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Magnetization Relaxation and Magnetization Dynamics

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A phenomenological description of magnetization relaxation and dynamics is briefly reviewed and discussed. Inconsistency in spin wave damping calculation by means of the phenomenological approach in comparison with one using a microscopic approach leads naturally to a new form of the magnetization relaxation in which the modulus of magnetization is not conserved. Consideration of the nonconservation of magnetization modulus results in a new relaxational magnetization dynamic equation. Application of this new dynamic equation enables us to give a sartisfactory explanation why the ferromagnetic relaxation in domain is always at variance with that in the domain wall.

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I. Introduction

Marcroscopic description of magnetization dynamics is generally given in terms of $\dot{\mathrm{M}} = -\gamma[\mathrm{M} \times \mathrm{F}]$ where $\gamma = g\mu_B/\hbar$, $\mathbf{F}$ is the effective field $\mathbf{F}(\mathbf{r},\mathbf{t}) = -\delta\,\omega/\delta$ M and ω the energy density of the system concerned. At a temperature far below critical one, the relativistic interaction is weak and the modulus of magnetization remains effectively constant. We have $\mathrm{M}^2 = |M_S^2| =$ const. For a system with energy density ω:

$$\omega = \frac{1}{2}\alpha_{ij}\frac{\partial M}{\partial x_i}\frac{\partial M}{\partial x_j} + f(\mathrm{M}^2) - \frac{1}{2}\beta(\mathrm{M}\dot{\mathrm{n}}_A)^2 - \mathrm{M}\cdot\mathrm{H}_{ext} \tag{1}$$

where α_{ij} is the inhomogeneous exchange interaction constant, $f(\mathrm{M}^2)$ is the homogeneous part of the exchange interaction. Note that $f(\mathrm{M}^2)$ term is strictly a constant if the modulus of magnetization M is unchanged, and that $f(\mathrm{M}^2)$ may not contribute to spin wave spectrum even if it is not a constant.

Crystalline symmetry and wave vector dependence of the spin wave relaxation is closely related. Two types of homogeneous ground state of Eq. (1) are known if $\mathbf{H}_{ext} = 0$. They are of the "easy-axis"type if $\beta > 0$ in which the magnetization vector is either parallel or antiparallel to the anisotropy axis; and the "easy-plane" type if $\beta < 0$ in which the magnetization vector is normal to the anisotropy axis.

Spin wave in long wave length limit exhibits the following characteristics depending upon crystalline symmetry. For an isotropic ferromagnet, the dispersion relation is $\omega_{sw}(k) =$

Dk^2 where $D = \gamma M_S \alpha, \alpha_{ij} = \alpha\delta_{ij}$. For an easy-axis ferromagnet with $\mathbf{H}_{ext} \| \mathrm{n}_A$, along the easy direction, we have $\omega_{sw}(k) = \gamma(H_{ext} + H_A) + Dk^2$ where $H_A = \beta M_S$. For an easy-plane ferromagnet with $\mathbf{H}_{ext} \perp \mathrm{n}_A$, normal to the easy direction, the dispersion relation is $\omega_{sw}(k) = \{[Dk^2 + \gamma H_{ext}][Dk^2 + \gamma(H_{ext} + H_A)]\}^{1/2}$. In the absence of magnetic field and in long wave length limit, we have $\omega_{sw}(k) = C_{SW}\ k$ where $C_{SW} = \sqrt{\gamma D H_A}$ is the "spin wave velocity". Note that the appearance of minimum activation energy for spin wave in the above expressions is due to breaking of symmetry by the introduction of H_{ext} to the Hamiltonian of the system.

II. Magnetization relaxation

The phenomenological description of magnetization relaxation was first introduced by Landau and Lifshitz in 1935 by adding a term R to the magnetization dynamic equation [1] :

$$\dot{\mathbf{M}} = -\gamma\,[\mathbf{M} \times \mathbf{F}] + \mathbf{R}. \tag{2}$$

In the language of Landau-Lifshitz, the equilibrium value of **M(r,t)** (the ground state) can be approached by means of relativistic interactions only. The relaxation term $\mathbf{R}_{LL}$ has the form,

$$\mathbf{R}_{LL} = -\gamma\lambda_L M_S^{-1}[\mathbf{M} \times [\mathbf{M} \times \mathbf{F}]] \tag{3}$$

where $\mathrm{M}^2 = M_S^2$=constant and λ_L is assumed to be small in accordance with the fact that the relativistic interaction is weak.

Gilbert [2] on the other hand intoduced a slightly different relaxation form,

$$\dot{\mathbf{M}} = -\gamma^*\,[\mathbf{M} \times \mathbf{F}] + \lambda_G M_S^{-1}[\mathbf{M} \times \dot{\mathbf{M}}] \tag{4}$$

where $\lambda_G = \lambda_L, \gamma^* = \gamma(1 + \lambda_G^2)$ and the dissipative force is proportional to M and opposite in direction to that of the effective field F. Various form of relaxation term has been reported in literature.

II- 1. Wangsness approach

Over the years, the structure of the relaxation term went through various stage of evolution.Wangsness [3], for example, described the marcroscopic dissipation function in the form

$$\mathbf{Q} = \frac{1}{2}\sum_{ij} \mathcal{L}_{ij} X_i X_j \tag{5}$$

where $\mathcal{L}_{ij}$ is the Onsager's coefficient, X_i is a force field corresponding to a time rate change of the parameters which describes the state of the system. Marcroscopic consideration of Wangsness formulation leads directly to the L-L relaxation but, microscopically, it is not valid for ferromagnet, though is acceptable for paramagnet. The symmetric and antisymmetric parts of the $\mathcal{L}_{ij}$ tensor describes respectively the relaxation and dynamic part of the magnetization dynamic equation.

The symmetric Onsager's coefficients have an axial symmetry, $\mathcal{L}_{ij}=\mathcal{L}_i\delta_{ij},(\mathcal{L}_1=\mathcal{L}_2=\mathcal{L}_\perp$ and $\mathcal{L}_3=\mathcal{L}_{\|})$ which, for an isotropic ferromagnet placed in a constant magnetic field, is exactly the same as in the case of uniaxial ferromagnet.

II-2. Bar'yakhtar's approach

During the last few decades, a commendable progress has been achieved by the introduction of a new relaxation term $\mathbf{R}_{ABP}$[4] to Eq. (2) based on two assumptions. First, the relaxation term is perpendicular to the dynamic part of the equation, that is, $\mathbf{R}_{ABP}\perp[\mathrm{M}\times\mathrm{F}]$. Second, the relaxation term is a linear function of the effective force field F, $(R_{ABP})=a_{ij}F_j$. The tensor a_{ij} may be expressed as $a_{ij}=\frac{1}{\tau_2}\delta_{ij}-\frac{1}{\tau_1}(n_in_j-\delta_{ij})$ where $\mathrm{n}=\mathrm{M}\cdot M_S^{-1}$. In the L-L equation we have

$$\mathbf{R}_{ABP}=\frac{1}{\tau_2}\mathbf{F}-\frac{1}{\tau_1}[\mathrm{n}\times[\mathrm{n}\times\mathbf{F}]] \tag{6}$$

where τ_1 is the relaxation time of the perpendicular component of the resultant magnetization, and τ_2 is the corresponding time for the magnetization to relax to its equilibrium value $\mathrm{M}_S(\mathrm{T})$ at a given temperature.

11-3. **Callen's** approach

By assuming that $\dot{\mathbf{M}}$ can be expanded in terms of M, [M x F] and [M x [M x F]] the relaxation equation of magnetization can be readily expressed as [5]

$$\dot{\mathbf{M}}=\lambda_1\mathbf{M}-\lambda_2[\mathbf{M}\times\mathrm{F}]-\lambda_3[M\times[\mathrm{M}\times\mathbf{F}]] \tag{7}$$

where λ_1,λ_2, and λ_3 are the unknown scalar functions of M and F and are the intrinsic characteristics of the ferromagnet.

The above equation reduces readily to the LL equation if $\lambda_1=0$, that is, if the rate change of the M is purely rotational. Consequently, the form of the relaxation equation is similar to that of the Wangsness and Bar'yakhtar.

Summarizing, one notices an important common feature of the equations. The modulus of the magnetic moment in these equations are not conserved while it is so for the LL or the LLG equation.

III. Spin wave damping

Inconsistency in the spin wave damping calculation using either microscopic or hydrodynamic theory in comparison with a phenomenological model using the LLG equation in isotropic ferromagnets is obvious. For example, spin wave calculation in an exchange approximation in isotropic ferromagnet reveals that all the phenomenological parameters of the relativistic nature in the LLG equation has to be put to zero. That is to say, the relaxation term in the LLG equation has to be dropped and spin wave spectrum in this model becomes damping free. On the other hand, spin wave damping calculation using microscopic [6] and hydrodynamic theory [7] both yield results which is directly in contradiction with the one obtained by using the traditional LLG equation. A microscopic calculation result shows that there is a damping, giving $\gamma(k)\propto k^4$ in the long wave length limit, For an easy-plane ferromagnet, a hydrodynamic theory of spin wave result indicates that spin wave damping is non-zero, giving $\gamma(k) \propto \omega_{sw}(k)^2 = C_{SW}^2 k^2$. On the other hand, the damping obtained by using the LLG equation is

$$\gamma_{SW}(k) = \frac{\lambda_G}{2}\gamma H_A + \lambda_G(\gamma H + Dk^2) \tag{8}$$

which is finite in the long wave length limit at H=O.

Consideration of magnon interaction with one another and with other elementary excitations, and above all, with exchange interaction, leads to dispersion in damping $\gamma_{SW}(k)$. Spatial dispersion dependence of relaxation generated by the exchange interaction was first discussed Bar'yakhtar [8] based on Onsager's kinetic equation.

Let $\partial \mathbf{M}/\partial t$ be the generalized flux, F the generalized force, the correct relaxation term may be expressed as [8],

$$\mathbf{R}_i^e = -\gamma\lambda_{kl}^e M_S \frac{\partial^2 \mathbf{F}_i}{\partial x_k \partial x_l} \tag{9}$$

where λ_{kl}^e is the phenomenological relaxation constants of the exchange contribution.

For an isotropic ferromagnet? the new relaxational dynamic equation has the form

$$\dot{\mathbf{M}} = -\gamma\,[\mathbf{M} \times \mathbf{F}] - \gamma M_S \lambda_e \nabla^2 \mathbf{F} \tag{10}$$

where $\lambda_{kl}^e = \lambda_e \delta_{kl}$. The spin wave damping is

$$\gamma(k) \propto \omega_{sw}(k)^2 \lambda_e k^2 = D\lambda_e k^4 \tag{11}$$

which corresponds well with the previous microscopic and hydrodynamic theory of spin wave calculations as remarked above.

Symmetry consideration for an easy-plane ferromagnet reveals that it possesses an axial symmetry with respect to the anisotropy axis $\mathbf{n}_A$ and behaves like a uniaxial ferromagnet. The relaxation term in the LLG equation on the other hand fails to exhibit any characteristics of this nature.

For an uniaxial ferromagnet, the relaxation term has the form $R_i = \lambda_{ik} F_k$ where λ_{ik} is dictated by crystal symmetry. Thus, the full relaxation term can be expressed as [9]

$$R_i = \gamma M_S \lambda_r (\delta_{ik} - n_A^i n_A^k) F_k \tag{12}$$

IV. New relaxational dynamic equation

A generalized magnetization dynamics equation taking into account crystalline symmetry and spatial dispersion of relaxation caused by exchange interaction for the case of a uniaxial ferromagnet can be expressed as [9]

$$\dot{\mathbf{M}} = -\gamma\,[\mathbf{M} \times \mathbf{F}] + \gamma M \lambda_r [\mathbf{F} - \mathbf{n}(\mathbf{n} \cdot \mathbf{F})] - \gamma M \lambda_e \nabla^2 \mathbf{F}, \tag{13}$$

where $\lambda_r = \lambda_L$ (the relaxation due to relativistic interaction), λ_e is a phenomenological constant describing spatial dispersion of relaxation caused by exchange interaction, n is the unit vector along the anisotropy axis. Major differences between this and the LLG Eq., in addition to introducing a new X,-term, is that the modulus of magnetization in Eq. (13) is no longer conserved while it remains so in the LLG Eq.

Expressing the magnetization vector in terms of its magnitude and unit vector in the form, M = Mm, Eq. (13) can be rewritten as follows

$$\begin{aligned}\gamma^{-1}\cdot\mathbf{m} = &- [\mathbf{m} \times \mathbf{F}] + \lambda_r [\mathbf{m} \times [\mathbf{F} \times \mathbf{m}]] \\ &+\lambda_r (\mathbf{n}\cdot\mathbf{F})[\mathbf{m}\times[\mathbf{m}\times\mathbf{n}]] \\ &+\lambda_e [\mathbf{m}\times[\mathbf{m}\times\nabla^2\mathbf{F}]] ,\end{aligned} \tag{14}$$

$$\gamma^{-1}\dot{M} = \lambda_r M (\mathbf{m}\cdot[\mathbf{n} \times [\mathbf{n} \times \mathbf{F}]]) - M\lambda_e\left(\mathbf{m}\cdot\nabla^2\mathbf{F}\right). \tag{15}$$

Equations (14) and (15) can be used to describe damping related to the domain wall motion and ferromagnetic resonance (FMR) 1'inewidth in the presence of a drive and transverse field [10].

V. Domain wall dynamics

Energy of a planar domain wall energy in the presence of a constant external magnetic field normal to the anisotropy axis may be expressed as

$$\begin{aligned}w\{\mathbf{m}\} = 2\pi Q\{&\Delta_B^2[\vartheta'^2 + \sin^2\vartheta\varphi'^2] + \varepsilon(\sin\vartheta \sin\varphi \\ &+\sin^2\vartheta - \sin\vartheta_0 \sin\psi_H)^2 - 2h_{||}\cos\vartheta - \\ &2h_t \sin\vartheta\cos(\varphi-\psi_H)\} - w_D\{\vartheta_0,\psi_H\},\end{aligned} \tag{16}$$

where h_t and $h_{||}$ are the transverse and drive field normalized by $H_K = 2K/M$, and $\mathbf{H}_t = H_t\{\cos\psi_H, \sin\psi_H, 0\}$, and ϑ_0 and $\varphi = \psi_H$ are the polar and azimuthal magnetization angles inside domains. The polar angle ϑ_0 is determined by the relation

$$\sin\vartheta_0 \cos\vartheta_0 - h_t \cos\vartheta_0 - h_{||}\sin\vartheta_0 = 0, \tag{17}$$

and $w_D\{\vartheta_0,\psi_H\}$ is energy density inside domains.

Derivation of the Slonczewski's type equations [11] can be obtained by carrying out the integration of $\vartheta(y,t)$ and $\varphi(t)$ over the domain wall proper, taking into account the dependency of changes of the modulus of magnetization $\mu = M - M_s$ upon $\vartheta(y,t)$ and $\varphi(t)$. The structure of the moving domain wall is assumed to be unchanged from its static shape given in Ref. [12], that is,

$$\begin{aligned}&\sin\vartheta = \sin\vartheta_0 + \frac{\cos^2\vartheta_0}{\cosh u + \sin\vartheta_0} \quad u = \frac{y - q(t)}{\mathrm{A}}, \\ &\varphi(t) = \psi_H + \psi(t)\left\{U\left(u + \frac{\pi}{2}\right) - U\left(u - \frac{\pi}{2}\right)\right\},\end{aligned} \tag{18}$$

where $q(t)$ is the coordinate of the domain wall center; Δ is the domain wall width determined by the minimum energy condition of the domain wall,

$$\Delta^2 = \Delta_B^2 r(\vartheta_0) a^{-2}(\vartheta_0, \psi, \psi_H), \tag{19}$$

$a(\vartheta_0, \psi, \psi_H)$ is the domain wall structural factor,

$$\begin{aligned} & a^2(\vartheta_0, \psi, \psi_H) \\ = \; & f(\vartheta_0)[1 + \varepsilon \sin^2(\psi + \psi_H)] \\ & + \frac{\pi}{2} \tan \vartheta_0 [2h_t + \varepsilon \sin \vartheta_0 \sin^2 \psi_H - \sin \vartheta_0] \\ & - 2g(\vartheta_0)[h_t \cos \psi + \varepsilon \sin \vartheta_0 \sin^2 \psi_H \sin(\psi + \psi_H)] \end{aligned} \tag{20}$$

where the functions $f(\vartheta_0)$, $g(\vartheta_0)$, and $r(\vartheta_0)$ are determined by the polar angle $\psi(0) = 0$; $U(u)$ is a symmetric step function, $U(u) = 1$ when $u > 0$; $U(u) = 1/2$ when $u = 0$ and $U(u) = 0$ when $u < 0$.

The Slonczewski's type equations for the case of the transverse field parallel to the domain wall plane $\psi_H = 0$ can now be written as

$$\Delta \dot{\psi} + \lambda_r \dot{q} R = \omega_A \Delta h_{\|},$$

$$\dot{q} - \Delta \dot{\psi} \lambda_r \frac{4}{v_*^2} R_2 = [f(\vartheta_0) \sin \Psi \cos \Psi \tag{21}$$

$$+ g(\vartheta_0) h_t \sin \Psi] \tag{22}$$

where

$$R \equiv R(\vartheta_0, \psi) = r(\vartheta_0) + \frac{4}{v_*^2} R_1(\vartheta_0, \psi), \quad v_* = \lambda_r/(4\pi \chi_{\|} Q) \tag{23}$$

where $\chi_{\|}$ is the longitudinal susceptibility and

$$\begin{aligned} R_1(\vartheta_0, \psi) &= \frac{1}{2 \cos \vartheta_0} \int du \Psi_M^2 \sin^2 \vartheta, \\ R_2(\vartheta_0, \psi) &= \frac{1}{2 \cos \vartheta_0} \int du \Phi_M^2 \sin^2 \vartheta. \end{aligned} \tag{24}$$

and $\Psi_M(u; \vartheta_0, \psi)$ and $\Phi_M(u; \vartheta_0, \psi)$ are the appropriate functions defined elsewhere [13,14] and expression for $a^2(\vartheta_0, \psi)$ is obtained from (20) at $\psi_H = 0$. From Eqs. (21) and (22) one obtains

$$V(H_{\|}, H_t) = \frac{\gamma \Delta_B H_{\|} \sqrt{r(\vartheta_0)}}{a(\vartheta_0, \psi) \lambda_r R(\vartheta_0, \psi)}. \tag{25}$$

describing the dependency of the steady state domain wall velocity upon the drive and transverse fields. The corresponding expressions derived from the Slonczewski-like equations based on the LLG equation (2) has the following form

$$\bar{V}(H_{\|}, H_t) = \frac{\gamma \Delta_B H_{\|}}{a(\vartheta_0, \psi)\lambda_r \sqrt{r(\vartheta_0)}}. \tag{26}$$

Comparison of the two velocity expressions shows clearly that the contribution of the domain wall drag caused by relaxational dynamics of the magnetization modulus leads to the qualitative changes in the the dependency of the domain wall velocity upon drive field.

Linear mobility of the domain wall is one of the most important parameters in characterizations of magnetic thin films [11, 13]. Linear domain wall mobility is determined by the expression

$$\begin{aligned}
\mu_{DW}(H_t) &= \lim_{H_{\|}\to 0} \frac{dV(H_{\|}, H_t)}{dH_{\|}} \\
&= \begin{cases} \dfrac{\frac{\gamma\Delta_B}{\lambda_{mob}}[3v_*^2+16]}{3\cos\vartheta_0\left[v_*^2 r(\vartheta_0)+4R_{10}(\vartheta_0)\right]}, & \text{from (25)} \\[2ex] \dfrac{\gamma\Delta_B}{\lambda_r}\dfrac{1}{\cos\vartheta_0 r(\vartheta_0)}, & \text{from (26)} \end{cases}
\end{aligned} \tag{27}$$

where

$$\lambda_{mob} = \lambda_r + \frac{16(4\pi Q \chi_{\|})^2}{3\lambda_r} \tag{28}$$

$$\begin{aligned}
R_{10}(\vartheta_0) = \frac{4}{3\cos\vartheta_0} - \frac{\sin\vartheta_0}{\cos^2\vartheta_0}\Big[&\frac{34}{3}\sin\vartheta_0\cos\vartheta_0 + \\
&(\pi - 2\vartheta_0)(2+\cos 2\vartheta_0) + 12\sin\vartheta_0 \ln\left|\tan\frac{\vartheta_0}{2}\right|\Big]
\end{aligned} \tag{29}$$

The above equation describes an important relation between the relaxation constant λ_r in domains (related to ferromagmnetic resonance width) and that in the domain wall, λ_{mob}. Based on the equation one can have for the first time in several decades a clear interpretation why the damping constant related domain wall mobility is always at variance, and frequently one to two order of magnitude larger than that obtained by ferromagnetic resonance. [10, 13, 14].

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