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QCD and Nuclear Physics

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Upon a brief explanation of what nuclear physics and QCD is all about, I would like to discuss how the imminent future of nuclear physics hinges on progresses in QCD. In addition, I wish to describe my recent work that a closed set of coupled differential equations for nonlocal condensates in QCD (which characterize the nontrivial feature of the QCD ground state or vacuum) may be derived in the large N_c limit (with N_c the number of color). As a specific example, the leading-order equations for the nonlocal condensates appearing in the quark propagator are derived and explicit solutions are obtained.

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I. Introduction

One common characterization of "nuclear physics" is the "physics of strong interactions" and in this sense the problem of strong interaction physics has been around for more than half a century (since early thirties when the question regarding the origin of nuclear force was first raised in a serious manner). The very nature of the problem varies considerably with the so-called "underlying theory", especially when quantum chromodynamics or QCD is taken universally to be such underlying theory. Thanks to its asymptotically free nature, QCD as the candidate theory of strong interactions has been tested to an accuracy of about 10 % at high energies.

QCD as the foundation of nuclear physics presents us a challenge of some unusual kind, as the basic degrees of freedom (DOF's) of QCD are quarks and gluons, remarkably different from nuclei and hadrons – the building blocks of nuclear physics. QCD is a nonlinear quantum field theory, the so-called nonabelian gauge theory or Yang-Mills theory, and it is the quantum version of such theory which matters. QCD at low energies is dictated by the various nonperturbative aspects associated with the theory.

Perhaps because of our inability to deal with nuclear waste coming out of nuclear power plants or due to the horror picture that an atomic bomb has the potential to incur, some nuclear physicists have for some time tried to do away possible associations which the name "nuclear physics" may bring about and in some instances do not want to be called "nuclear physicists" any longer. Indeed, I think that there is some need to deal with the social impact or the animosity which a "poor" or "bad" name may have caused. Unfortunately or fortunately, the Sun will continue to shine on us for billions of years to come because the interior of the Sun is a gigantic nuclear power reactor, a reactor which is far from being free of the nuclear waste problem. Nuclear physics is the most central piece of knowledge if we can ever understand the Sun in some of its entirety. To say it in fair terms, a good portion of progresses in astrophysics depends on nuclear physics. On the other hand, quarks, leptons, and many other advanced objects of less animosity would not have been with us, should we have been deprived of the ability of using nuclei as a laboratory to produce or access them.

As of today, it remains almost impossible to solve problems related to hadrons or nuclei, except perhaps through lattice simulations of QCD but it is unlikely that, through the present-day computation power which is still not quite adequate, simulations would give us any good part of what we wish to know. Nevertheless, the need of solving QCD directly has gained tremendous importance over the last decade and, to this end, any fruitful attempt in confronting directly with QCD should be given due attention.

As I see it, the benefit to be gained from basing nuclear physics on QCD is really immense. First of all, it is a quantized nonlinear phenomenon which we are trying to grapple with. The lesson will help us to tackle many other nonlinear phenomena, either quantum or classical. I think that the modern physics era, with relativity and quantum principle as two pillars, is closing its glory chapter at the turn of the century – one of the new directions is, in my opinion, something which has to do with nonlinear phenomena. Second, it is "nuclear physics in its new form and new life" which we are trying to come up with. Any science that has life of some sort must, from time to time, be endowed with some drastically new elements, either experimental or purely conceptual. Finally, perhaps as a wishful thinking, deeper understanding of the subject may someday enable us to deal with the nuclear waste problem, or other problem of similar magnitude, in a technically efficient manner and the positive picture towards the term "nuclear physics" may thus be evolved from there.

In the rest of my talk (this paper), I wish to describe my recent research on QCD, especially in explaining how a closed set of coupled differential equations for nonlocal condensates in QCD (which characterize the nontrivial feature of the QCD ground state or vacuum) may be derived in the large N_c limit (with N_c the number of color). As a specific example, the leading-order equations for the nonlocal condensates appearing in the quark propagator are derived and explicit solutions are obtained. In choosing to do so, I am inclined more as a conscientious effort to avoid talking about QCD without specifics while paying little attention, perhaps I should have done so, to doing proper justice to what numerous other nuclear physicists (better than I am) have accomplished in the recent past. Well, at this 50th Anniversary Celebration of our department, I may have the uniquue excuse in doing what I am trying to do here.

II. Introduction to QCD

The ground state, or the vacuum, of QCD is known to be nontrivial, in the sense that there are non-zero condensates, including gluon condensates, quark condensates, and perhaps infinitely many higher-order condensates. In such a theory, propagators, i.e. causal Green's functions, such as the quark propagator

$$iS_{ij}^{ab}(x) \equiv < 0 \mid T(q_i^a(x)\bar{q}_j^b(0)) \mid 0 >, \tag{1}$$

carry all the difficulties inherent in the theory. Higher-order condensates, such as a four-quark condensate,

$$< 0 \mid T(\bar{\psi}(z)\gamma_\mu\psi(z)q_i^a(x)\bar{q}_j^b(0)) \mid 0 >,$$

with $\psi(z)$ also labeling a quark field, represent an infinite series of unknowns unless some useful ways for reduction can be obtained. As the vacuum, $\mid 0 >$, is highly nontrivial, there is little reason to expect that Wick's theorem (of factorization), as obtained for free quantum field theories, is still of validity. Thus, we must look for alternative methods in order to obtain useful results.

It is known that the equations for Green functions up to a certain order usually involve Green functions of even higher order, thereby making such hierarchy of equations often useless in practice. In what follows, however, we wish to show that, provided that we may use the large N_c approximation to treat condensates of much higher order, there is in fact a natural way of setting up closed sets of differential equations which govern the inter-related Green functions to a given order. We consider this as an important accomplishment, both because we can always go over to the next level of sophistication in order to improve the approximation and because the large N_c expansion has been shown to yield desirable results for describing hadron physics.

In light of the nontrivial QCD vacuum, we shall begin by considering the feasibility of working directly with the various matrix elements such as the quark propagator of Eq. (1). Useful relations may be derived if we regard the equations for interacting fields [1],

$$\left\{i\gamma^\mu\left(\partial_\mu + ig\frac{\lambda^a}{2}A_\mu^a\right) - m\right\}\psi = 0; \tag{2}$$

$$\partial^\nu G_{\mu\nu}^a - 2gf^{abc}G_{\mu\nu}^b A_\nu^c + g\bar{\psi}\frac{\lambda^a}{2}\gamma_\mu\psi = 0, \tag{3}$$

as the equations of motion for *quantized* interacting fields, subject to the standard rule for quantization that the equal-time (anti-)commutators among these quantized interating fields are identical to those among non-interacting quantized fields. As our basic example, we allow the operator $\{i\gamma^\mu\partial_\mu - m\}$ to act on the matrix element defined by Eq. (1) and obtain

$$\{i\gamma^\mu\partial_\mu - m\}_{ik} iS_{kj}^{ab}(x) = i\delta^4(x)\delta^{ab}\delta_{ij} + < 0 \mid T\left(\left\{g\frac{\lambda^n}{2}A_\mu^n\gamma^\mu q(x)\right\}_i^a \bar{q}_j^b(0)\right) \mid 0 > . \tag{4}$$

We should always keep in mind that the QCD vacuum $\mid 0 >$ is a nontrivial ground state which is in general not annihilated by operating on it the annihilation operators.

Eq. (4) can be solved by splitting the propagator into a singular, perturbative part and a nonperturbative part:

$$iS_{ij}^{ab}(x) = iS_{ij}^{(0)ab}(x) + i\tilde{S}_{ij}^{ab}(x), \tag{5}$$

where

$$iS_{ij}^{(0)ab}(x) \equiv \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} iS_{ij}^{(0)ab}(p), \tag{6a}$$

$$iS_{ij}^{(0)ab}(p) = \delta^{ab} \frac{i(\hat{p}+m)_{ij}}{p^2 - m^2 + i\epsilon}, \tag{6b}$$

with $\hat{a} \equiv \gamma^\mu a_\mu$ for a four-vector a_μ. The nonperturbative part then satisfies the equation:

$$\{i\gamma^\mu \partial_\mu - m\}_{ik} i\tilde{S}_{kj}^{ab}(x) = \left\langle 0 \mid T\left(\left\{g\frac{\lambda^n}{2} A_\mu^n \gamma^\mu q(x)\right\}_i^a \bar{q}_j^b(0)\right) \mid 0 \right\rangle. \tag{7}$$

This would be pretty much the end of the story unless we could find some way to proceed. We should note that Eq. (4) may also be derived by making use of, e.g., the path-integral formulation, and the issue of how to define renormalized composite operators, i.e. products of field operators, is by no means trivial (and fortunately we need not worry about such problem for the sake of this paper).

As a useful benchmark, we note that, with the fixed-point gauge,

$$A_\mu^n(x) = -\frac{1}{2} G_{\mu\nu}^n x^\nu + \cdots, \tag{8}$$

we may solve the nonperturbative part $i\tilde{S}_{ij}^{ab}(x)$ as a power series in x^μ,

$$\begin{aligned} i\tilde{S}_{ij}^{ab}(x) = & -\frac{1}{12}\delta^{ab}\delta_{ij} < \bar{q}q > + \frac{i}{48} m\hat{x}_{ij}\delta^{ab} < \bar{q}q > \\ & + \frac{1}{192} < \bar{q}g\sigma \cdot Gq > \delta^{ab} x^2 \delta_{ij} + \cdots. \end{aligned} \tag{9}$$

The first term is the integration constant which defines the so-called "quark condensate", while the mixed quark-gluon condensate appearing in the third term arises because of Eqs. (7) and (8). It is obvious that the series (9) is a short-distance expansion, which converges for small enough x_μ. We note that it is just the standard quark propagator cited in most papers in QCD sum rules [2].

The approach which we suggest here [3] is based upon two key elements, namely, the set of interacting field equations *plus* the rule of canonical quantization (for interacting fields). The equations which we obtain, such as Eq. (5), are much the same as the set of Schwinger-Dyson equations (for the matrix elements). An important aspect in our derivation is that the nontriviality of the vacuum $\mid 0 >$ is observed at every step – a central issue in relation to QCD.

As another important exercise, we may split the gluon propagator into the singular, perturbative part and the nonperturbative part. The nonperturbative part is given by [3]

$$g^2 < 0 \ |: G^n_{\mu\nu}(x) G^m_{\alpha\beta}(0) : | 0 >= \frac{\delta^{nm}}{96} < g^2 G^2 > (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha})$$
$$- \frac{\delta^{nm}}{192} < g^3 G^3 > \{ x^2 (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \qquad (10)$$
$$- g_{\mu\alpha} x_\nu x_\beta + g_{\mu\beta} x_\nu x_\alpha - g_{\nu\beta} x_\mu x_\alpha + g_{\nu\alpha} x_\mu x_\beta \}$$
$$+ O(x^4),$$

with

$$< g^2 G^2 > \equiv < 0 |: g^2 G^n_{\mu\nu}(0) G^{n\mu\nu}(0) : | 0 >, \qquad (11a)$$

$$< g^3 G^3 > \equiv < 0 \ |: g^3 f^{abc} g^{\mu\nu} G^a_{\mu\alpha}(0) G^{b\alpha\beta}(0) G^c_{\beta\nu}(0) : | 0 > . \qquad (11b)$$

Again, the first term in Eq. (10) is the integration constant for the differential equation satisfied by the gluon propagator. Note that inclusion of the gluon condensate $< g^2 G^2 >$ in Eq. (10) is standard but the triple gluon condensate $< g^3 G^3 >$ is a new entry required by the interacting field equation (3). Our approach indicates when condensates of entirely new types should be introduced as we try to perform operator-product expansions to higher dimensions.

III. Condensates in QCD

To give a pedagogical and specific example, we wish to focus on the quark propagator, as specified by Eqs. (1) and (4)-(7). We write

$$i \tilde{S}^{ab}_{ij}(x) = \delta^{ab} \{ \delta_{ij} f(x^2) + i \hat{x}_{ij} g(x^2) \}, \qquad (12)$$

with $x^2 \equiv x_0^2 - \mathbf{x}^2$. $f(x^2)$ and $g(x^2)$ are what we refer to as "nonlocal condensates" in connection with the quark propagator. We note that

$$<: \mathrm{Q(x)\ q(0)} :>= -12 f(x^2), \qquad (13)$$

which is the nonlocal quark condensate in the standard sense. To proceed further, we work only with the leading term in the fixed-point gauge and introduce

$$<: \{ g G_{\mu\nu} q(x) \}^a_i \bar{q}^b_j(0) :> = \delta^{ab} \{ (\gamma_\mu x_\nu - \gamma_\nu x_\mu) A(x^2)$$
$$+ i \sigma_{\mu\nu} B(x^2) \ \mathrm{t} \ (\gamma_\mu x_\nu - \gamma_\nu x_\mu) \hat{x} C(x^2) \ \mathrm{t} \ i \sigma_{\mu\nu} \hat{x} D(x^2) \}, \qquad (14)$$

with $G_{\mu\nu} \ \square \ (\lambda^n/2) G^n_{\mu\nu}$, an antisymmetric operator. The invariant functions $A(x^2), B(x^2)$, $C(x^2)$, and $D(x^2)$ are additional nonlocal condensates which we deal with explicitly in this paper.

Under the assumption that we keep only the leading term in the fixed-point gauge (a simplifying assumption which can be removed whenever necessary), we have

$$\{i\gamma^\alpha\partial_\alpha - m\}_{ik} <:\{gG_{\mu\nu}q(x)\}_k^a\bar{q}_j^b(0):>$$

$$= -\frac{1}{2}x^\beta <:\{g^2G_{\mu\nu}G_{\alpha\beta}\gamma^\alpha q(x)\}_i^a\bar{q}_j^b(0):> \tag{15a}$$

$$= -\frac{1}{2}\cdot\frac{1}{96}\cdot\frac{4}{3}\cdot <g^2G^2><:\{(\gamma_\mu x_\nu - \gamma_\nu x_\mu)q(x)\}_i^a\bar{q}_j^b(0):> \tag{15b}$$

$$= \frac{1}{144} <g^2G^2> \delta^{ab}(\gamma_\mu x_\nu - \gamma_\nu x_\mu)\{f(x^2) + i\hat{x}g(x^2)\}. \tag{15c}$$

Here the second line, (15a), follows from the field equation and the third line, (15b), is based on the large N_c approximation that the contribution in which $G_{\mu\nu}$ and $G_{\alpha\beta}$ do not couple to color-singlet is suppressed by a factor of $1/N_c^2$. Thus, the factorization in the present case is justified to order $1/N_c^2$, rather than just order l/N,. We also note that we have used Eq. (10) to leading order in obtaining Eq. (15b), but this approximation can be relaxed if necessary.

Now, we may use Eqs. (7) and (15c) and obtain a closed set of equations:

$$2f'(x^2) - mg(x^2) = -\frac{3}{2}i(B - x^2C), \tag{16a}$$

$$2x^2g'(x^2) + 4g(x^2) + mf(x^2) = \frac{3}{2}x^2(A - D), \tag{16b}$$

$$4iB' - 2iC - 2ix^2C' - mA = -\frac{1}{144} <g^2G^2> f(x^2), \tag{16c}$$

$$2iA + 2ix^2D' - mB = 0, \tag{16d}$$

$$-2iA' + 4iD' - mC = -\frac{i}{144} <g^2G^2> g(x^2), \tag{16e}$$

$$2iB' + 2iC - mD = 0, \tag{16f}$$

where the derivatives are with respect to the variable x^2.

Treating m as an expansion parameter,

$$F(x^2) = \sum_{k=0}^{\infty} m^k F_k(x^2), \tag{17}$$

we may solve the coupled equations, (16a)-(16f), order by order in m. To leading order in m, we obtain

$$x^2f_0''' + 3f_0'' - \xi_0^2x^2f_0' - 2\xi_0^2f_0 = 0, \tag{18}$$

$$(x^2)^3g_0''' + 5(x^2)^2g_0'' + \{2x^2 - \xi_0^2(x^2)^3\}g_0' - \{2 + 2\xi_0^2(x^2)^2\}g_0 = 0, \tag{19}$$

with $\xi_0^2 \equiv \langle g^2G^2 \rangle /384$. The equations for A_0, B_0, C_0, and D_0 can easily be solved once we obtain f_0 and g_0.

Eq. (19) can be simplified considerably by introducing

$$g_0(x^2) \equiv (x^2)^{-2}\tilde{g}_0(x^2), \tag{20a}$$

which leads to the equation:

$$x^2\tilde{g}_0''' - \tilde{g}_0'' - \xi_0^2 x^2 \tilde{g}_0' = 0. \tag{20b}$$

Eqs. (17) and (20) can be solved by iteration, leading to the result:

$$f_0(t) = a_0\left\{1 + \frac{1}{1\cdot 3}(\xi_0 t)^2 + \frac{1}{1\cdot 3^2\cdot 5}(\xi_0 t)^4 + \cdots\right\}$$

$$+a_1 t\left\{1 \; t \; \frac{1}{2\cdot 4}(\xi_0 t)^2 \quad \frac{1}{2\cdot 4^2\cdot 6}(\xi_0 t)^4 + \cdots\right\}, \tag{21}$$

$$\tilde{g}_0'(t) = c_2 t^2\left\{1 + \frac{1}{2\cdot 4}(\xi_0 t)^2 \; t \; \frac{1}{2\cdot 4^2\cdot 6}(\xi_0 t)^4 + \cdots\right\}, \tag{22}$$

with $t \equiv x^2$ and

$$a_0 = -\frac{1}{12} < \bar{q}q >, \qquad a_1 = \frac{1}{192} < \bar{q}g_c\sigma\cdot Gq >, \qquad c_2 = \frac{g_c^2 < \bar{q}q >^2}{2^5\cdot 3^4}. \tag{23}$$

Eq. (23) is obtained by comparing to the well-known series expansion for the quark propagator (see, e.g., [4]). Note that there are two integration constants, a_0 and a_1, for $f(x^2)$ but there is only one permissible constant for $g(x^2)$ (and c_2 is in fact a four-quark condensate taken in the large N_c limit).

For a number of applications, it is useful to obtain analytic expressions for $f_0(t)$ and $g_0(t)$. This turns out to be possible by way of Laplace transforms.

$$\bar{f}_0(s) \equiv \int_0^\infty ds e^{-st} f_0(t), \qquad \bar{\tilde{g}}_0'(s) \equiv \int_0^\infty ds e^{-st}\tilde{g}_0'(t). \tag{24}$$

We obtain

$$\bar{f}_0(s) = -\frac{2a_1}{\xi_0^2} - \frac{a_0}{\xi_0}\frac{s}{\sqrt{s^2-\xi_0^2}}\sec^{-1}\frac{s}{\xi_0} + \frac{\gamma_0 s}{\sqrt{s^2-\xi_0^2}}, \tag{25}$$

$$\bar{\tilde{g}}_0'(s) = \frac{2c_2}{(s^2-\xi_0^2)^{3/2}}, \tag{26}$$

with $\gamma_0 = 2a_1/\xi_0^2$. Looking up the table for Laplace transforms, we find

$$\tilde{g}_0' = \frac{2c_2}{\xi_0^2}\cdot \xi_0 t \cdot I_1(\xi_0 t), \tag{27}$$

$I_1(z)$ the modified Bessel function of the first kind, to order one. It is straightforward to show that Eq. (27) yields the series expansion in Eq. (22). Also, the function $I_1(\xi_0 t)$ enters the second series in $f_0(t)$ as in Eq. (21).

To close our presentation of the explicit solution to leading order in m, we note that Eqs. (16) yields

$$f_0'(t) = -i\tfrac{3}{4}(tB_0(t))', \quad (28a)$$
$$C_0(t) = -B_0'(t), \quad (28b)$$
$$tg_0'(t) + 2g_0(t) = -\frac{3}{4}t(tD_0(t))', \quad (28c)$$
$$A_0(t) = -tD_0'(t). \quad (28d)$$

Thus, the functions A_0, B_0, C_0, and D_0 can be solved explicitly once f_0 and go are known.

On the other hand, we may go beyond the leading order in m and obtain, as example,

$$tf_1''' + 3f_1'' - \xi_0^2 tf_1' - 2\xi_0^2 f_1 = \frac{1}{2}tg_0'' + \frac{3}{2}g_0' - \frac{3}{8}\{tA_0' + 2A_0 + t(tD_0)'' + 2(tD_0)'\}; \quad (29a)$$

$$t^3 g_1''' + 5tg_1'' + (2t - \xi_0^2 t^3)g_1' - (2 + 2\xi_0^2 t^2)g_1 = -\frac{1}{2}(t^2 f_0'' + tf_0' - f_0) + \frac{3t^2}{8}(-iB_0' + 2iC_0 + itC_0'). \quad (29b)$$

These equations can also be solved by Laplace transforms. In other words, the nonlocal condensate functions $f(x^2)$ and $g(x^2)$ can be analytically solved order by order in m.

IV. Discussion and summary

Much of the material which I have described so far has been presented elsewhere in a preprint [5], in which I have also discussed a couple of problems where application of our results is most relevant. In the context of the standard QCD sum rules (such as the Belyaev-Ioffe nucleon mass sum rules [6,4]), our analytical results on nonlocal condensates may not be very useful if the resultant series converges rapidly, and in general our analytical expressions may be used to perform further analytical analysis of the problems as a way to improve the results obtained via short-distance expansions (in x_μ). In the case when one considers the response of the QCD vacuum to some external fields, such as the method of QCD sum rules in the presence of an external axial field $Z_\mu(x)$ [7], our analytical expressions for nonlocal condensates help to determine the induced condensates previously treated as new parameters, thereby making the external-field QCD sum rule method more powerful than what it used to be.

The specific way of obtaining a closed set of coupled differential equations for the nonlocal condensates in relation to the quark propagator may be generalized to other condensates, such as those involved in the gluon propagator. Such approach may be very useful for understanding, in terms of QCD, a large number of problems in hadron physics or nuclear physics.

To sum up, I wish to re-iterate that problems in nuclear physics must now be synthesized starting from QCD and in return QCD is giving a new life to nuclear physics.

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