CHINESE JOURNAL OF PHYSICS VOL. 32, NO. 1 FEBRUARY 1994

Inclusion of Parton Transverse Momentum Effects in the Proton Structure Functions

Jiunn-Wei Chen ’ and W-Y. P. Hwang[1,2]

[1] *Department of Physics, National Taiwan University, Taipei, Taiwan 106, R.O.C.*

[2] *Institut für Kemphysik der Kernforschungsanlage Jülich, D-51 70 Jülich, F. R. G.*

(Received November 4, 1993)

We consider the possibility of incorporating the parton transverse momenta into the definition of parton distributions. In particular, we investigate a possible formulation by defining parton distributions as functions of the light-cone variables $x(\equiv p^+/P^+)$ and $\vec{p}_\perp$ and obtain modifications to the proton spin and spin-averaged structure functions. As a numerical example, it is seen that inclusion of parton transverse momentum effects may allow an increase of the value of the sum rule $\int dx g_1^{(0)}(x, Q^2)$ at $Q^2 = 10.7 GeV^2$ by 11.7% where $g_1^{(0)}(x, Q^2)$ refers to the structure function ***without*** transverse momentum effects ’ and is the one relevant for extracting the fractions of the proton spin carried by quarks or antiquarks. Accordingly, the statistical significance of the experimental implication on the amount of the polarized strange sea (or on other interpretations) is somewhat reduced.

PACS. 13.60.Hb – Total and inclusive cross sections (inc. deep-inelastic processes).
PACS. 12.38.Lg – Other nonperturbative calculations.
PACS. 13.88.+e – Polarization in interactions and scattering.

I. INTRODUCTION

Recently, an incredible amount of theoretical interest has been stirred up by the celebrated experiments [1,2] in measuring the proton spin structure function $g_1(x, Q^2)$. If we define

$$\Delta q = \int_0^1 dx \{q^\uparrow(x) - q^\downarrow(x) + \bar{q}^\uparrow(x) - \bar{q}^\downarrow(x)\}, \tag{1}$$

and use the naive parton model, we have [1], at $Q^2 = 10.7 GeV^2$,

$$\begin{aligned} \int_0^1 dx g_1(x,\ Q^2) &= \frac{1}{2}\left(\frac{4}{9}\Delta u \text{ t } \frac{1}{9}\Delta d \text{ t } \frac{1}{9}\Delta s\right) \\ &= 0.126\ \pm 0.010(stat.) \pm 0.015(syst.). \end{aligned} \tag{2}$$

On a different front, there has also been considerable theoretical interest concerning the significance of the finding [3] by the New Muon Collaboration (NMC) on the violation of the Gottfried sum rule (GSR) [4]. The GSR has to do with the integral:

$$S_G \equiv \int_0^1 \frac{dx}{x}\{F_2^{ep}(x) - F_2^{en}(x)\}, \tag{3}$$

which is, in the context of the quark parton model,

$$\begin{aligned} S_G &= \frac{1}{3}\int_0^1 dx\{u_p(x) - d_p(x) + \bar{u}_p(x) - \bar{d}_p(x)\} \\ &= \frac{1}{3} + \frac{2}{3}\int_0^1 dx\{\bar{u}_p(x) - \bar{d}_p(x)\}, \end{aligned} \tag{4}$$

where $u_p(x)$ is the u-quark distribution in the proton and similarly for the other quark distributions $d_p(x), \bar{u}_p(x)$, and $\bar{d}_p(x)$. In obtaining Eq. (4), one has used *isospin invariance* in relating the distributions in a neutron to those in a proton to obtain $u_n(x) = d_p(x)$, $d_n(x) = u_p(x)$, $s_n(x) = s_p(x)$, etc. The Gottfried sum rule [4] may then be derived by assuming *isospin independence* of the sea distributions in the proton:

$$\bar{u}_p(x) = \bar{d}_p(x), \tag{5}$$

so that $S_G = \frac{1}{3}$. Here we emphasize that *isospin invariance,* or *isospin symmetry,* is *not* violated even if Eq. (5) is *not* true, since as a member of an isospin doublet the proton already has different valence u and d quark distributions. Nevertheless, the finding [3] of the NMC group, at $< Q^2 >= 4\,GeV^2$, has aroused considerable theoretical interest:

$$\int_{0.004}^{0.8} \frac{dx}{x}\{F_2^{ep}(x) - F_2^{en}(x)\} = 0.230 \pm 0.013(stat.) \pm 0.027(syst.), \tag{5a}$$

primarily because it is against the standard practice of assuming the validity of Eq. (5) in extracting parton distributions from experiments. Using only the NMC F_2^n/F_2^p ratio and the world average fit to F_2 on deuterium, the NMC group has also obtained [5]:

$$\int_0^1 \frac{dx}{x}\{F_2^{ep}(x) - F_2^{en}(x)\} = 0.240 \pm 0.016, \tag{5b}$$

a slightly improved value as compared to Eq. (5a).

It is clear that the physics of great interest, concerning both the spin sum rule, Eq. (2), and the measured Gottfried sum, Eq. (5b), involves an implicit assumption that the definition of parton distributions is extendable in some way to very small x. With $x \equiv Q^2/(2m_p v)$ (i.e., the Bjorken x, to be distinguished from the light-cone x introduced later) and ν the energy transfer to the target nucleon, it is expected that, for the Q^2

relevant for the cited experiments, the transverse momentum of the very small-~ parton cannot be neglected in comparison with its longitudinal momentum (which defines x). To ensure that we can address the physics issues as related to these experiments, there is thus a need to quantify the uncertainty as caused by the neglect of the parton transverse momentum especially at very small x. At this juncture, it is worth mentioning that the x region kinematically accessible at the upcoming HERA (at DESY, Hamburg) could be as small as $(10^{-4}-10^{-5})$, again making it extremely important to address the question which we have posed.

The purpose of this paper is to consider the possibility of incorporating the parton transverse momenta into the definition of parton distributions. In particular, we wish to consider parton distributions defined as functions of x $\equiv p^+/P^+$ and pi, with p^+, P^+, and $\vec{p}_\perp$ the light-cone (or null-plane) variables, and assume that such notion is related to the parton distributions of Feynman (in the "infinite-momentum" frame) by a Lorentz transformation in a limiting sense. As a very useful example, we investigate the proton spin structure functions $g_1(x,Q^2)$ and $g_2(x,Q^2)$ to illustrate possible role played by inclusion of parton transverse momenta in the definition of parton distributions.

We would like to caution that the importance of taking into account the effects of parton transverse momenta in the naive parton model has already been recognized by many people, but with mixed blessings - some think that it might spoil the consistency of the parton model idea while others just put it in by hand on phenomenological considerations (e.g., by using Gaussian-type distributions in $\vec{p}_\perp$). In a recent work, Jackson, Ross, and Roberts [6] (JRR) pointed out some of the deep-rooted problems. Later in this paper, we shall touch upon these issues and we wish to caution our reader ahead of time that the line of thinking which we adopt in this work is less restricted than that used in the JRR paper.

II. ON A POSSIBLE FORMULATION

For a composite system (such as a nucleon) which is made of different constituents, we label the four-momentum of a specific constituent by $(\vec{p}_i, ip_{i0})$ and that of the entire system by $(\vec{P}, iP_0)$. We choose to work with the light-cone (or null-plane) dynamics and introduce

$$x \equiv \frac{p_i^+}{P^+} = \frac{p_{i0}+p_{i3}}{P_0+P_3}. \tag{6}$$

We shall then count the number of constituents (of a specific flavor) with a given x and $\vec{p}_\perp$ $[\equiv (p_{i1}, p_{i2})]$, leading to a possible definition of parton distributions as functions of x and $\vec{p}_\perp$. It is straightforward to show that the light-cone variables x and $\vec{p}_\perp$ are invariant under an arbitrary Lorentz transformation in the chosen z direction. In other words, different observers which move relative to one another with velocities in the z direction will use the same $(x, \vec{p}_\perp)$ to characterize the same constituent (parton) and they should be able to communicate with one another.

We may then introduce a set of "parton" distributions for all the observers who are related by Lorentz boosts in the z direction. The variables are $(x, \vec{p}_\perp)$ plus the indices to label flavor and other discrete degrees of freedom. One can then let $P_z \rightarrow oo$ and define the parton model of Feynman as the limit. The light-cone variable x approaches the Bjorken variable x in the limit, and under normal circumstances we may neglect $\vec{p}_\perp$ in comparison with xP_3. We are thus led to the first postulate for our formulation of parton distributions including transverse momentum effects.

Postulate I: *For a given hadron, it is possible to introduce "parton distributions" which are functions of the light-cone variable x of Eq. (6) and the parton transverse momentum* $\vec{p}_\perp$ *and which are invariant under Lorentz boosts in the z-direction.*

Note that, on the basis of this postulate, we do not necessarily assume the existence of "parton distributions" as Lorentz invariants, i.e. which are invariant under *any* Lorentz transformation. Nevertheless, it is already a severe assumption in the sense that the "parton distributions" for a hadron at rest already exist, since we may bring the system gradually from a large and finite velocity to at rest.

What we have said so far may not contain much new to an expert but it serves the purpose of characterizing the role of the parton transverse momentum in a set of suitably defined "parton distributions". Such parton distributions are invariant among observers who are related to one another by Lorentz boosts in the z direction. The direction z can of course be chosen at the convenience of simplifying a given problem - in this sense it is arbitrary.

Next, we consider as a definitive example the inclusion of transverse momentum effects in the derivation of the proton spin structure functions. We shall briefly sketch the derivation before presenting the final formulae on $g_1(x, Q")$ and $g_2(x, Q^2)$.

The differential cross section for the deep inelastic scattering $\vec{e}(\ell)+\vec{p}(P,S) \rightarrow e(\ell')+X$ is given by

$$d\sigma = \frac{d^3\ell'}{2\ell'_0(2\pi)^3} \frac{1}{((\ell \cdot P)^2 - m_\ell^2 M^2)^{1/2}} \frac{e^4}{q^4} L_{\mu\nu} W_{\mu\nu}, \tag{7}$$

where $L_{\mu\nu}$ $W_{\mu\nu}$ are specified by

$$L_{\mu\nu} = \delta_{\mu\nu}(\ell \cdot \ell' + m_\ell^2) - \ell_\mu \ell'_\nu - \ell_\nu \ell'_\mu + m_\ell \epsilon_{\mu\nu\rho\sigma}(\ell'_\rho s^\ell_\sigma - \ell_\rho s^\ell_\sigma), \tag{8}$$

$$\begin{aligned} W_{\mu\nu} = \; & W_1(q^2,\nu)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ & + \frac{1}{M^2} W_2(q^2,\nu)\left(P_\mu - \frac{(P\cdot q)q_\mu}{q^2}\right)\left(P_\nu - \frac{(P\cdot q)q_\nu}{q^2}\right) \\ & + M\epsilon_{\mu\nu\rho\sigma} q_\rho \{M^2 S_\sigma G_1(q^2,\nu) + ((P\cdot q)S_\sigma - (S\cdot q)P_\sigma)G_2(q^2,\nu)\}. \end{aligned} \tag{9}$$

Here we have adopted the notations $q \equiv \ell - \ell'$, $\nu \equiv -(P\cdot q)/M$, and $q^2 \equiv \vec{q}^{\,2} - q_0^2 = Q^2$.

The probability assumption underlying the quark parton model of Feynman yields

$$W_{\mu\nu} = \int d^2 p_\perp \frac{dx}{x} \sum_i f_i(x, \vec{p}_\perp, s) Q_i^2 \delta(2p_i\cdot q + q^2) W^i_{\mu\nu}, \tag{10}$$

where

$$W^i_{\mu\nu} = \delta_{\mu\nu} p_i\cdot q - 2p_{i\mu}p_{i\nu} - p_{i\mu}q_\nu - p_{i\nu}q_\mu + m_i \epsilon_{\mu\nu\rho\sigma} q_\rho s_{i\sigma}. \tag{11}$$

Eq. (10) may be viewed as a defining equation for the parton distribution $f_i(x, \vec{p}_\perp, s)$ with i and s specifying the flavor and spin of a given parton. (Note that $Q_i e$ is the electric charge carried by the parton.) By equating the spin-dependent parts in Eqs. (9) and (11) for two linearly independent configurations, we may express $G_1(q^2,\nu)$ and $G_2(q^2,\nu)$ in terms of parton distributions. First, we consider the configuration in which the proton is longitudinally polarized:

$$q_\mu = (0,0,q_3,iq_0), \quad P_\mu = (0,0,P_3,iE), \quad S_\mu = (0,0,E/M,iP_3/M). \tag{12}$$

Let $h_i^+(x,\vec{p}_\perp)$ denote the probability of finding the quark (antiquark) in the helicity state specified by

$$p_{i\mu} = (\vec{p}_\perp, p_{i3}, ip_{i0}), \quad s_{i\mu} = m_i^{-1}\left(\frac{p_{i0}}{|\vec{p}_i|}\vec{p}_\perp, \frac{p_{i0}}{|\vec{p}_i|}p_{i3}, i\,|\vec{p}_i|\right). \tag{13}$$

Note that $h_i^-(x,\vec{p}_\perp)$ is defined accordingly, i.e. $s_{i\mu} \to -s_{i\mu}$. Note also that the polarization 4-vector s_i is defined such that $s_i\cdot s_i = 1$ and s; $\cdot p_i$ = 0. Choosing $(\mu,\nu) = (1,2)$ in equating Eqs. (9) and (11), we find, with $m_i \to 0$ taken at the end,

$$\begin{aligned} M^2(M\nu G_1 + q^2 G_2) & = A \\ & \equiv \int d^2 p_\perp \frac{dx}{x} \sum_i \Delta h_i(x,\vec{p}_\perp) Q_i^2 \delta(2p_i . q + q^2)\left(\frac{q^2}{2} + p_\perp^2\right), \end{aligned} \tag{14}$$

with $\Delta h_i(x,\vec{p}_\perp) \equiv h_i^+(x,\vec{p}_\perp) - h_i^-(x,\vec{p}_\perp)$.

As suggested by Feynman [7], we may take the other independent configuration as the one in which the proton is transversally polarized:

$$q_\mu = (0,0,q_3,iq_0), \quad P_\mu = (0,0,P_3,iE), \quad S_\mu = (1,0,0,0). \tag{15}$$

The helicity distributions $k^{\pm}(x,\vec{p}_\perp)$ are defined in the same way as $h^{\pm}(x,\vec{p}_\perp)$. Choosing (μ,ν) to be $(2,4)$ or $(2,3)$ in equating Eqs. (9) and (11), we find

$$\begin{aligned} M^2(M^2G_1 - M\nu G_2) &= B \\ &\equiv \int d^2p_\perp \frac{dx}{x} \sum_i \Delta k_i(x,\vec{p}_\perp) Q_i^2 \delta(2p_i \cdot q + q^2) M p_{i1}. \end{aligned} \tag{16}$$

Eqs. (14) and (16) may be used to solve G_1 and G_2 but in terms of two unknowns Δh and Δk - it is not at all useful unless we can relate Δk to Δh in some way. To this end, we note that the integrand of Eq. (16) may be split into two factors: $d^4p_i\delta(p_i^2+m_i^2)\delta(2p_i\cdot q+q^2)$ and $\Delta k_i(p_{i1},p_{i2},p_{i3})Q_i^2 M p_{i1}$. The first factor is Lorentz invariant so that, if we insist that the overall result is Lorentz invariant, the second factor must also be Lorentz invariant. We are thus led to the second postulate for our formulation of parton distributions, which enables us to take into account spin-dependent effects.

Postulate II: *It is possible to rotate "parton distributions" of a nucleon polarized in the x direction to obtain parton distributions of a nucleon polarized in the z direction. Specifically, it is assumed that, after the integration d^4p_i is carried out, $\Delta k_i^{(0)}(p_{i1},p_{i2},p_{i3})Mp_{i1}$ is the same as $\Delta h_i^{(0)}(p_{i1},p_{i2},p_{i3})Mp_{i3}$ upon the rotation which brings a nucleon polarized in the x direction to the nucleon polarized in the z direction.*

Again, we note that, on the basis of our second postulate, we do not necessarily assume the existence of "parton distributions" as Lorentz invariants, i.e. which are invariant under *any* Lorentz transformation. Nevertheless, it is also a nontrivial assumption in the sense that the "parton distributions" for a nucleon polarized in the transversal direction are now assumed to exist and are related to those which are postulated to exist via Postulate I.

We may contrast the two postulates of ours with that of Jackson, Ross, and Roberts [6] (JRR), who propose adoption of a completely Lorentz-invariant parton distributions, i.e. invariant under *any* Lorentz transformation as contrast to our case of being invariant only under Lorentz boosts in the z direction. The parton distributions as in the original quark parton model of Feynman may exist *only* in a single hypothetical frame - the so-called "infinite-momentum frame". It is clearly a nontrivial assumption to assert that such distributions are in fact invariants of some sort - either in the spirit of our formulation or in the spirit of the JRR postulate [6].

Let us assume for the moment the JRR postulate [6]. The parton distributions must be functions of inner products among P, S, p_i, and s_i. Should the partons be on-shell, only two invariants $P \cdot p_i$ and $S \cdot p_i$ are linearly independent - leading to the conjectured form of the parton distributions:

$$f_i(x,\vec{p}_\perp) = \frac{p_i \cdot S}{M}\,\xi_i(P\cdot p_i). \tag{17}$$

The form follows from consideration of symmetry under parity and linearity in S. (Note that the function ξ is a function of $P\cdot p_i$ alone.) As a result, we obtain

$$\begin{aligned}\Delta h_i(x,\vec{p}_\perp) &= \frac{p_{i3}}{M}\,\xi_{ii}(P\cdot p_i),\\ \Delta k_i(x,\vec{p}_\perp) &= \frac{p_{i1}}{M}\,\xi_{ii}(P\cdot p_i).\end{aligned} \tag{18}$$

Note that Eqs. (18) lead to our second postulate. We therefore conclude that the JRR postulate [6] is more stringent than our formulation which is based on Postulates I and II. The converse is not true since our formulation does not necessarily imply the existence of Lorentz-covariant parton distributions.

We shall now return to our formulation. Note that, in front form in which $x\equiv p_i^+/P^+$ is used, Lorentz boosts in the z direction is kinematical, that is, no particles are created or destroyed (on top of the fact that x has an invariant meaning). Thus, we may perform a specific Lorentz boost in the z direction to bring the proton to the rest frame. The parton distributions $h_i^\pm(x,\vec{p}_\perp)$, as functions of the invariants x and pi, remain unchanged:

$$h_i^{(0)\pm}(\vec{p}_\perp,x) = h_i^\pm(\vec{p}_\perp,x). \tag{19}$$

In the rest frame, however, we have, with P the unitary operator characterizing the parity transformation,

$$\begin{aligned}h_i^{(0)+}(\vec{p}_\perp,p_{i3}) &\equiv |<\vec{p}_i,h_i=+\,|\,S_z^{(+)},\vec{P}=0>|^2\\ &=|<\vec{p}_i,h_i=+\,|\,P^\dagger P\,|\,S_z^{(+)},\vec{P}=0>|^2\\ &=|<-\vec{p}_i,h_i=-\,|\,S_z^{(+)},\vec{P}=0>|^2\\ &=h_i^{(0)-}(-\vec{p}_\perp,-p_{i3}).\end{aligned} \tag{20}$$

Using Eq. (20), we establish the important result that $\Delta h_i(\vec{p}_\perp,p_{i3})$, which is just $\Delta h_i^{(0)}(\vec{p}_\perp,p_{i3})$, must be an odd function as $\vec{p}_i\to-\vec{p}_i$. Nevertheless, azimuthal symmetry for the longitudinally polarized proton configuration implies that $\Delta h_i(\vec{p}_\perp,p_{i3})$ must be even under $\vec{p}_\perp\to-\vec{p}_\perp$ and so it must be odd under $p_{i3}\to-p_{i3}$ which can easily shown to be the same as $x\to p_\perp^2/(xM^2)$.

For the distributions $k_i^\pm(x,\vec{p}_\perp)$, we can again do a specific Lorentz de-boost to bring out $k_i^{(0)\pm}(x,\vec{p}_\perp)$, which are however related to $h_i^{(0)\pm}(x,\vec{p}_\perp)$ by a 90" rotation on the (xz) plane. Thus, we expect

$$k_i^\pm(p_{i1},p_{i2},p_{i3}) = h_i^\pm(-p_{i3},p_{i2},p_{i1}). \tag{21}$$

Postulate II serves as an additional constraint on $\Delta k_i (\equiv k_i^+ - k_i^-)$. Note that we have

$$p_{i0}^2 - p_{i3}^2 = m_i^2 + p_\perp^2, \tag{22}$$

so that, together with Eq. (6),

$$\begin{aligned} p_{i0} &= \frac{1}{2}\left(xP^+ + \frac{m_i^2 + p_\perp^2}{xP^+}\right), \\ p_{i3} &= \frac{1}{2}\left(xP^+ - \frac{m_i^2 + p_\perp^2}{xP^+}\right). \end{aligned} \tag{23}$$

To reinforce the S-function $\delta(2p_i \cdot q + q^2)$ in Eqs. (14) and (16), we obtain, to a sufficient approximation,

$$x = x_0\left(1 + \frac{m_i^2 + p_\perp^2}{q^2}\right), \tag{24}$$

with $x_0 \equiv -q^+/P^+$.

Noting that $p_{i3} = (x^2M^2 - p_\perp^2)/(2xM)$, we shall in fact replace Eq. (16) as follows:

$$\begin{aligned} M^2(M^2G_1 - M\nu G_2) &= B \\ &\equiv \int d^2p_\perp \frac{dx}{x} \sum_i \Delta h_i(x, \vec{p}_\perp) Q_i^2 \delta(2p_i \cdot q + q^2) \frac{x^2M^2 - p_\perp^2}{2x}. \end{aligned} \tag{25}$$

It is then straightforward to solve Eqs. (14) and (25):

$$\begin{aligned} g_1(x, Q^2) &\equiv M^3\nu G_1(q^2, \nu) \\ &= \frac{1}{2}\int d^2p_\perp dx \delta\left(x - x_0\left(1 + \frac{m_i^2 + p_\perp^2}{q^2}\right)\right) \\ &\quad \cdot \sum_i \Delta h_i(x, \vec{p}_\perp) Q_i^2 \left(1 - 2\frac{p_\perp^2}{q^2} - 2\frac{x^2M^2}{q^2}\right) \\ &= \left(1 - 2\frac{< p_\perp^2 >}{q^2} - 2\frac{x^2M^2}{q^2}\right) g_1^{(0)}(x, Q^2) \\ &\equiv (1 + \epsilon_1(x)) g_1^{(0)}(x, Q^2). \end{aligned} \tag{26}$$

$$\begin{aligned} g_2(x, Q^2) &\equiv -M^2\nu^2 G_2(q^2, \nu) \\ &= -\frac{1}{4}\int d^2p_\perp dx \delta\left(x - x_0\left(1 + \frac{m_i^2 + p_\perp^2}{q^2}\right)\right) \sum_i \Delta h_i(x, \vec{p}_\perp) Q_i^2 \\ &\quad \cdot \left(1 + 2\frac{p_\perp^2}{q^2} + \frac{p_\perp^2}{x^2M^2} - 4\frac{x^2M^2}{q^2}\right) \\ &= \left(1 + 2\frac{< p_\perp^2 >}{q^2} + \frac{< p_\perp^2 >}{x^2M^2} - 4\frac{x^2M^2}{q^2}\right) g_2^{(0)}(x, Q^2) \\ &\equiv (1 + \epsilon_2(x)) g_2^{(0)}(x, Q^2). \end{aligned} \tag{27}$$

Eqs. (26) and (27) are our central results for the proton spin structure functions. We shall use them to estimate the importance of incorporating parton transverse momentum effects into the definition of parton distributions. We note that our expression for $g_2(x,Q^2)$, Eq. (27), is of some interest in light of a recent review article by Jaffe [8], especially in connection with why there is lack of a sum rule for $\int_0^1 dx g_2(x,Q^2)$ upon inclusion of transverse momentum effects. The parton model as based on Postulates I and II may be regarded as a microscopic model of some general sort. It is possible that a microscopic model violates some general principle but, at the end, such violation gets restored in some way. (An example is that quark model calculations of electromagnetc form factors often violate gauge invariance but the problem can be remedied.) In our case, it remains to be of critical importance to further investigate [8] if there is indeed some sum rule which should not be violated. In any event, it appears that a parton model as based on Postulates I and II deserves much of future investigation.

It is relatively straightforward to obtain similar results on the proton spin-averaged structure functions. Skipping the derivation, we quote the final formulae, with $f_i(x,\vec{p}_\perp) \equiv h_i^+(x,\vec{p}_\perp) + h_i^-(x,\vec{p}_\perp)$,

$$\begin{aligned} F_1(x,Q^2) &\equiv -W_1(q^2,\nu) \\ &= \frac{1}{2}\int d^2p_\perp dx \sum_i f_i(x,\vec{p}_\perp)Q_i^2\delta\left(x - x_0\left(1+\frac{m_i^2+p_\perp^2}{q^2}\right)\right) \\ &\equiv F_1^{(0)}(x,Q^2), \end{aligned} \tag{28}$$

$$\begin{aligned} F_2(x,Q^2) &\equiv -\frac{\nu}{M}W_2(q^2,\nu) \\ &= \frac{q^2}{2M\nu}\int d^2p_\perp dx \sum_i f_i(x,\vec{p}_\perp)Q_i^2\delta\left(x - x_0\left(1+\frac{m_i^2+p_\perp^2}{q^2}\right)\right) \\ &\quad \cdot \left(1+\frac{q^2}{\nu^2}\right)^{-1}\left(1+4\frac{p_\perp^2}{q^2}\right) \\ &\equiv \left(1+\frac{q^2}{\nu^2}\right)^{-1}\left(1+4\frac{< p_\perp^2 >}{q^2}\right)F_2^{(0)}(x,Q^2). \end{aligned} \tag{29}$$

III. RESULTS AND DISCUSSION

Having established the role of parton transverse momenta in the proton structure functions, we may look into its numerical significance in some detail.

First, Eq. (29) indicates that the measured Gottfried sum S_G should be larger than that entering GSR by the factor $\beta \equiv 1 + 4 < p_\perp^2 > /Q^2$. In other words, the RHS of

(5a) or (5b) should be reduced further by $1/\beta$, making deviation of S_G from the GSR value of $1/3$ even larger than before. It is not clear what value of $< p_\perp^2 >$ should be adopted but a very sensible value of $(0.5\ \mathrm{GeV})^2$ already yields a correction of great numerical significance.

After having pointed out a relatively trivial result on the Gottfried sum, we wish to focus our attention primarily on the spin structure functions. Let us first note that it is the quantity $g_1(x,Q^2)$ which is measured experimentally but it is the quantity $g_1^{(0)}(x,Q^2)$ which enters the sum rule such as the first equality in Eq. (2) or, more precisely, the QCD-corrected and anomaly-corrected Ellis-Jaffe sum rule [9].

One possibility is to treat x and $p_\perp^2$ as independent variables. Terms of the form

$$x^\alpha - \left(\frac{p_\perp^2}{xM^2}\right)^\alpha$$

observe symmetry properties discussed earlier. In particular, we have used

$$\exp(-p_\perp^2/0.3)\left(x^{0.1} - \left(\frac{p_\perp^2}{xM^2}\right)^{0.1}\right) + C\exp(-p_\perp^2/0.2)\left(x^{0.0995} - \left(\frac{p_\perp^2}{xM^2}\right)^{0.0995}\right), \quad (30)$$

as a candidate choice of $\sum_i \Delta h_i(x,p_\perp^2)Q_i^2$ and obtain an excellent fit to the EMC data of $xg_1(x,Q^2)$. The result is displayed in Fig. 1. Note that x and Q^2 are correlated in the data, i.e. $Q^2 \approx 40\sqrt{x}\ \mathrm{GeV}^2$, which we take into account in the fitting. In Fig. 1, the quantity $xg_1(x,Q^2)$ is shown as a solid curve while $xg_1^{(0)}(x,Q^2)$ is depicted as a dashed curve. In Fig. 2, we display the difference between the two curves in a magnified scale, with the quantity $\epsilon_1(x)$ of Eq. (26) shown in percentage.

In the present case, the integrated value $\int dx g_1^{(0)}(x,Q^2)$ is larger than $\int dx g_1(x,Q^2)$ by 11.7%, i.e. it is closer to the Ellis-Jaffe sum rule by that amount. If we follow the analysis of the EMC group (the second paper in Ref. 1), we will find $\Delta s = -0.135 \pm 0.07$ instead of their $\Delta s = -0.19 \pm 0.07$, thereby making the finding of less statistical significance. This result alone is of great significance as far as the question of proton spin is concerned.

Nevertheless, we should mention that a different picture emerges if we adopt the hypothesis of Jackson, Ross, and Roberts [6], Eq. (17). We may write

$$y \equiv -\frac{P\cdot p_i}{M^2} = \frac{x^2M^2 + p_\perp^2}{2xM^2}. \quad (31)$$

We find that a choice of the form

$$\frac{1}{y}\exp(-y/0.19)\left(\frac{x^2M^2 - p_\perp^2}{2xM^2}\right)^{2.5} \quad (32)$$

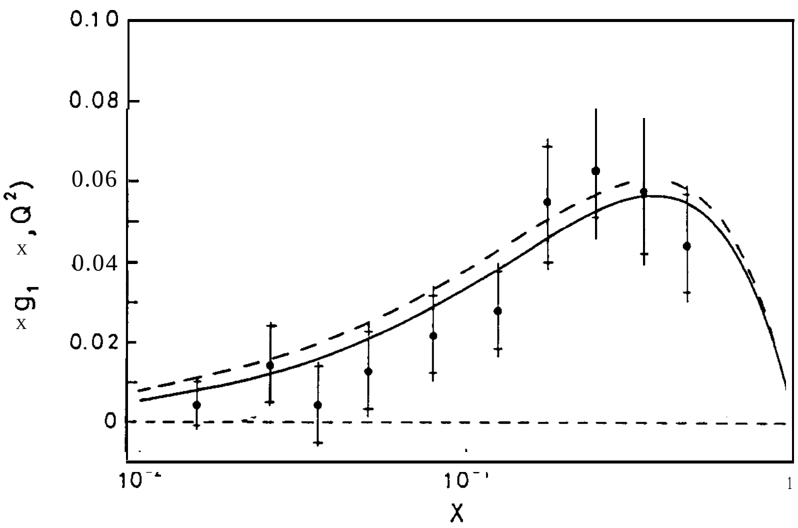

FIG. 1. The quantity $xg_1(x, Q^2)$ is shown as a solid curve while $xg_1^{(0)}(x,Q^2)$ is depicted as a dashed curve, when Eq. (30) is used as a candidate choice of $\sum_i \Delta h_i(x,p_\perp^2)Q_i^2$. The data points are from Ref. 1.

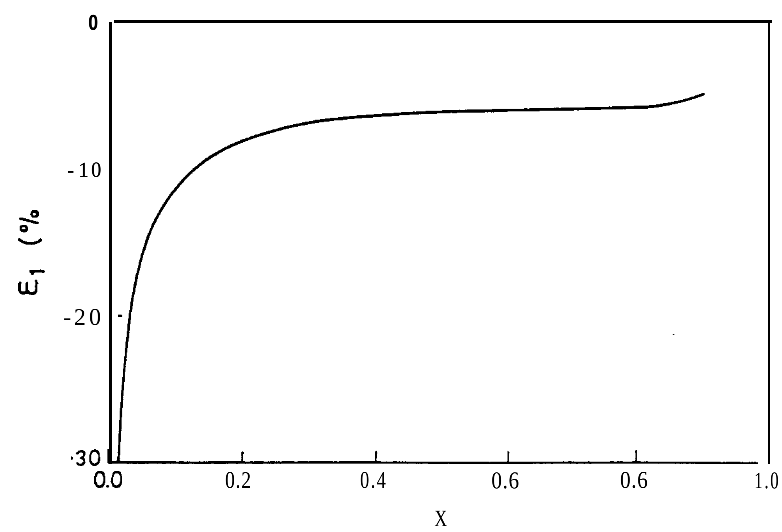

FIG. 2. The quantity $\epsilon_1(x)$ as defined by Eq. (26) is shown to exhibit the difference between $xg_1(x,Q^2)$ and $xg_1^{(0)}(x,Q^2)$ of Fig. 1, when Eq. (30) is used as a candidate choice of $\sum_i \Delta h_i(x,p_\perp^2)Q_i^2$.

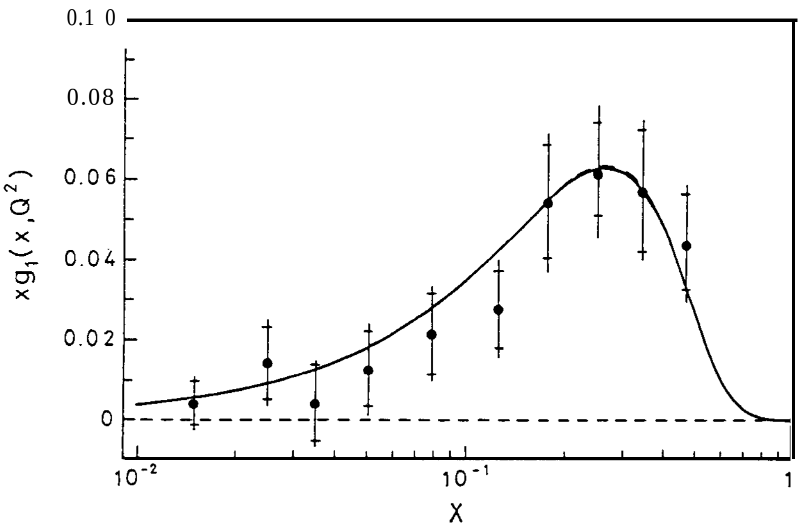

FIG. 3. The quantity $xg_1(x,Q^2)$ or $xg_1^{(0)}(x,Q^2)$ is shown (with the latter overlapping almost completely with the former one), when Eq. (32) is used as a candidate choice of $\sum_i \Delta h_i(x,p_\perp^2)Q_i^2$. The data points are from Ref. 1.

for $\sum_i \Delta h_i(x,p_\perp^2)Q_i^2$ leads to an excellent fit as illustrated by Fig. 3, where the curve for $xg_1(x,Q^2)$ does not differ visibly from the curve for $xg_1^{(0)}(x,Q^2)$. However, the smallness of $\epsilon_1(x)$ in this case can be attributed to the fact that using Eq. (17) one can easily show, as $x \rightarrow 0$, $< p_\perp^2 > \leq 2xM^2 < y >$. Or, Eq. (34) indicates that $< p_\perp^2 >$ must vanish with x as x $\rightarrow$ 0, an artifact of the fact that x and $p_\perp^2$ are not treated as independent variables. In comparing the two cases, we believe that the results shown in Figs. 1 and 2 represent what we may expect in the general case.

In summary, the numerical results which we have obtained using Eqs. (26)-(29) suggest the importance of taking into account effects of parton transverse momenta into the definition of parton distributions.

ACKNOWLEDGMENT

W-Y. P. Hwang wishes to acknowledge the Alexander von Humboldt Foundation for a fellowship to visit Jülich for conducting research. His research work was also supported in part by the National Science Council of the Republic of China under the grant NSC82-0208-M002-036.

REFERENCES

[1] J. Ashman et al., EMC Collaboration, Phys. Lett. B206, 364 (1988); Nucl. Phys. **B328**, 1 (1989).

[2] M. J. Alguard et al., Phys. Rev. Lett. 37, 1261 (1976); M. J. Alguard et al., ibid. 41, 70 (1978); G. Baum et al., Phys. Rev. Lett. 51, 1135 (1983).

[3] The NMC Group, in Proceedings of the Twenty-Fifth Rencontre de Moriond (Hadronic Interactions), March 1990 (to be published); R. Windmolders, The NMC Group, in Proceedings of the Twelfth International Conference on Particles and Nuclei (PANIC XII), Cambridge, Massachusetts, June 1990.

[4] K. Gottfried, Phys. Rev. Lett. 18, 1154 (1967).

[5] P. Amaudruz et *al.*, New Muon Collaboration, Phys. Rev. Lett. 66, 2712 (1991).

[6] J. D. Jackson, G. G. Ross, and R. G. Roberts, Phys. Lett. B226, 159 (1990).

[7] R. P. Feynman, Photon *Hadron Interactions* (W. A. Benjamin, New York, 1971).

[8] R. L. Jaffe, Comments Nucl. Part. Phys. 19,239 (1990); R. L. Jaffe and X. Ji, Phys. Rev. D43, 724 (1991).

[9] J. Ellis and R. L. Jaffe, Phys. Rev. **D9**, 1444 (1974); **D10**, 1669 (1974).