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On Understanding QCD Condensates from Hadron Spectroscopy

W-Y. P. Hwang[1], A. N. Mitra[1,2], and Kwei-Chou Yang[1]

[1] *Department of Physics, National Taiwan University, Taipei, Taiwan 107, R.O.C.*

[2] *Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India*

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We investigate in a specific and systematic manner the possibility of understanding some of the principal QCD condensates $\langle \bar{q}\mathcal{O}q\rangle$, which are traditionally associated with QCD sum rules, directly in terms of their definition, viz., $-\int d^4p\mathrm{Tr}\{\tilde{S}_F^A(p)\mathcal{O}\}$ where the quark propagator $\tilde{S}_F^A(p)$ is so defined that the perturbative part is suitably subtracted. To this end, we relate the mass function $m(p^2)$ to the pion-quark vertex function in the chiral limit. This last aspect provides a concrete handle for its determination through the vehicle of the Bethe-Salpeter equation (BSE) for $q\bar{q}$ hadrons. Since the latter is directly adaptable to spectroscopic studies, the method provides a clear linkage between the high-energy and low-energy descriptions of hadrons in QCD. The gluon condensate which is related to the same $q\bar{q}$ interaction in the confining region (the infrared domain of the gluon propagator) may also be calculated in a similar fashion. The results for most condensates are in good overlap with the values employed in the method of QCD sum rules.

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I. STATEMENT OF THE PROBLEM

It is believed that quantum chromodynamics (QCD), an $SU(3)$ gauge theory constructed out of the *exact* color $SU(3)$ symmetry, describes strong interactions among quarks, antiquarks, and gluons. At high energies (i.e., large momentum transfer squared, $Q^2 >> 1$ GeV^2), QCD is asymptotically free, allowing perturbative treatment of physical processes involving hadrons. At low energies (i.e. $Q^2 \sim 1$ GeV^2), however, the nonperturbative physics dominates such that the physical ground state (i.e., the physical vacuum) differs in general from the trivial ground state (i.e., the trivial vacuum in which all field variables vanish identically). Indeed, hadrons, including baryons, mesons, and glueballs all of which are color-singlet objects consisting of quarks, antiquarks, and gluons, act as the effective degrees of freedom in hadron physics (strong interaction physics).

The study of hadron physics is essential for progresses in many research areas, ranging from nuclear physics, to particle physics, to collapse of heavy stars, or even to the hadron era or nucleo-synthesis in the early universe. Our understanding of the structure of a nucleon, a proton or neutron, is still not very adequate as compared to the structure of a hydrogen atom in atomic physics (where the nucleus can be treated to a very good approximation as a pointlike object). Yet, the behavior of a nucleon may vary from one environment to another. The diverse nonperturbative aspects of QCD have by and large defied many attempts to find universally acceptable solutions to strong interaction problems, including the problem of the nucleon structure. The QCD condensates are among the most fundamental parameters of this strong-interaction theory designed to bridge the huge gap between its perturbative and non-perturbative regimes through the powerful analytical tool of the operator product expansions (OPE) [1,2]. The method of QCD sum rules represented the first practical attempt [3] in this direction by employing a duality principle between the quark-gluon language and the meson-baryon picture.

To recapitulate, the method of QCD sum rules is based on the idea of finding a Q^2 region (≈ 1 GeV^2) where one may incorporate nonperturbative physics, through Wilson's short-distance operator product expansion, into the perturbative QCD treatment of physical processes involving hadrons. The ansatz is, on the one hand, to replace, in the evaluation of a certain correlation function (or Green's function) $\Pi(p)$, the free quark (or gluon) propagator by the one suitable in the case of the nontrivial vacuum while, on the other hand, to express, via dispersion relations, the same correlation function in terms of the variables in the meson-baryon picture. The result at the quark level is then equated with that obtained in the meson-baryon picture, yielding sum rules which allow for determination of the variables adopted in the meson-baryon picture.

At the hands of the Russian School [4,5], the method was considerably developed and applied to a large class of observable (hadronic) amplitudes with great success, and is now regarded as one of the most efficient methods available for strong interaction theory, whose inputs are the condensates themselves. In the language of field theory, the quark condensates are formally defined as

$$\begin{aligned} \langle \bar{q}\mathcal{O}_i q \rangle &\equiv \sum_{a,j} \langle 0 \mid : \bar{q}^a_j(0)\mathcal{O}_i q^a_j(0) : \mid 0 \rangle \\ &= -\int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\{\tilde{S}^A_F(p)\mathcal{O}_i\}, \end{aligned} \tag{1}$$

where $\mathcal{O}_i$ is an operator representing the nature of condensate, the index A represents the effect of a background field, and $\tilde{S}^A_F(p)$ is the quark propagator with the perturbative part suitably subtracted.

At this stage, it is useful to distinguish between the gluonic background field and other external ones such as electromagnetic, axial, etc. The latter can be treated perturbatively, but the former, with its characteristic problem of color gauge invariance, must be addressed in a more substantial manner. While it is not the purpose of this paper to go into the technical details of this subject (on which there already exists a vast literature [6]), it is nevertheless possible to incorporate in practice a major fraction of this effect through the simple expedient of changing the variable of integration in Eq. (1) from p_μ to II, $= p_\mu - \frac{1}{2}g_c\lambda^a G^a_\mu$ with G^a_μ the gluon field. [In an abelian gauge theory such as QED, the procedure has been referred to as the principle of "minimum substitution" – a substitution principle which has been used for amplitudes at different levels and which generates proper gauge invariant results.] This "minimum-substitution" procedure would in general not be possible if one were to evaluate more complicated integrals involving more propagators and vertex functions, but since the integral in Eq. (1) "sees" only one such quantity, the substitution should give rise to proper color-gauge invariant results, especially because we are primarily interested in a constant background $G_{\mu\nu}$–field, i.e. $G^a_\mu(x) = -\frac{1}{2}x^\mu G^a_{\mu\nu}$. Note that this is basically a non-abelian adaptation of the celebrated Schwinger method [7] to the present situation but the details of the available methods [6] are not necessary for justifying the above step. With this understanding, we shall not use any additional subscript or superscript in Eq. **(1)** to specify the gluonic background, but rather regard the integration variable p_μ to represent II, $= p_\mu - \frac{1}{2}g_c\lambda^a G^a_\mu$.

Clearly there are a large variety of condensates depending on the choice of the operator $\mathcal{O}_i$ and/or the external field **A,,** the principal one being the main quark condensate $\langle\bar{q}q\rangle_0$ (i.e. with $\mathcal{O}_i$ = 1 in the absence of an external field). The corresponding gluon condensate may be formally defined in coordinate space as

$$\langle g_c^2 G^2\rangle_0 = \mathrm{Tr}\{(\nabla_\mu\nabla_\nu - \delta_{\mu\nu}\nabla^2)g_c^2 D_{\mu\nu}(0)\}\,, \tag{2}$$

where ∇_μ is the gauge covariant derivative and $D_{\mu\nu}(x)$ is the infrared (non-perturbative) part of the gluon propagator. In principle, this quantity can be accessed from the infrared part of the $q\bar{q}$ potential, though this requires some careful treatment of its heavy flavor content since an extrapolation to the small x regime is involved – but see Sec. II for more discussions together with an explicit calculation.

Now if the strong interaction problem of QCD were fully soluble, the condensates in Eqs. (1) and (2) would also be calculable to any desired accuracy and therefore devoid of any special significance. In the absence of such a facility, however, these quantities (which are often regarded as free parameters in the QCD sum rule treatments) are the only concrete handles available for a 'glimpse' into the (low energy) non-perturbative domain of QCD from its (high energy) perturbative end. On the other hand, except for the two principal condensates $\langle\bar{q}q\rangle_0$ and $\langle g_c^2 G^2\rangle_0$, the self-consistency of whose determination is fairly well established through cross checks against a large variety of data, the same cannot be said of the higher order ones whose determinations often leave much ambiguities. A partial list is [4]

$$\langle \bar{q} i \gamma_\mu \gamma_5 q \rangle_A, \quad \langle \bar{q} \sigma_{\mu\nu} q \rangle_F, \quad \langle \bar{q} \frac{1}{2} \lambda \, \sigma \cdot G q \rangle_0, \quad \langle \bar{q} \frac{1}{2} \lambda G_{\mu\nu} q \rangle_F . \tag{3}$$

In the method of QCD sum rules [3-5], there is no intrinsic mechanism to evaluate them from first principles but only an extrinsic 'matching' between the two sides of the duality relation with the help of suitable parameters. And for condensates of still higher dimensions, additional assumptions, such as factorization, are also introduced. Clearly, additional principles (not just ad hoc parameters) are needed to give more teeth to the method of QCD sum rules.

In this paper, we propose a new principle which enables us to calculate all the condensates (1)-(3) and many more, through the linkage to the *spectroscopy sector* which is perhaps the weakest link in the method of QCD sum rules, except for the ground states of the principal hadrons [3,4]. Of course, the spectroscopy sector has its own language showing up through the $q\bar{q}$ potential, or in a better term the kernel of the Bethe-Salpeter equation (BSE), within a Bethe-Salpeter equation and Schwinger-Dyson equation (BSE-cum-SDE) framework which may be regarded as a description complementary to the method of QCD sum rules, and it should be of sufficient physical interest to show its links with the vacuum amplitudes Eqs. (1) and (2).

At this juncture, it is essential to reflect upon the paradoxical situation which we are facing. All the condensate parameters, as employed in the method of QCD sum rules, are characteristics of the nonperturbative aspect of QCD and are therefore not calculable in any perturbative manner. On the other hand, the spectroscopy sector via, e.g., the BSE-cum-SDE framework must, to some extent, incorporate nonperturbative physics properly, especially if it yields successful spectroscopic predictions. Unless the condensate parameters are determined via lattice simulations of QCD, therefore, we should try our best to exploit any possible connection between the QCD sum rule method and the spectroscopy. Indeed the connection between the two methods comes about from the naturalness with which the condensates can be calculated from the BSE-cum-SDE premises of field theory which maintains the links with spectroscopy. A partial attempt was recently made in this direction [8], using the chiral property of the effective $q\bar{q}$ interaction kernel (vector-like) which provides, among other things, a concrete structure for the quark mass function, the "key ingredient" of the SF-function in Eq. (1), through its complete equivalence with the pion-quark vertex function in the chiral limit ($M_\pi \to 0$) brought about via the Ward-Takahashi identity for axial currents [9].

Specifically we shall make a systematic attempt to calculate several quark condensates in Eq. (3), as well as the gluon condensate. The central ingredient for the former is the quark mass function deduced as the chiral limit of the pion-quark vertex function [8], and that for the latter is the kernel of the corresponding BSE which is vector-like and hence serves as the effective gluonic propagator in the infrared regime. Its 3D support also provides an exact interconnection [10] between the 4D and 3D forms of the BSE, the latter being the right vehicle for making contact with spectroscopy [11]. Its agreement with the data for both $q\bar{q}$ [12] and qqq [13] hadrons with just three basic constants provides the rationale for this exercise. In Sec. II we shall collect the main ingredients of the BSE-formalism and also work out the gluon condensate which stands out from the other (quark) condensates in terms of its mode of evaluation. In Sec. III we first develop a systematic method for the evaluation of the quark condensates and then use it for determining the various quark condensates listed in Eqs. (1) and (3). In Sec. IV we discuss the significance of our results *vis-a-vis* their QCD sum rule counterparts.

II. FORMALISM

As noted previously in Sec. I, the proposed procedure to determine the various condensates is related to the spectroscopic studies of hadrons through the Bethe-Salpeter equation (BSE) so that the method provides a linkage between the high and low energy descriptions of hadrons via QCD. To that end we recall a specific BSE treatment of the $q\bar{q}$ [12] and qqq [13] spectra within a unified framework based on the ansatz of covariant instantaneity [10] which can also be motivated on several other grounds [14], as summarized in a recent paper [15]. The covariant instantaneity ansatz (CIA) is expressed by the statement that the kernel $K(q,q')$ of the 4D BSE representing the internal dynamics of a $q\bar{q}$ system, where q and q' are the internal 4-momenta, has a 3D support [10],

$$K(q,q') \Rightarrow K(\hat{q},\hat{q}'); \qquad \hat{q}_\mu = q_\mu - \frac{q\cdot P}{P^2}P_\mu\,, \tag{4}$$

where P_μ is the total 4-momentum related to the individual momenta $(p_{1,2},p'_{1,2})$ for equal mass kinematics by

$$p^\mu_{1;2} = \frac{1}{2}P^\mu \pm q^\mu; \qquad p'^\mu_{1;2} = \frac{1}{2}P^\mu \pm q'^\mu\,. \tag{5}$$

Note that $\hat{q}\cdot P = 0$. The CIA provides an interconnection between the 4D and 3D forms of the BSE [10], and gives a concrete structure for the hadron-quark vertex function Γ_H in the form [10]

$$\Gamma_H(\hat{q}) = \hat{D}(\hat{q})\phi(\hat{q})\,, \tag{6}$$

$$\hat{D}(\hat{q}) = 4\hat{\omega}\left(\hat{\omega}^2 - \frac{1}{4}M^2\right); \qquad \hat{\omega}^2 = m_q^2 + \hat{q}^2\,, \tag{7}$$

where $\hat{D}(\hat{q})$ is a (covariantly expressed) 3D denominator function and $\phi(\hat{q})$ the corresponding wave function. These two quantities appear in the 3D BSE structure as $\hat{D}\phi = \int K\phi$ with K given by Eq. (4), so that a 3D Schroedinger-like equation (albeit fully covariant) which is appropriate for making contact with the hadronic spectra in view of their known 3D character [11]. The fact that the CIA treatment [10] of the spectroscopy yields both $q\bar{q}$ [12] and qqq [13] spectra in good agreement with the data [11], with a common set of (three) parameters, constitutes our assertion on the spectroscopic link of the SDE-BSE framework via the 3D CIA description. To see this link more quantitatively, we first recall the Schwinger-Dyson equation (SDE) for the mass operator C(p) and the Bethe-Salpeter equation (BSE) for the vertex function $\Gamma_H(\hat{q})$, especially the structure of such SDE-cum-BSE framework as adapted to the 3D CIA treatment:

$$\Sigma(p) = \frac{4i}{3}\int \frac{d^4k}{(2\pi)^4}\mathcal{D}_{\mu\nu}(k)\gamma_\mu S_F(p-k)\gamma_\nu \,, \tag{8}$$

$$\begin{aligned}\Gamma_H(\hat{q}) = \; & \frac{4i}{3}\int \frac{d^4q'}{(2\pi)^4}\mathcal{D}_{\mu\nu}(q-q')\gamma_\mu S_F\left(\frac{1}{2}P+q'\right)\Gamma_H(\hat{q}') \\ & \cdot S_F\left(-\frac{1}{2}P+q'\right)\gamma_\nu \,,\end{aligned} \tag{9}$$

where the hat notation signifies perpendicularity to the hadron momentum P_μ. At this stage, we shall simultaneously specialize to the pion in the chiral limit ($P_\mu \to 0$), for which it can easily be checked [8,9] that Eqs. (8) and (9) become identical. In this limit, the vertex function becomes γ_5 times a scalar $m(\hat{q}^2)$, while the mass operator is expressible in terms of the same quantity as:

$$\Sigma(p) + i\gamma\cdot p = \frac{1}{i}S_F^{-1}(p) = i\gamma\cdot p + m(\hat{p}^2)\,, \tag{10}$$

where we have adopted the Landau gauge which permits setting the overall normalization to unity, $A(p^2) = 1$ [16] (no renormalization of the vector current: or CVC), and also emphasized its dependence on $\hat{p}^2$, in the sense of Eq. (4).

It is clear that the days of the 'naive' constituent quark model are long over and that any serious investigation at the quark level today must work with standard, current, quarks of QCD. On the other hand, it is also true that a solution to QCD in the nonperturbative regime is not yet in sight. It is this fact that provides a sort of locus standin, hence legitimacy, however temporary, to *effective* QCD motivated approaches in which the starting Lagrangian is still defined in terms of current quarks (almost zero mass m_c) but the propagators of the latter in their nonperturbative form contain the dynamically generated mass function $m(\hat{p}^2)$. The mass function $m(\hat{p}^2)$, which is supposed to arise as the "dynamically broken" solution of the SDE corresponding to the original chirally symmetric QCD

Lagrangian with (almost zero mass) $(u$, d) quarks, is the key to a proper understanding of the constituent mass m_q via the Politzer relation [2] $m_q = m_c +$ m(0) which for u or d quarks amounts essentially to m_q = m(0). It is this interpretation that has been behind our approach so that the 'constituent' mass m_q for the u/d quarks $(m_c = 0)$ is automatically accounted for as the low momentum limit ($\hat{p}^2 = 0$) of the mass function $m(\hat{p}^2)$, without the need to employ, e.g., a constituent quark field of fixed mass m_q.

Since in the chiral limit $(M_\pi = 0)$ the BSE for the $q\bar{q}$ pion reduces exactly to the SDE [9,8] for a single quark, we are able to derive [8] the mass function in terms of pion's BS wave function in the chiral limit $M_\pi = 0$. Admittedly, the function $m(\hat{p}^2)$ does have an empirical content, but the empiricity lies in the infrared part of the gluon propagator and not in any other ad hoc definition of the quark field.

Analogously, the gluon propagator in the Landau gauge has the form

$$\mathcal{D}^{ab}_{\mu\nu}(k) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\mathcal{D}(\hat{k})\,, \tag{11}$$

where a, b are the color indices in the adjoint representation.

At this stage, we are in a position to explicitly state the connection between Eqs. (8)-(11) and our spectroscopy-oriented BSE formalism [11-13] under CIA [10], whose fermionic kernel $K(q,q')$ is expressible in terms of $\mathcal{D}_{\mu\nu}(k)$ as

$$K(q,q') \Leftrightarrow \gamma_\mu \mathcal{D}_{\mu\nu}(q-q')\gamma_\nu\,. \tag{12}$$

The scalar part $\mathcal{D}(\hat{k})$ in Eq. (11) in the infrared region may be identified with the confining part of the K-function as [8,17]:

$$\mathcal{D}(\hat{k}) = \frac{3}{4}(2\pi)^3\omega_0^2 2m_q\alpha_c(4m_q^2)\left[\frac{\nabla_{\hat{k}}^2}{\sqrt{1 - A_0 m_q^2\nabla_{\hat{k}}^2}} + \frac{C_0}{\omega_0^2}\right]\delta^3(\hat{k})\,. \tag{13}$$

Here we have employed the full $q-\bar{q}$ potential which fits the spectroscopy for all flavors (light and heavy) [17], but specialized to the equal mass (m_q) case. The constants C_0, ω_0, A_0 are given by [12,17]:

$$C_0 = 0.27, \qquad \omega_0 = 158\,\text{MeV}, \qquad A_0 = 0.0283, \tag{14}$$

while the ('constituent') values of m_q for different flavors are [12]:

$$m_{ud} = 265\text{ MeV}, \quad m_s = 415\text{ MeV}, \quad m_c = 1530\,\text{MeV}, \quad m_b = 4900\,\text{MeV}. \tag{15}$$

The QCD coupling constant α_c is given by [12,17]:

$$\alpha_c(Q^2) = \frac{4\pi}{11 - \frac{2}{3}N_f}\,\frac{1}{\ln\frac{Q^2}{\Lambda_c^2}}\,; \qquad \Lambda_c = 200\text{ MeV}\,. \tag{16}$$

The coordinate representation $\mathcal{D}(\hat{R})$ of the gluon propagator (13) is

$$\mathcal{D}(\hat{R}) = \frac{3}{4}\omega_0^2 \cdot 2m_q\alpha_c(Q^2)\left[\frac{C_0}{\omega_0^2} - \frac{\hat{R}^2}{\sqrt{1 + A_0 m_q^2 \hat{R}^2}}\right]. \tag{17}$$

Note the, in the $\hat{R} \to \infty$ limit, $\mathcal{D}(\hat{R})$ is linear in $\hat{R}$ as well as flavor independent (the m_q–factor cancels out), except for the $\alpha_c(Q^2)$ effect. Thus the structure (17), despite its empiricity, respects the standard QCD expectation, but only in the strict confining region. On the other hand, the smallness of A_0 (= 0.0283) ensures that for light flavors its structure is dominated by the harmonic form, which amounts to setting $A_0 = 0$. This is an excellent approximation for the pion-vertex function in the chiral limit ($M_\pi = 0$), and hence for the quark mass function $m(\hat{p}^2) = m(\hat{q}^2)$, which, according to Eqs. (6)-(7), has the form [18]

$$m(\hat{q}^2) = m_q^{-2}\hat{\omega}^3\phi(\hat{q}), \tag{18}$$

in this limit, $\phi(\hat{q})$ being the 3D wave function for the pion [10]. Here we have normalized the mass function to m(0) = m_q and identified this dynamical quantity as the constituent mass for the ud–quarks only (ignoring their small ‘current’ values). The 3D wave function $\phi(\hat{q})$ may be given by [8]

$$\phi(\hat{q}) = \exp\left(-\frac{1}{2}\hat{q}^2/\beta^2\right); \qquad \beta^4 = \frac{2m_q^2\omega_0^2\alpha_c(4m_q^2)}{1 - 2\alpha_c(4m_q^2)C_0}; \tag{19}$$

$$\beta = 0.060 \text{ GeV}^2; \qquad m_q = m_{ud} = 265 \text{ MeV}. \tag{20}$$

We shall make use of this mass function (18)-(20) in Sec. III for the evaluation of the various quark condensates.

Before closing this section we indicate briefly a derivation of the gluon condensate, Eq. (2), by inserting the gluon propagator (17) in its definition. First, we may remark that the evaluation of the gluon condensate may be obtained in principle by averaging over the ‘background’ gluon field G_{ext}, but here the problem should be viewed in another way since our parametrization of the infrared part of the gluon propagator was designed to incorporate this background effect explicitly. Indeed as is known from the work of M. Shifman [19] and M. Voloshin [20], the small distance behaviour of the infrared part of the gluon propagator, due to the presence of background fields, must go like $\hat{R}^2$, which is explicitly seen in our gluon propagator structure, Eq. (17), with a detailed QCD check described elsewhere [21]. In other words, we have conciously attempted to make up for the empirical structure of the infrared part of the gluon propagator, Eq. (17), by making it conform to as many general principles as possible (Voloshin, Shifman-like), so as not to violate the QCD constraints at long and short distances.

We proceed to insert the gluon propagator (17) into Eq. (2) in order to obtain an estimate of the gluon condensate. The Co-term may be dropped as it will not survive the subsequent differentiations in Eq. (2). For the main term, the following integral representation is employed:

$$\frac{\hat{R}^2}{\sqrt{1+A_0 m_q^2 \hat{R}^2}} = \frac{2m_q\sqrt{A_0}}{2\pi i}\oint dR_0 \frac{\hat{R}^2}{1+A_0 m_q^2 R^2}, \tag{21}$$

where $R^2 = \hat{R}^2 - R_0^2$ (Lorentz-invariant). The 4D expression $D(R)$ may now be inferred from its definition in terms of the 3D quantity $\mathcal{D}(\hat{R})$.

$$V(R) = \frac{\alpha_c(Q^2)}{\pi}\frac{2m_q^2\omega_0^2\sqrt{A_0}\,\hat{R}^2}{1+A_0 m_q^2 R^2}. \tag{22}$$

This is as far as one may go if we adopt the 3D form (17) for $\mathcal{D}(\hat{R})$. However, it is sufficiently suggestive of the extrapolation needed to make it fully covariant, viz. $\hat{R}^2 \to R^2$ in the numerator of Eq. (22). We adopt this 'natural' extrapolation in what follows [22]. The full propagator in the Landau gauge is already given in Eq. (11) where k_μ should be read as $k_\mu = -i\partial_\mu^R$. To evaluate the gluon condensate we first note the result:

$$\frac{\alpha_c}{\pi}\langle G_{\alpha\mu}^a G_{\beta\nu}^b\rangle = \{-2\partial_\alpha^R\partial_\beta^R D_{\mu\nu}^{ab}(R) + 2\partial_\alpha^R\partial_\nu^R D_{\mu\beta}^{ab}(R)\}|_{R=0}, \tag{23}$$

and obtain by straightforward differentiation

$$\langle g_c^2 G^2\rangle = \sqrt{A_0}\,4\pi\alpha_c(4m_q^2)(6m_q\omega_0)^2/\pi^2. \tag{24}$$

The remaining question concerns what value of the quark mass m_q, i.e. what flavor, should be employed for evaluating the gluon condensate. The structure (17) does exhibit the desired features of linear confinement and flavor independence, but the extrapolation of these features in the opposite limit $(R \to 0)$, as demanded by Eq. (23), brings in an "effective flavor dependence" of the final formula (24). The heavier the flavor, the more important is the corresponding mass (m_q), *vis-a-vis* the Au-term in the $q\bar{q}$ potential (17). Since, on the other hand, the full potential (17) fits all the flavor sectors rather well [17,12], we have chosen to employ a simple "weighting" procedure involving only the three flavor sectors with a nontrivial flavor mass, viz. $s\bar{s}, c\bar{c}$, and $b\bar{b}$ with equal weights (in the sense of a geometric mean), taking account of the m_q-dependence $m_q^2\alpha_c(4m_q^2)$ of Eq. (24). This gives the result

$$\langle m_q^2\alpha_c(4m_q^2)\rangle = 13.91\{m_u^2\alpha_c(4m_u^2)\}, \tag{25}$$

in units of its value in the (ud)-region, and its substitution in (24) yields the final estimate

$$\langle g_c^2 G^2\rangle = 0.502\ \mathrm{GeV}^4, \tag{26}$$

which is compared well with the value of 0.47 GeV^2 adopted in the QCD sum rule literature [4]. [Note that, should we incorporated the recent discovery of the top quark, the factor 13.91 in Eq. (25) may be replaced naively by 24.4, yielding a somewhat larger value of $\langle g_c^2 G^2 \rangle$ at 0.88 GeV^4. Nevertheless, it is expected that the simple "weighting" procedure should be modified when one of the quarks, the top, is unusually heavy – the top mass is 179 GeV, far above the ordinary hadronic mass scale.]

III. $\langle \bar{q} \mathcal{O}_i q \rangle$ CONDENSATES

In this section we shall substitute the mass function $m(\hat{p}^2)$, obtained in Sec. II, Eqs. (18-20), viz.,

$$m(\hat{p}^2) = m_q^{-2}(m_q^2 + \hat{p}^2)^{3/2} \exp\left(-\frac{\hat{p}^2}{\beta_i^2}\right), \tag{27}$$

into the general formula (1), to derive the various condensates for different choices of $\mathcal{O}_i$. As already noted in Sec. I (in light of color gauge invariance), the quantity p_μ in Eq. (27) and everywhere else in the following, must be read as Π_μ [7], with appropriate non-abelian corrections. The formula (1) now reads as

$$\langle \bar{q} \mathcal{O}_i q \rangle_0 = \mathrm{Tr} \frac{-i}{(2\pi)^4} \int d^4\Pi \frac{m(\hat{\Pi}) - i\gamma \cdot \Pi}{m^2(\hat{\Pi}) + (\gamma \cdot \Pi)^2} \mathcal{O}_i \tag{28}$$

in the absence of external fields. Note that the subtracted part with $m(\hat{\Pi})=0$ in Eq. (1) gives no effect on tracing in the absence of external fields.

Some general remarks may be in order concerning the use of the integration variable II, instead of p_μ. Note that we are following closely the method of Schwinger's celebrated paper [7] by using explicitly gauge-invariant quantities during the integration process itself, but now with the added the complexities associated with the non-abelian colour matrices. In particular, one must exercise caution in regard to the non-commutativity of the Π_μ variables, even in the presence of a constant background field, and indeed many important terms might be missed without proper precaution of this aspect. Nevertheless the correct procedure needs only a few standard tricks, most of which have been already indicated by [e.g. Eqs. (29)-(33)] given immediately below. For example, the gauge invariant measure $d^4\Pi$ can be expressed in terms of commuting longitudinal and transverse variables Π_ℓ and $\hat{\Pi}^2$ plus the (largely passive) angular variables. The further observation that most of the non-commuting terms in the integrand involve odd angular functions which do not survive their integration is enough to ensure a straightforward evaluation along the lines outlined immediately below. (The only nontrivial problem of commutation involves the composite operator Σ_g for whose square we have adopted an obvious "colour-averaging approximation", Eq. (33), a *la* standard methods.)

We first express the denominator in the alternative form:

$$m^2(\hat\Pi) + (\gamma \cdot \Pi)^2 = \hat\omega^2 - \Sigma_g - \Pi_l^2 \equiv \Delta - \Sigma_g \,, \tag{29}$$

$$\hat\omega^2 = m^2(\hat\Pi) + \hat\Pi^2 \,; \qquad \Pi^2 = \hat\Pi^2 - \Pi_l^2 \,, \tag{30}$$

$$\Sigma_g = \frac{1}{2} g_c \frac{1}{2} \lambda^a G^a_{\mu\nu} \sigma_{\mu\nu} \,, \tag{31}$$

where Π_l is the longitudinal component of Π,, $d^4\Pi = d^3\hat\Pi d\Pi_l$, and the integration must first be carried out over Π_l. Because of the presence of the Σ_g−term in Eq. (29), however, a further “ rationalization” of Eq. (28) is necessary according to the identity

$$\frac{1}{\Delta - \Sigma_g} \equiv \frac{1}{\hat\omega^2 - \Pi_l^2 - \Sigma_g} = \frac{\Delta + \Sigma_g}{\Delta^2 - \Sigma_g^2} \,. \tag{32}$$

At this stage it is probably adequate to replace Σ_g^2 in the denominator of Eq. (32) by its spin-color-averaged value $\langle \Sigma_g^2 \rangle$:

$$\Sigma_g^2 \to \langle \Sigma_g^2 \rangle = \frac{1}{12} \langle g_c^2 GG \rangle \equiv \mu^4 (= 8.48 m_q^4) \,. \tag{33}$$

after the necessary substitutions have been made from Eqs. (26) and (15). Thus $\langle \Sigma_g^2 \rangle$ contains a strong signature of the gluon condensate whose large value introduces some bad analyticity properties in the denominator of the integrand in Eq. (28) or (32), for purposes of III-integration, since the $\hat\omega^2$−term is numerically much smaller than μ^2. We should like to emphasize that this feature has nothing to do with our 3D BSE treatment, since we have not yet passed the barrier of the orthodox 4D quark propagator in the background of the gluon field. It is rather a very general manifestation of the strong spin-color effect of the quark-quark interaction via the color magnetic field. The problem is not so serious in QED [7] where the smallness of the coupling constant leaves the counterpart of the μ^2 term well below the positivity limit (i.e., $\hat\omega^2 - \mu^2 > 0$), but the large value of μ^2 in the present (QCD) case tends to invalidate the standard analyticity structure of Eq. (28) for purposes of further integration with respect to $d^3\hat\Pi$. We have not been able to solve this problem here but it seems to deserve more serious attention from a wider community. (Taken literally, it would imply the introduction of a complex ***phase*** in the condensates!) In the meantime we take a conservative view that the maximum allowed value of $\langle \Sigma_g^2 \rangle$ (consistent with the positivity of the denominator after III-integration) should not exceed $\hat\omega^4$ for all values of $\hat\Pi^2$, i.e.

$$\langle \Sigma_g^2 \rangle = \sigma^2 \le m_q^4 \,. \tag{34}$$

Thus we shall understand Eq. (32) as

$$\frac{1}{\mathrm{A} - \Sigma_g} \Rightarrow \frac{\Delta \ \mathrm{t}\ \Sigma_g}{\Delta^2 - \sigma^2}; \qquad \Delta \equiv \hat{\omega}^2 - \hat{\Pi}_l^2 . \tag{35}$$

For the numerator of Eq. (35) which still carries the spin-dependent quantity Σ_g, Eq. (31), there is no restriction of magnitude for one X,-factor only, since it contributes to condensates only after contracting with another C-factor in Eq. (36). [However, other factors which come from the rationalization of the denominator with higher powers of Σ_g must be subject to the same restriction.] With this precaution, Eq. (28) serves to define two condensates simultaneously, viz, these with, $\mathcal{O}_i$=1 and $\mathcal{O}_i = g_c(\lambda^a/2)G^a_{\mu\nu}\sigma_{\mu\nu}$, where the latter is expressible in the notation of Ref. [4] as

$$\langle 0|\bar{q}2\Sigma_g q|0\rangle \equiv m_0^2\langle\bar{q}q\rangle_0 . \tag{36}$$

To evaluate the integral over $d\Pi_l$, we have

$$\frac{1}{2\pi i}\int d\Pi_l \frac{\Delta;\sigma}{\Delta^2 - \sigma^2} = [I(\sigma); J(\sigma)] , \tag{37}$$

where

$$I(\sigma); J(\sigma) \equiv \frac{1}{2}\left(\frac{1}{\sqrt{\omega^2 - \sigma}} \pm \frac{1}{\sqrt{\omega^2 + \sigma}}\right) . \tag{38}$$

After collecting the necessary trace factors the final result for the two condensates is expressible as a simple quadrature $(q = u$ or d):

$$\langle\bar{q}q\rangle_0[1;\, m_0^2] = -\frac{3}{\pi^2}\int_0^\infty \hat{\Pi}^2 d\Pi\, m(\hat{\Pi})\left[I(\sigma);\ \frac{2\langle\Sigma_g^2\rangle}{\sigma}J(\sigma)\right] . \tag{39}$$

On insertion of the structure Eq. (27) for the mass function, and putting the "maximum allowed value" of σ, viz, m_q^2, Eq. (34), the results are [23]

$$\langle\bar{q}q\rangle_0 = -(266\,\mathrm{MeV})^3; \qquad (c.f. - (240 \pm 25\,\mathrm{MeV})^3; \text{ see [4]}); \tag{40}$$

$$m_0^2 = 0.130\,\mathrm{GeV}^2; \qquad (c.f.\ 0.8\ \mathrm{GeV}^2; \text{ see [4]}). \tag{41}$$

We next calculate three induced condensates X, κ, and J, due to a constant external $e.m.$ field $F_{\mu\nu}$, which are defined as [4]:

$$\langle\bar{q}\sigma_{\mu\nu}q\rangle_F \equiv ee_q\chi F_{\mu\nu}\langle\bar{q}q\rangle_0 , \tag{42a}$$

$$g_c\langle\bar{q}(\lambda^a/2)G^a_{\mu\nu}q\rangle_F \equiv ee_q\kappa F_{\mu\nu}\langle\bar{q}q\rangle_0 , \tag{42b}$$

$$g_c\langle\bar{q}(\lambda^a/2)\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}q\rangle_F \equiv ee_q\zeta F_{\mu\nu}\langle\bar{q}q\rangle_0 . \tag{42c}$$

In these equations the *relative* phases of the induced condensates are defined with respect to the main condensate $\langle \bar{q}q\rangle_0$, in accordance with our formula (28), and we keep systematic track of this feature. Like the two condensates Eq. (39), the quantities X and κ are in a sense dual to each other, and are best described together. The *e.m.* field is introduced through the substitution

$$[\hat{m} + i\gamma \cdot \Pi]^{-1} \rightarrow [\hat{m} + i\gamma_\mu(\Pi_\mu - eA_\mu)]^{-1} \tag{43}$$

in the propagator Eq. (1) and keeping only the first order term in A,. Thus we have to calculate

$$\int \mathrm{Tr}\, S_F(\Pi)(ie\gamma \, . \, A)S_F(\Pi)\Big[\sigma_{\mu\nu}; \frac{1}{2}\lambda^a G^a_{\mu\nu}\Big]. \tag{44}$$

This is facilitated, for a constant e.m. field, by the representation

$$A, = -\frac{1}{2}x_\nu F_{\mu\nu}\,; \qquad x_\mu = i\frac{\partial}{\partial \Pi_\mu}\,. \tag{45}$$

The substitution in Eq. (44) and subsequent trace evaluation is routine but lengthy. However certain precautions are necessary in the matter of extraction of two groups of terms, proportional to $\sigma_{\mu\nu}$ and $G_{\mu\nu}$ respectively, before the trace evaluation, which will survive contraction with the external e.m. field $F_{\mu\nu}$. Thus,

$$\Pi_\mu \Pi_\nu \Rightarrow \frac{i}{2}g_c\frac{1}{2}\lambda_a G^a_{\mu\nu}\,; \qquad \gamma_\mu\gamma_\nu \Rightarrow i\sigma_{\mu\nu}\,. \tag{46}$$

In terms like $i\sigma_{\mu\lambda}\Pi_\lambda\Pi_\nu$, additional survivors come from the symmetrized product $\{\Pi_\lambda, \Pi_\nu\}$ for which we make the standard isotropy ansatz. In this respect, their association with (space-like) magnetic effects makes it more meaningful to do an effectively 3D averaging, viz. $\Pi_\mu\Pi_\nu \Rightarrow \frac{1}{3}\hat{\Pi}^2(\delta_{\mu\nu} - \hat{\eta}_\mu\hat{\eta}_\nu)$ where $\hat{\eta}_\mu$ is a unit vector whose direction need not be specified too precisely. After this step, the tracing process is straightforward, and we omit the details. A useful formula is

$$\mathrm{Tr}\Big\{\frac{1}{2}\lambda^a G^a_{\mu\nu} g_c \Sigma_g \sigma_{\alpha\beta} F_{\alpha\beta}\Big\} = \frac{1}{3}\langle g_c^2 G^2\rangle F_{\mu\nu}\,. \tag{47}$$

The results for the three quantities X, κ, and ζ are expressed as:

$$\begin{aligned} \chi\langle\bar{q}q\rangle = & -\frac{i}{(2\pi)^4}\int d^4\Pi\Big\{12\frac{\Delta^2+\sigma^2}{(\Delta^2-\sigma^2)^2}\Big(\hat{m}+\frac{2\hat{m}'\hat{\Pi}^2}{3}\Big) \\ & -16\hat{m}\hat{\Pi}^2\frac{(\Delta^2+3\Delta\sigma^2)}{(\Delta^2-\sigma^2)^3} \\ & +\frac{\langle g_c^2G^2\rangle}{3}\Big[\frac{\hat{m}'\Delta}{(\Delta^2-\sigma^2)^2}+\frac{\hat{m}-2\hat{m}^2\hat{m}'}{(\Delta^2-\sigma^2)^3}(3\Delta^2+3\sigma^2)\Big]\Big\}, \end{aligned} \tag{48a}$$

$$\begin{aligned}\kappa\langle\bar{q}q\rangle = & -\frac{i}{(2\pi)^4}\frac{\langle g_c^2G^2\rangle}{3}\int d^4\Pi\Big[\frac{\Delta}{(\Delta^2+\sigma^2)^2}\Big(\hat{m}+\frac{2\hat{m}'\hat{\Pi}^2}{3}\Big)\\ & -2\hat{m}\Pi^2\frac{(\Delta^2+\sigma^2)}{(\Delta^2-\sigma^2)^3}+\frac{1}{2}\hat{m}'\frac{\Delta^2+\sigma^2}{(\Delta^2-\sigma^2)^2}\\ & +(\hat{m}-2\hat{m}^2\hat{m}')\frac{\Delta^3+3\Delta\sigma^2}{(\Delta^2-\sigma^2)^2}\Big],\end{aligned} \tag{48b}$$

$$\begin{aligned}\zeta\langle\bar{q}q\rangle = & \frac{2i}{(2\pi)^4}\frac{\langle g_c^2G^2\rangle}{3}\int d^4\Pi\Big[\frac{\Delta}{(\Delta^2-\sigma^2)^2}\Big(\frac{\hat{m}+2\hat{m}'\hat{\Pi}^2}{3}\Big)\\ & -2\hat{m}\hat{\Pi}^2\frac{\Delta^2+\sigma^2}{(\Delta^2-\sigma^2)^3}\Big].\end{aligned} \tag{48c}$$

After integration $w.r.t.$ Π_l, the resulting expressions are:

$$\begin{aligned}\chi\langle\bar{q}q\rangle = & \frac{1}{(2\pi)^3}\int d^3\hat{\Pi}\Big\{12\Big(\hat{m}+\frac{2\hat{m}'\hat{\Pi}^2}{3}\Big)J'-8\hat{m}\hat{\Pi}^2I''\\ & +\frac{\langle g_c^2G^2\rangle}{3\sigma}\Big[\hat{m}'\frac{I'}{2}+\frac{3}{4}(\hat{m}-2\hat{m}^2\hat{m}')\Big(J''-\frac{J'}{\sigma}+\frac{J}{\sigma^2}\Big)\Big]\Big\},\end{aligned} \tag{49a}$$

$$\begin{aligned}\kappa\langle\bar{q}q\rangle = & \frac{1}{(2\pi)^3}\frac{\langle g_c^2G^2\rangle}{3}\int d^3\hat{\Pi}\Big[\frac{I'}{2\sigma}\Big(\hat{m}+\frac{2\hat{m}'\hat{\Pi}^2}{3}\Big)\\ & +\frac{\hat{m}'J'}{2}(\hat{m}-2\hat{m}^2\hat{m}')\frac{I''}{2}-\frac{\hat{m}\hat{\Pi}^2}{2}\Big(J''-\frac{J'}{\sigma}+\frac{J}{\sigma^2}\Big)\Big],\end{aligned} \tag{49b}$$

$$\zeta\langle\bar{q}q\rangle = -\frac{2}{(2\pi)^3}\frac{\langle g_c^2G^2\rangle}{3}\int d^3\hat{\Pi}\Big[\frac{I'}{2\sigma}\Big(\hat{m}+\frac{2\hat{m}'\hat{\Pi}^2}{3}\Big)\frac{\hat{m}'J'}{2}\Big]. \tag{49c}$$

The complementary nature of the quantities χ and $\kappa(\zeta)$ is quite manifest from a comparison of their dependece on the dual functions $I(\sigma)$ and $J(\sigma)$ which are respectively even and odd $w.r.t.$ σ; while their successive derivatives alternate in parity. (σ is of course put equal to m_q^2 after performing the indicated differentiations; see Eq. (34).) For brevity, we have omitted the argument σ from the $I(\sigma)$ and $J(\sigma)$ functions in Eqs. (49).

The numerical results are as follows:

$$\chi = -3.56\,\mathrm{GeV}^{-2}; \qquad (c.f.\ -(6\pm 2)\ \mathrm{GeV}^{-2};\ [4]); \tag{50a}$$

$$\kappa = -0.11; \qquad \text{(sign negative !)}; \tag{50b}$$

$$\zeta = +0.06\,\mathrm{GeV}^{-2}; \qquad \text{(sign according to the definition of } \gamma_5\text{)}. \tag{50c}$$

Axial Condensates

So far there has been no explicit need to subtract the perturbative contribution ($\hat{m} = 0$) to the condensates calculated above, since their traces are zero. We now consider the *axial* condensate ($\mathcal{O}_i = i\gamma_\mu\gamma_5$) in a constant external axial field A_μ [24], where an explicit subtraction is necessary to ensure convergence of the integral. This condensate is connected with the axial *isoscalar* coupling which enters the Bjorken sum rule [25] for DIS of polarized electrons on a polarized proton [26]. It is defined through the relation

$$\langle \bar{q} i\gamma_\mu\gamma_5 q\rangle_A = \mathcal{A}_s A_\mu \,, \tag{51}$$

and its value was calculated in Ref. [26] as $f_\eta^2 \approx f_\pi^2$, on the assumption that the axial field, interacts with the 8-th component (isoscalar) of the unitary octet current. In the present treatment it does not need any such extra assumption but can be simply calculated from Eq. (1) with ($\mathcal{O}_i = i\gamma_\mu\gamma_5$), and introducing the axial field by the gauge substitution $\Pi_\mu \to \Pi_\mu - \gamma_5 A_\mu$ in the propagator, and keeping only the first order term in the expansion. The result is expressed by

$$\mathcal{A}_s A_\mu = \frac{-i}{(2\pi)^4}\mathrm{Tr}\int [S_F(\Pi) i\gamma\cdot A\gamma_5 S_F(\Pi) i\gamma_\mu\gamma_5] d^4\Pi - (\hat{m} = 0 \text{ term}) \,. \tag{52}$$

Evaluating the trace and using the isotropy condition $< \Pi_\mu\Pi_\nu > = \delta_{\mu\nu}\Pi^2/4$ we obtain

$$\mathcal{A}_s = \frac{-3i}{(2\pi)^4}\mathrm{Tr}\int d^4\Pi \Big[\frac{\hat{m}^2 - \Pi^2/2 + \Sigma_g}{(\Delta - \Sigma_g)^2} + \frac{\Pi^2/2 - \Sigma_g}{(\Pi^2 - \Sigma_g)^2}\Big] \,. \tag{53}$$

In this case however it is perhaps not as meaningful to keep track of the Σ_g-terms for numerical purposes as for the *e.m.* case, we shall drop them at this stage. Then with a simple rearrangement $\hat{m}^2 - \Pi^2/2 = 3\Pi^2/2 - \Delta/2$ the $\Delta/2$ term can be combined with the last term through a Feynman variable u ($0 \le u \le 1$) and the pole integration carried out. The final result is

$$\begin{aligned}\mathcal{A}_s &= \frac{3}{4\pi^2}\int_0^\infty \hat{\Pi}^2 d\hat{\Pi} \int_0^\infty du\, \hat{m}^2 \Big[\frac{3}{(\hat{m}^2 + \hat{\Pi}^2)^{3/2}} + \frac{1}{(\hat{m}^2 u + \hat{\Pi}^2)^{3/2}}\Big] \\ &= 0.021 \text{ GeV}^2 \,,\end{aligned} \tag{54}$$

which may be compared with $f_\pi^2 \approx 0.018$, or perhaps better with f_η^2 which is the relevant isoscalar quantity [26] having a larger value [11] than f_π^2. For completeness we record also the corresponding result for an external pseudoscalar field Z [27] on the condensate $< \bar{q}\gamma_5 q >$. The result is

$$\begin{aligned}\langle \bar{q}\gamma_5 q\rangle_Z &= \frac{3}{2\pi^2}\int_0^1 du \int_0^\infty \hat{\Pi}^2 d\Pi\, (\hat{m}^2 u + \hat{\Pi}^2)^{-3/2} \\ &= 0.017 \text{ GeV}^2 \,,\end{aligned} \tag{55}$$

which is closer to f_π^2. Since this field is not a gauge-covariant insertion, unlike the axial field A,, *we* refrain from making further comments here and refer the reader to [27] for more details.

IV. DISCUSSION

We are now in a position to compare and discuss our results given in Sec. III *vis-a-vis* those normally assumed for the QCD sum rule analysis [4]. The comparison made in Sec. III shows by and large a rather good overlap between our results and those obtained in connection with the QCD sum rule studies, except for the case of m_0^2 (see below). In particular, the two main condensates $\langle \bar{q}q \rangle$ and $\langle g_c^2 G^2 \rangle$ leave little to be desired, in view of the parameter-free nature of the entire calculation.

As regards the induced condensates only X seems to have been estimated in the QCD sum rule literature with some degree of confidence while a corresponding determination of the other two (κ, ζ) remains quite tentative. Our value of X (-3.56) is somewhat smaller than the ITEP result [4], but it seems to tally quite well with another determination (-3.3) by Balitsky et al. [28], as quoted in Chiu et al. [29]. (The sign is of course negative.) As regards κ (-0.11) and $\zeta(+0.06)$, our values are admittedly small, but they are more difficult to compare with the literature [4,28,29] (both in magnitude and in sign) than in the case of X which is somewhat sensitive to the magnetic moment determination of the nucleon.

Our value for the axial condensate (A, $=0.021 \mathrm{GeV}^2$) is also in good agreement with the result of ref. 27, but the larger value than f_π^2 is probably not unwelcome since an isoscalar axial condensate should stand a better comparison with f_η^2 which has higher value [11] than f_π^2.

Since the agreement of our condensate values with those assumed for QCD sum rule studies is quite good on the whole, it is qualitatively clear, even without a detailed calculation, that our values used as input will equally well reproduce the results of QCD sum rule analysis whether it is the proton magnetic moment [30], or the nucleon mass itself [31]. A comparison of the work of [30,31] *vis-a-vis* [28,29] seems to indicate the possibility of a wider variation in the input value of X.

The only visible discrepancy has to do with the $\langle \bar{q}\sigma . G^a \lambda^a q \rangle$ condensate m_0^2 (0.13 GeV^2) which comes out a bit low compared to the value adopted earlier (0.8 GeV) [31]. Nevertheless, the (extrinsic) determination of this quantity from QCD sum rule analysis is probably not as "rigid" as those of the other condensates, as can be seen immediately below. To this end, we may follow the analysis of [31] in search of the best values for (M_N, W^2, β_N^2), which enter the two Belyaev-Ioffe sum rules for the determination of the nucleon mass, with the first one given by

$$\frac{M_B^6}{8L^{4/9}}E_2 + \frac{M_B^2}{32L^{4/9}}\langle g_c^2 G^2\rangle E_0 + \frac{1}{6}a^2 L^{4/9} - \frac{1}{24M_B^2}a^2 m_0^2$$
$$= \beta_N^2 \exp(-M_N^2/M_B^2)\,, \tag{56}$$
$$L = \frac{\ln(M_B^2/\Lambda^2)}{\ln(\mu^2/\Lambda^2)}\,,$$

where a = $-(2\pi)^2\langle\bar{q}q\rangle$ with the Bore1 mass, M_B, in GeV and $\beta_N^2 \equiv (2\pi)^4\lambda_N^2/4$. μ is the renormalization point taken to be 0.5 GeV and A is the QCD scale parameter taken to be 0.2 GeV. The factors $E_0 = 1 - e^{-x}, E_1 = 1 - (1 + x)e^{-x}$, and $E_2 = 1 - (1 + x + \frac{1}{2}x^2)e^{-x}$, with $x \equiv W^2/M_B^2$ are used to correct the sum rule to obtain consistent M_B^2 dependence for contributions from excited states through perturbative QCD techniques [30,31]. They also serve to restrict the range of the integration and increase the weight given to the nucleon. The best values are given by

$$M_N = 918 \text{ MeV}; \qquad W^2 = 2.7 \text{ GeV}^2. \tag{57}$$

Note that the observed nucleon mass is well reproduced and the value of the threshold is only slightly higher (W^2 = 2.3 GeV^2 in [31]), despite the fact that the new value of m_0^2 (and all the other calculated condensates) is used as the basic input.

Analogously, we recall the proton magnetic moment sum rule [30]:

$$\frac{e_u M_B^4}{L^{4/9}}E_2 + \frac{a^2}{3M_B^2}L^{4/9}\left\{-\left(e_d + \frac{2}{3}e_u\right) + \frac{1}{3}e_u(\kappa - 2\zeta)\right.$$
$$\left. -2e_u\chi\left(\frac{M_B^2}{L^{16/27}} - \frac{1}{8}\frac{m_0^2}{L^{28/27}}\right)\right\} = 2\beta_N^2 e^{-M_N^2/M_B^2}\left(\frac{\mu_p}{M_B^2} + A_p\right), \tag{58}$$
$$e_u = \frac{2}{3}\,, \qquad e_d = -\frac{1}{3}\,;$$

where the constant *A,* is introduced to represent the residual continuum contribution to the dispersion integral [30]. This formula serves to remind us of the role played by the various induced condensates κ, ζ, and X, which we have already calculated. Typical numerical anaylysis of this sum rule is displayed in Fig. 1, where we have adopted our calculated condensates as the input. It is seen that the predicted proton magnetic moment is in the vicinity of $2.2\mu_N$ for the higher Bore1 mass range, but its value drops if a lower Bore1 mass range is used. We have also found that, by choosing a larger value for the threshold W^2, the predicted proton magnetic moment could easily be brought into agreement with the observed value.

On the whole it appears that the model has reproduced the QCD sum rule values of the condensates fairly well, despite some minor discrepancies. Of the two factors $\hat{\omega}^3$ and $\phi(\hat{q})$ of the mass function, the former is on more solid theoretical foundation since it can be

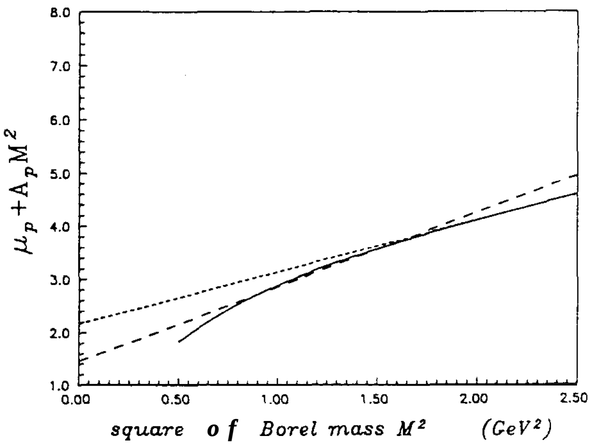

FIG. 1. Linear analysis of the proton magnetic moment sum rule, Eq. (56), using the condensate parameters calculated in this paper.

traced to the structure of the propagators in the 4D BSE, when treated (covariantly) in the instantaneous approximation [10], but does not contain any parameters of the $q\bar{q}$ interaction as such. Any change in this component would involve a major modification, such as a full-fledged 4D form of parametrization of the mass function and the gluon propagator, within an otherwise SDE-cum-BSE framework [32], were it not for its $O(4)$-like implications on the hadron spectra [33]. As regards the other factor $\phi(\hat{q})$, any change in this component would involve a different structure for the infrared part of the gluon propagator, viz. Eq. (13), with its own ramifications on the entire hadronic spectra [12,13]. Observational checks on the mass function $m(\hat{p}^2)$, such as f_π or $\Gamma(\pi \to \gamma\gamma)$, have provided satisfactory results [10], but clearly more are needed. A recent estimate of the pion electromagnetic form factor $F(Q^2)$ for large Q^2 indicates a value of about 0.7 GeV^2 for $Q^2F(Q^2)$, somewhat larger than the experimental value of about 0.5 GeV^2, indicating scope for corrections such as one-gluon-exchange effects.

V. SUMMARY

To summarize, we have investigated in a specific and systematic manner the possibility of understanding some of the principal QCD condensates $\langle \bar{q}\mathcal{O}q \rangle$, which are traditionally associated with QCD sum rules, directly in terms of their definition, viz., Eq. (1). To this end, we have related the mass function $m(p^2)$ to the pion-quark vertex function in the chiral limit [8,9]. This last aspect provides a concrete handle for its determination through the vehicle of the Bethe-Salpeter equation (BSE) for $q\bar{q}$ hadrons. Since the latter is directly adaptable to spectroscopic studies, the method provides a clear linkage between the high-energy and low-energy descriptions of hadrons in QCD. The gluon condensate which is related to the same $q\bar{q}$ interaction in the confining region (the infrared domain of the gluon propagator) have also been calculated in a similar fashion. The results for most condensates are in good overlap with the values employed in the method of QCD sum rules.

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REFERENCES

[1] K. G. Wilson Phys. Rev. **179, 1499** (1969).

[2] H. D. Politzer, Nucl. Phys. **B117, 397 (1976).**

[3] M. A. Shifman, A. J. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385, 448 (1979).

[4] B. L. Ioffe, Acta Phys. Polonica B16, 543 (1985).

[5] V. L. Chernyark and A. R. Zhitnitsky, Phys. Rep. **112C**, 174 (1984).

[6] Some typical references are: B. S. Dewitt, Phys. Rev. 162, 1195, 1239 (1967); J. Honerkamp, Nucl. Phys. **B48**, 269 (1972); R. Kallosh, Nucl. Phys. **B78**, 293, (1974); S. G. Matinyan and G. K. Savvidy, Nucl. Phys. B134, 539 (1978); H. Leutwyler, Nucl. Phys. B179, 129 (1981); L. F. Abbot, ibid, B185, 189 (1982).

[7] J. Schwinger, Phys. Rev. 82, 664 (1951).

[8] **A. N.** Mitra and B. M. Sodermark, Intl. J. Mod. Phys. **A9**, 915 (1994).

[9] R. Delbourgo and M. D. Scadron, J. Phys. **G5**, 1621 (1979); S. Adler and A. C. Davis, Nucl. Phys. B244, 469 (1984).

[10] A. N. Mitra and S. Bhatnagar, Intl. J. Mod. Phys. **A7**, 121 (1992); A. N. Mitra and I. Santhanam, Few-Body Syst. 12, 41 (1992).

[11] Particle Data Group, Phys. Rev. D45, Part II, June (1992).

[12] K. K. Gupta et al., Phys. Rev. D42, 1604 (1990).

[13] A. Sharma, S. R. Choudhury, and A. N. Mitra, Phys. Rev. **D50**, (1994); to appaer.

[14] Yu. L. Kalinovsky, L. Kaschluhn, and V. N. Pervushin, Phys. Lett. **231B**, 288 (1989), on Markov-Yukawa conditions; H. Sadzian, J. Bijtebier, and H. C. Crater, in *Proc. Intl. Symp. Extended* Objects, Karuizawa, 1992, Eds. S. Ishida et al. (World Scientific, Singapore, 1993), on constrained dynamics.

[15] W. Y. P. Hwang and A. N. Mitra, Few-Body Syst. 15, 1 (1993).

[16] See, e.g., D. Atkinson and R. Johnson, Phys. Rev. D41, 4165 (1990).

[17] A. Mittal and A.N. Mitra, Phys. Rev. Lett. 57, 290 (1986).

[18] The parametrization for β^4, Eq. (19), was made in K. K. Gupta et al., Ref. [12], on the basis of NPA which amounted to the replacement of one m_q−factor in Eq. (19) by $M/2$, where M is the hadron mass in accordance with the correspondence $2m_q$(CIA)$\leftrightarrow M$ (NPA), as explained elsewhere [S. Bhatnagar et al., Phys. Lett. B263, 485 (1991)]. This makes little difference for most hadrons (down to the kaon), excepting the pion for which this replacement $h4 \rightarrow 2m_q$ is now formally necessary before going to the chiral limit ($M_\pi \rightarrow 0$).

[19] M. Shifman, Nucl. Phys. B173, 13 (1980).

[20] M. Voloshin, Nucl. Phys. **B154**, 365 (1979).

[21] Y. K. Mathur and A. N. Mitra, Sov. J. Nucl. Phys. 49, 336 (1989).

[22] If we do not adopt the replacement $\hat{R}^2 \rightarrow R^2$ in the numerator, then the result on the gluon condensate should be reduced by a factor of $\frac{3}{4}$.

[23] The difference from the value obtained in Ref. [8] is partly due to the change in β^2 as a result of the replacement indicated in Ref. [18] to obtain a non-zero chiral limit for this quantity and partly due to the effect of the Σ_g−term not considered in Ref. [8].

[24] V. M. Belyaev and Ya. I. Kogan, Pis'ma Zh. Eksp. Teor. Fiz. 37, 611 (1983) [JETP Lett. 37, 730 (1983)]; C. B. Chiu, J. Pasupathy, and S. J. Wilson, Phys. Rev. D32, 1786 (1985).

[25] J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[26] V. M. Belyaev, B. L. Ioffe, and Ya. I. Kogan, Phys. Lett. **151B**, 290 (1985).

[27] E. M. Henley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Rev. D46, 431 (1992).

[28] I. I. Balitsky and A. V. Yung, Phys. Lett. **129B**, 328 (1983).

[29] C. B. Chiu, J. Pasupathy, and S. J. Wilson, Phys. Rev. D33, 1961 (1986).

[30] B. L. Ioffe and A. V. Smilga, Nucl. Phys. B232, 109 (1984).

[31] B. L. Ioffe, Nucl. Phys. **B188**, 317 (1981); [E] **B191**, 591 (1981); V. M. Belyaev and B. L. Ioffe, Zh. Eksp. Teor. Fiz. 83, 876 (1982) [Sov. Phys. JETP 56, 493 (1982)].

[32] J. Praschifka, R. T. Cahill, and C. D. Roberts, Intl. J. Mod. Phys. **A4, 4929** (1989); K. I. Aoki, T. Kugo, and M. G. Mitchard, Phys. Lett. **266B**, 467 (1991); H. J. Munczek and P. Jain, Phys. Rev. D46, 438 (1992).

[33] For a more detailed discussion on this issue, see Ref. [8].