148	0.204	0.092	0.20
$C_{\overline{15}}^T$	0.151	0.012	0.152	0.012	0.15
$\delta_{\bar{3}}$	46.80^0	32.24^0	40.09^0	18.92^0	3.3^0
δ_6	83.46^0	29.48^0	75.97^0	19.28^0	7.9^0
$\delta_{\overline{15}}$	-3.68^0	18.98^0	-10.39^0	15.78^0	-2.8^0

From Table VI, we see that the size of the hadronic parameters C_i are not affected very much, but the CP conserving phase δ_i can vary quite a lot, especially for $\delta_{\overline{15}}$. In terms of the branching ratios and CP asymmetries, we find that the branching ratios are similar, but the CP asymmetries can be quiet different, which can be seen from Tables VII and VIII. The differences are largely caused by the differences in δ_i. It is therefore very important to have good CP asymmetry measurements, which not only provide information for CP violation but also information for the detailed dynamics of hadronic physics.

IV. EFFECTS OF ANNIHILATION CONTRIBUTIONS

In the analyses of the previous sections we have neglected annihilation contributions to the $B \to PP$ decays. In this section we study the effects of the annihilation terms on the $B \to PP$ decays. The inclusion of the annihilation contributions introduce 6 more hadronic parameters. They are

$$A_{\bar{3}}^T e^{i\delta_{A_{\bar{3}}^T}}, A_{\bar{3}}^P e^{i\delta_{A_{\bar{3}}^P}}, A_{\overline{15}}^T e^{i\delta_{A_{\overline{15}}^T}}. \quad (9)$$

In total we would have 13 parameters.

Theoretical calculations of the annihilation contributions in the QCD improved factorization method are complicated, because of the end point divergences involved [22]; they can in principle be treated by the PQCD method [22], but still with large uncertainties. Here we take a more phenomenological approach to determine them by fitting available experimental data. The results obtained can provide important information about the size of these contributions. We will also study how the annihilation contributions affect the branching ratios and CP asymmetries.

As can be seen from Table II, there are 16 experimental data points. In principle, the 13 hadronic parameters under consideration can be determined. In Tables IX, X, and

TABLE VII: The prediction of the branching ratio without annihilation terms and γ=59°, using data on branching ratios and CP asymmetry in $B_d \to K^+\pi^-$.

Branching ratio	SU(3) Exact		SU(3) break	
	central value	Error(Max , Min)	central value	Error(Max , Min)
$B_u \to \pi^-\pi^0$	5.9	(6.9 , 4.9)	6.0	(7.0 , 5.1)
$B_u \to K^-K^0$	0.7	(1.1 , 0.6)	1.1	(1.5 , 1.0)
$B_d \to \pi^+\pi^-$	5.2	(5.7 , 4.6)	5.1	(5.7 , 4.6)
$B_d \to \pi^0\pi^0$	2.0	(2.6 , 1.3)	2.0	(2.6 , 1.3)
$B_d \to \bar{K}^0K^0$	0.7	(1.1 , 0.5)	1.0	(1.4 , 0.9)
$B_u \to \pi^-\bar{K}^0$	19.5	(20.6, 18.4)	19.6	(20.6 , 18.5)
$B_u \to \pi^0K^-$	10.8	(11.3, 10.4)	11.1	(11.6 , 10.6)
$B_d \to \pi^+K^-$	18.9	(19.3, 18.1)	18.8	(19.6 , 17.9)
$B_d \to \pi^0\bar{K}^0$	8.7	(8.9 , 8.3)	8.5	(8.8 , 8.1)
$B_s \to K^+\pi^-$	4.9	(5.4 , 4.4)	7.2	(8.0 , 6.4)
$B_s \to K^0\pi^0$	1.9	(2.3 , 1.2)	2.8	(3.5 , 1.9)
$B_s \to K^+K^-$	17.8	(18.7, 16.9)	27.2	(28.5 , 25.9)
$B_s \to K^0\bar{K}^0$	17.2	(18.3, 16.2)	26.5	(27.7 , 25.1)

TABLE VIII: The prediction of the CP asymmetry without annihilation terms and γ=59°, using data on branching ratios and CP asymmetry from $B_d \to K^+\pi^-$.

Asymmetry	SU(3) Exact		SU(3) break	
	Central value	Error (Max,Min)	Central value	Error (Max,Min)
$B_u \to \pi^-\pi^0$	0	(0 , 0)	0	(0 , 0)
$B_d \to \pi^+\pi^-$	0.33	(0.48 , 0.19)	0.23	(0.33 , 0.13)
$B_u \to \pi^-\bar{K}^0$	0.0002	(0.06 , -0.06)	0.0004	(0.04 , -0.04)
$B_u \to \pi^0K^-$	0.02	(0.13 , -0.11)	0.07	(0.16 , -0.05)
$B_d \to \pi^+K^-$	-0.09	(-0.05 , -0.13)	-0.10	(-0.06 , -0.13)
$B_u \to K^-K^0$	-0.01	(0.96 , -0.97)	-0.01	(0.81 , -0.81)
$B_d \to \pi^0\pi^0$	0.43	(0.61 , 0.00)	0.42	(0.53 , 0.17)
$B_d \to \bar{K}^0K^0$	-0.01	(0.96 , -0.97)	-0.01	(0.81 , -0.81)
$B_d \to \pi^0\bar{K}^0$	-0.10	(0.00 , -0.17)	-0.15	(-0.04 , -0.22)
$B_s \to K^+\pi^-$	0.33	(0.48 , 0.19)	0.23	(0.33 , 0.13)
$B_s \to K^0\pi^0$	0.43	(0.61 , 0.00)	0.42	(0.53 , 0.17)
$B_s \to K^+K^-$	-0.09	(-0.05 , -0.13)	-0.10	(-0.06 , -0.13)
$B_s \to K^0\bar{K}^0$	0.0002	(0.06 , -0.06)	0.0004	(0.04 , -0.04)

TABLE IX: The best fit values and their errors for the hadronic parameters with annihilation terms and $\gamma = 59°$.

Invariant Parameters	SU(3) exact		SU(3) break		QCD Factorization
	central value	error range	central value	error range	
$C_{\bar{3}}^P$	0.139	0.004	0.114	0.003	0.08
$C_{\bar{3}}^T$	0.181	0.171	0.192	0.115	0.35
C_6^T	0.042	0.165	0.086	0.132	0.20
$C_{\overline{15}}^T$	0.152	0.012	0.155	0.012	0.15
$\delta_{\bar{3}}$	61.5^0	73.6^0	47.1^0	41.6^0	3.3^0
δ_6	90.3^0	163.9^0	89.0^0	60.4^0	7.9^0
$\delta_{\overline{15}}$	19.1^0	17.6^0	13.0^0	13.5^0	-2.8^0
$A_{\bar{3}}^P$	0.027	0.051	0.018	0.031	0.002
$A_{\bar{3}}^T$	0.061	0.161	0.045	0.099	0.006
$A_{\overline{15}}^T$	0.035	0.077	0.009	0.065	0.002
$\delta_{A_{\bar{3}}^P}$	26.0^0	103.9^0	-9.0^0	88.5^0	
$\delta_{A_{\bar{3}}^T}$	100.0^0	111.2^0	88.5^0	92.2^0	
$\delta_{A_{\overline{15}}^T}$	290.0^0	163.0^0	261.3^0	398.6^0	

XI we show the results on the hadronic parameters, $B \to PP$ branching ratios, and the CP asymmetries.

From Table IX, we see that the size of the best fit annihilation parameters A_i are small compared with the non-annihilation terms $C_{\bar{3},\overline{15}}$. This confirms the conjecture that the annihilation contributions are small. The allowed ranges are, however, large; therefore we can not rule out the possibility of having significant annihilation contributions. We have to wait for improved experiments to obtain more precise information. We note that the A_i actually have a similar size as C_6^T. The branching ratios for $B_d \to K^-K^+$, $B_s \to \pi^+\pi^-$, and $B_s \to \pi^0\pi^0$, which only receive contributions from annihilation, are no longer vanishing. The branching ratios are expected to be small. From Table X, we indeed find that these branching ratios are among the small ones.

It is interesting to note that although the annihilation amplitudes are small, in certain decay modes, such as $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$, the effects on the branching ratios can be significant. Because, although $A_{\bar{3}}^P$ is small compared with $C_{\bar{3},\overline{15}}$ and is comparable with C_6^T, yet it is enhanced by a KM factor $|V_{tb}V_{ts}^*/V_{ub}V_{us}^*|$. These modes provide good places to study the annihilation contributions. It can be seen that SU(3) breaking effects are also large in these decays. From Table XI, we also see that CP violation can be affected significantly. CP asymmetries in $B_d \to K^0\bar{K}^0$ can be more than 50%, with a not too small branching ratio.

TABLE X: The prediction of the branching ratios with annihilation terms and γ=59°

Branching ratio	SU(3) Exact		SU(3) break	
	central value	Error(Max , Min)	central value	Error(Max , Min)
$B_u \to \pi^-\pi^0$	6.0	(7.0 , 5.0)	6.2	(7.1 , 5.2)
$B_u \to K^-K^0$	0.7	(1.7 , -0.3)	1.0	(2.0 , 0.0)
$B_d \to \pi^+\pi^-$	5.2	(5.8 , 4.5)	5.1	(5.7 , 4.5)
$B_d \to \pi^0\pi^0$	1.9	(2.5 , 1.2)	1.7	(2.3 , 1.1)
$B_d \to K^-K^+$	0.1	(0.5 , -0.1)	0.2	(0.5 , -0.1)
$B_d \to \bar{K}^0K^0$	1.8	(4.3 , 0.3)	2.0	(4.4 , 0.6)
$B_u \to \pi^-\bar{K}^0$	19.5	(20.8 , 18.1)	19.2	(20.5 , 17.9)
$B_u \to \pi^0K^-$	10.7	(11.4 , 10.0)	10.9	(11.6 , 10.2)
$B_d \to \pi^+K^-$	19.0	(20.0 , 17.9)	19.1	(20.1 , 18.0)
$B_d \to \pi^0\bar{K}^0$	8.8	(9.3 , 8.3)	8.6	(9.1 , 8.1)
$B_s \to K^+\pi^-$	3.8	(6.7 , 2.4)	5.9	(8.6 , 4.3)
$B_s \to K^0\pi^0$	1.6	(2.9 , 0.6)	2.1	(3.4 , 1.1)
$B_s \to \pi^+\pi^-$	2.7	(11.3 , -0.1)	1.3	(5.7 , -0.1)
$B_s \to \pi^0\pi^0$	1.4	(5.7 , -0.1)	0.6	(2.8 , -0.1)
$B_s \to K^+K^-$	33.1	(56.4 , 5.8)	47.7	(77.6 , 15.3)
$B_s \to K^0\bar{K}^0$	32.1	(55.3 , 5.1)	45.6	(75.4 , 13.7)

V. DISCUSSIONS AND CONCLUSIONS

We have studied branching ratios and CP violating rate asymmetries in $B \to PP$ decays in the Standard Model using SU(3) flavor symmetry. In the SM, when annihilation contributions are neglected, only seven hadronic parameters are needed to describe $B \to PP$ decays, six more hadronic parameters are needed to include the annihilation contribution.

Great efforts have been made to understand the dynamics of low energy strong interactions, in order to calculate theoretically the decay amplitudes and the CP conserving FSI phases for $B \to PP$ decays, such as the factorization approximation with improvements from QCD corrections [22]. Yet we are still far away from being able to predict with a high confidence level the amplitudes. Nevertheless, factorization calculations may provide some idea about the order of magnitude. We have numerically studied the predictions of the factorization approximation for the size of the SU(3) invariant amplitudes. We found that the size of the hadronic amplitudes, listed in Tables III and VI, are on the same order of magnitude as those from factorization calculations [7], but the FSI phases, which can not be reliably calculated in the factorization approximation, can be very different and large. We also found that the annihilation contributions are generally small, as can be seen from Table IX. But one can not rule out possible significant effects on some decays, such as

TABLE XI: The prediction of the CP asymmetry with annihilation terms and γ=59°

Asymmetry	SU(3) Exact		SU(3) break	
	Central value	Error (Max,Min)	Central value	Error (Max,Min)
$B_u \to \pi^-\pi^0$	0	(0 , 0)	0	(0 , 0)
$B_d \to \pi^+\pi^-$	0.45	(0.64 , 0.28)	0.43	(0.60 , 0.28)
$B_u \to \pi^-\bar{K}^0$	0.01	(0.05 , -0.04)	0	(0.02 , -0.03)
$B_u \to \pi^0 K^-$	-0.08	(-0.02 , -0.16)	-0.07	(-0.01 , -0.15)
$B_d \to \pi^+ K^-$	-0.09	(-0.06 , -0.13)	-0.10	(-0.07 , -0.14)
$B_u \to K^- K^0$	-0.22	(0.78 , -0.85)	-0.05	(0.71 , -0.78)
$B_d \to \pi^0\pi^0$	0.16	(0.58 , -0.23)	0.12	(0.48 , -0.67)
$B_d \to K^- K^+$	0.64	(1.00 , -1.00)	0.99	(1.00 , -0.14)
$B_d \to \bar{K}^0 K^0$	0.95	(1.00 , -0.06)	0.72	(0.92 , -0.29)
$B_d \to \pi^0\bar{K}^0$	0.01	(0.11 , -0.08)	-0.01	(0.09 , -0.09)
$B_s \to K^+\pi^-$	0.43	(0.70 , 0.33)	0.31	(0.47 , 0.23)
$B_s \to K^0\pi^0$	-0.05	(0.44 , -0.65)	0.04	(0.34 , -0.14)
$B_s \to \pi^+\pi^-$	-0.03	(0.08 , -0.14)	-0.07	(0.07 , -0.15)
$B_s \to \pi^0\pi^0$	-0.03	(0.08 , -0.14)	-0.07	(0.07 , -0.15)
$B_s \to K^+K^-$	-0.07	(-0.03 , -0.24)	-0.10	(-0.05 , -0.18)
$B_s \to K^0\bar{K}^0$	-0.05	(0.06 , -0.23)	-0.03	(0.04 , -0.13)

$B_s \to K^+K^-, K^0\bar{K}^0$. We emphasize that, at present, due to large errors in the data, the allowed ranges for the annihilation contributions can vary. It can, however, be expected that in the future the error bars will be reduced significantly. The annihilation amplitudes obtained this way will provide good guidance for theoretical studies, which presently have difficulties in obtaining reliable results.

We attempted to study SU(3) breaking effects in the $B \to PP$ decays, by assuming a simple pattern for the breaking effects. We found that, although the general features are not changed very much, in certain decays the effects can be large, such as the branching ratios for $B_s \to K^+K^-, K^0\bar{K}^0$. Therefore these modes can be good modes in which to study SU(3) breaking effects.

We predicted branching ratios for several $B_s \to PP$ decays. These decay branching ratios can be measured at future hadron colliders. The SM and SU(3) flavor symmetry can be tested.

At present, CP violating rate asymmetries in $B \to PP$ have not been measured. The use of SU(3) flavor symmetry can also provide important information on CP violation in the Standard Model. Using the best fit values for the hadronic parameters, we also obtained CP violating rate asymmetries for the $B \to PP$ decays. We found that some of the asymmetries can be large and within the reach of the B factories. CP asymmetry in $B_d \to \pi^+\pi^-$ can be as large as 30% and even larger for $B_d \to KK$. It can be expected that, with more accurate experimental measurements, the study of CP violating rate asymmetries can provide crucial information about the dynamics for B decays in the Standard Model.

Before closing we would like to make the following remark on the method proposed in this paper. In order to determine the relevant hadronic parameters, and therefore to predict the branching ratios and CP violating asymmetries, we have carried out χ^2 analyses using the available experimental data. We have considered six different cases and obtained the best fit values of the hadronic parameters in Tables III, VI, and IX. The χ^2 per degrees of freedom are: 0.77 (exact SU(3)) and 0.99 (broken SU(3)) in Table III, 1.23 (exact SU(3)) and 1.41 (broken SU(3)) in Table VI, and 1.73 (exact SU(3)) and 1.39 (broken SU(3)) in Table IX. These numbers indicate that the fitting is reasonable. There is no conflict between the SM and the experimental data. We therefore expect that the method described in this paper can provide good estimates of $B \to PP$ branching ratios and CP violating asymmetries. Future improved data, with more detailed error correlations, will also provide more information about annihilation contributions and SU(3) breaking effects.

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