The Optical Conductivity of Gauge Field Model†

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The thermal Green's function method has been used to calculate the real part of the optical conductivity in the gauge-field model. The **gauge** invariant response functions were considered to the second order of the gauge field. We found that there is a structure in the frequecy range of a fraction of the Fermi energy. It is due to the excitation of particle-hole pair and a gauge field by a photon. In the small-frequency region, the optical conductivity is sensitive to temperature because of boson excitations. In the intermediate and high frequency region it is not sensitive because fermion excitations dominates.

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In the past few years the gauge field model has been studied extensively [1,2]. Lee and Nagoasa [1] showed that it can be derived from the t-J model using the slave-boson method. The temperature dependence of the resistivity can be explained by this model though there is problem in the Hall coefficient. The infrared reflection spectrum of the normal state of high temperature superconductors (HTCS) shows a non-Drude behavior [3] which may be an indication that these materials are not conventional Fermi liquid. Kim et al. [4] studied the optical conductivity of fermions interacting with gauge fields and found thit the gauge field has a profound effect on the optical conductivity. In our system there are fermions and bosons and the gauge field is related to both of them. However, the behavior of the gauge field is similar. Thus it is interesting to see if the gauge field model can produce a result which is in accord with expermental results. Our work is an attempt to clarify if it can give a correct description to HTSC. Since we are concerned with the normal state properties, we do the calculation at finite temperature.

In the continuum limit, the gauge field model has the Lagrangian density

$$L = f^*\partial_\tau f + b^*\partial_\tau b - \frac{1}{2m_f} f^* \nabla^2 f - \frac{1}{2m_b} b^* \nabla^2 b + \bar{L} \tag{1}$$

where

$$\bar{L} = -\vec{j}_f \cdot \vec{a} - \vec{j}_b \cdot (\vec{a} + \frac{e}{c}\vec{A}) + \frac{1}{2m_f}\rho_f \vec{a}^2 + \frac{1}{2m_b}\rho_b(\vec{a} + \frac{e}{c}\vec{A})^2, \tag{2}$$

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f and b are fermion and boson operators and $\vec{A}$ and $\vec{a}$ denote external and gauge field respectively. Here the Coulomb gauge had been chosen. We started with Eq. (1) and derived the form of the polarization function to the second order in gauge field.

The Fourier transform of the propagator of gauge field $< T[a_i^+(\tau)a_j(0)] >$ has the expression [1,3]

$$\begin{aligned} D(q,\omega) &= (\delta_{i,j} - \frac{q_i q_j}{q^2}) d(q,\omega) \\ &= (\delta_{i,j} - \frac{q_i q_j}{q^2}) \frac{1}{\Pi_f^o(q,\omega) + \Pi_b^o(q,\omega)} \end{aligned} \tag{3}$$

in the level of the random phase approximation. $\Pi^0_{f(b)}$ is the current-current polarization of free fermion (boson) shown in the Fig. 1. Since the real part of the optical conductivity is physically more interesting, we shall concentrate our attention to the imaginary part of polarization $\Pi(\Omega, Q \to 0)$, which is related to the real part of optical conductivity as

$$Re\sigma_{xx}(\Omega, Q \to 0) = \frac{e^2}{\Omega} Im\Pi_{xx}(\Omega, Q \to 0). \tag{4}$$

The photon momentum is very small compared to those of electrons so we shall consider only the polarization in the long wave-length limit. In the gauge field model the electron polarization function has the form:

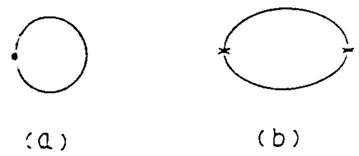
$$\Pi(\Omega) = \frac{\Pi_f(\Omega)\Pi_b(\Omega)}{\Pi_f(\Omega) + \Pi_b(\Omega)}. \tag{5}$$

Eq. (5) was obtained first by L. B. Ioffe and A. I. Larkian [3]. The current-current polarization of fermions and bosons have similar forms. The imaginary part of free fermion polarization, Π_f^0, vanishes in the frequency region $\Omega > v_f Q$, where v_f is the Fermi velocity, because energy and momentum conservations can not be simultaneously satisfied. (For finite temperature it is only approximately true.) For the same reason that of bosons vanishes when $\Omega > Q\sqrt{2m_b T}$. Therefore, we neglected the imaginary part of polarizations of both free fermions and free bosons. This is a good approximation in the temperature range of $100°K$. The imaginary part of polarization can be obtained from the next order diagrams shown in Fig. 2. The summation of diagrams of Fig. 2a, 2b, 2c, 2d, and 2e gives the imaginary part of polarization $\Pi^c_{f(b)}$ the in following expression (superscript c denotes the current polarization)

$$\begin{aligned} Im\Pi^c_{f(b),xx}(\Omega) &= \frac{1}{2\pi\Omega^2} \sum_q \frac{q_x q_x}{m^2_{f(b)}} \int_{-\infty}^{\infty} d\omega \left[\coth\left(\frac{\omega}{2T}\right) - \coth\left(\frac{\omega+\Omega}{2T}\right)\right] \\ &\quad \{Im[d^r(q,\omega)P_b^r(q,\omega)]Im[d^r(q,\omega+\Omega)P_f^r(\omega+\Omega)] \\ &\quad - Im[d^r(q,\omega)P_f^r(q,\omega)P_b^r(q,\omega)]Im[d^r(q,\omega+\Omega)]\}, \end{aligned} \tag{6}$$

where

$$P^r_{f(b)}(q,\omega) = \sum_p \left|\frac{p \times \hat{q}}{m_{f(b)}}\right|^2 \frac{n_{f(b)}(\epsilon_{p+q}) - n_{f(b)}(\epsilon_p)}{\omega + \epsilon_p - \epsilon_{p+q} + i\delta}, \tag{7}$$



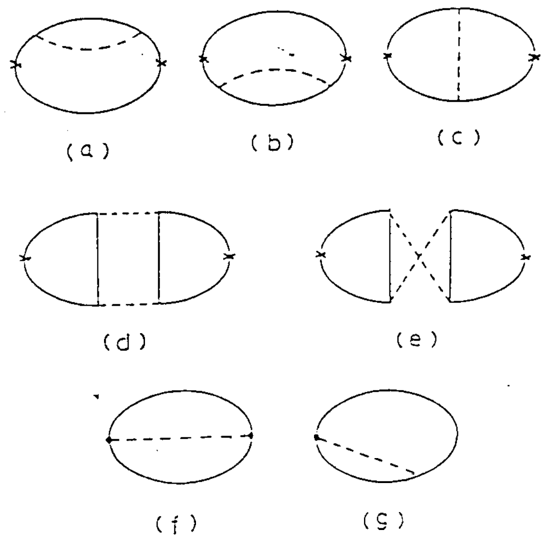


FIG. 1. The one loop diagrams for (a) diamagnetic diagram, (b) paramagnetic diagram. The cross and dot denote the current and density vertex respectively.

FIG. 2. The diagrams of the second order in gauge field.

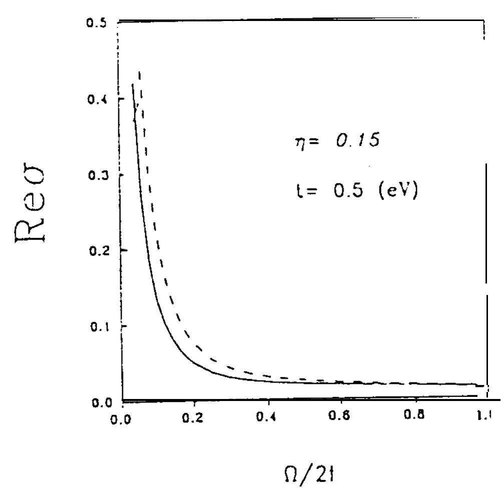


FIG. 3. The real part of σ as a function of frequency $\Omega/2t$ at (a) T = 130 °K (solid line) and (b) T = 200 °K (dashed line).

with ε being a postive infinitesimal value, and $\hat{q}$ a unit vector. $n_{f(b)}(\epsilon)$ is the Fermi (Bose-Eiensten) distribution function. The superscript r denotes the retarded function. The diagram of Fig. 2g vanishes due to the symmetry reason. The diagram of Fig. 2f with the density vertex (superscript n) has the form:

$$Im\Pi^{n}_{f(b),xx}(\Omega) = \frac{1}{2\pi m^2_{f(b)}}\sum_q \int_{-\infty}^{\infty} d\omega \left[\coth(\frac{\omega}{2T}) - \coth(\frac{\omega+\Omega}{2T})\right] Imd^r(q,\omega) Imp^r_{f(b)}(q,\omega,\Omega) \tag{8}$$

where

$$p_{f(b)} = \sum_p \frac{n_{f(b)}(\epsilon_{p+q}) - n_{f(b)}(\epsilon_p)}{\omega + \Omega + \epsilon_p - \epsilon_{p+q} + i\delta}. \tag{9}$$

The presence of the boson polarizations always makes the contirburion small because the bosons have a much smaller energy scale, T and momentum scale, $\sqrt{2m_bT}$ than fermions have. Therefore, only in the small frequency region ($\Omega \ll e_F$) does $Im\Pi^c_{f(b)}(\Omega)$ have significant contribution. The fermion part of $Im\Pi^n(\Omega)$ has the dominant contribution to the conductivity in the intermediate and high frequncg region.

The real part of the optical conductivity using the Eq. (5) is plotted in Fig. 3 under the condition $1/2m_ba^2$ = t = 0.5 (eV), $1/2m_fa^2 = J = 0.1eV$, (a is the lattice constant, t is the hopping energy and J is the exchange energyj and the concentration of dopping $\eta =$ 0.15. The solid line and the dashed line correspond to temperture $T = 150°K$ and T $= 200°K$ respectively. The result of Fig. 3 showed a sophisticated behavior of spectrum of optical conductivity. In the small-frequency region, the optical conductivity is sensitive to temperature because of boson excitations. The dominant contributution comes from $Im\Pi^c_{f(b)}(\Omega)$. In the intermediate and high frequency region it is not sensitive because fermion excitations dominates. The entire spectrum can be viewed as a curve proportional to $1/\Omega$ plus a structure given by $Im\Pi^n_{f(b)}(\Omega)$in the freqency range of a fraction of the Fermi energy. The latter is due to the excitation of a particle-hole pair and a gauge-field by a photon. Most HTSC material did show a non-Drude behavior in the infrared spectrum [3] and experimental data give σ a $\Omega^{-\alpha}$ where α is slightly greater than 1 while in the Drude model a = 2. Therefore the gauge field model is a candidate for HTSC.

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