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Polarization Correlations of Radiation from Electron-Impact Excited Atoms

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A kinematic analysis of radiation from atoms or ions after electron-impact excitation is carried out in a relativistic framework. The polarization state of the radiation is related to that of the incident electron by considering rotation and parity symmetries. Kinematic formulas for angular distribution and polarization of the radiation are presented in terms of angle-independent dynamic parameters. These dynamic parameters are given as linear sums of reduced matrix elements suitable for numerical computation. All electromagnetic multipole amplitudes are included in the calculation. Physical interpretations of the five independent dynamic parameters in the electric dipole approximation are given.

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I. INTRODUCTION

Studies of correlations in electron-atom collisions can provide much valuable information about the collision dynamics. Using polarized electron beam as a probe, we can investigate atomic structures by studying the angular distribution and polarization of the scattered electron or the emitted photon. An excellent discussion of the scattering of polarized electrons has been given [1] and the spin effects in inelastic electron-atom collisions have been reviewed [2]. For heavy atoms or ions, relativistic effects are important and should be taken into account. A relativistic theory of electron-atom scatterings has been reported for polarized incident electrons on unpolarized target atoms [3-5].

More detailed information is available by measuring the scattered electron in coincidence with the emitted photon. A review of electron-photon angular correlations has been reported [6]. A theory of electron-photon coincidence with polarized electrons was given [7]. Electron-impact excitations of mercury [8,9] and magnesium [10] have been studied by electron-photon coincidence techniques. Light polarizations in mercury [11,12] and in cesium [13] after impact excitation by polarized electrons have been measured.

In this paper, we will treat the radiative de-excitation process [14,15] of the target atom or ion after electron-impact excitation. It is useful to separate the kinematics and dynamics of the collision processes in the application of various dynamic theories. A kinematic analysis of radiation from atoms or ions after electron-impact excitation is carried out in a relativistic framework. The procedure is similar to that for the treatment of radiation after photoexcitation [14]. A formal scattering theory is given in Sec. II in terms of density matrices and a symmetry analysis is carried out in Sec. III. In Sec. IV, general formulas for angular distribution and polarization of the radiation are presented in terms of angle-independent dynamic parameters. The electric dipole transition is discussed in Sec. V as a special case. Finally we make a conclusion in Sec. VI.

II. FORMAL SCATTERING THEORY

We shall consider the angular distribution and polarization of radiation emitted by an atom or ion after excitation by a polarized electron. The excitation and subsequent radiative de-excitation are treated as two independent processes. We consider first the electron-impact excitation in Sec. II-1 and the radiative de-excitation in Sec. 11-2.

II-l. Electron-impact excitation

The relativistic wave equation for the composite system of an incident electron and an N-electron target is assumed to be

$$H_{N+1}|\Psi>= E|\Psi>, \tag{2.1}$$

where the $(N+1)$-electron Hamiltonian is given in atomic units by

$$H_{N+1} = H_N^T + H_1^e + V. \tag{2.2}$$

Here the Hamiltonian of the target atom or ion is

$$H_N^T = \sum_{i=1}^{N}(-ic\vec{\alpha}_i \cdot \vec{\nabla}_i + c^2\beta_i - Z/r_i) + \sum_{j>i=1}^{N} 1/r_{ij}, \tag{2.3}$$

and the Hamiltonian of the incident electron in the long-range Coulomb potential of the target is

$$H_1^e = -ic\vec{\alpha}_0 \cdot \vec{\nabla}_0 + c^2\beta_0 - (Z - N)/r_0. \tag{2.4}$$

The interaction Hamiltonian is given by

$$V = -N/r_0 + \sum_{j=1}^{N} 1/r_{0j}, \tag{2.5}$$

which is a short-range potential. In the the above equations, c is the speed of light, Z is the charge of the nucleus, and $\vec{\alpha}_i$ and β_i denote the Dirac matrices of electron i.

Now we consider a collision process of the type

$$e^- + A(o) \rightarrow e^- + A(\alpha), \tag{2.6}$$

in which an electron with linear momentum $\vec{k}_i$ and helicity μ_i is incident upon a target atom or ion A in the eigenstate with angular momentum J_0 and energy We. After the collision, a scattered electron with linear momentum $\vec{k}_\alpha$ emerges, and the target is left in the eigenstate with angular momentum J_α and energy W_α. From energy conservation we have

$$E = E_i + W_0 = E_\alpha + W_\alpha, \tag{2.7}$$

where E_i and E_α are the energies of the incident and scattered electrons, respectively.

The wave function of an eigenstate of the total Hamiltonian is given asymptotically by

$$|\Psi_i^+ > \underset{r\rightarrow\infty}{\longrightarrow} |\phi_i^+ > |J_0 M_0 > + \sum_\alpha \frac{e^{iy_\alpha \ln 2k_\alpha r}}{r} |\hat{r} k_\alpha \mu_\alpha > |J_\alpha M_\alpha > f_{\alpha i}(\hat{r}; \hat{k}_i), \tag{2.8}$$

where $y_\alpha = (Z - N)E_\alpha/(c^2 k_\alpha), r = |\vec{r}_0|$, and $\hat{r} = \vec{r}_0/|\vec{r}_0|$. Here the subscript i in $|\Psi_i^+ >$ and $|\phi_i^+ >$ refers to the initial state, and the superscript "+" indicates that they satisfy the outgoing-wave boundary condition. The second term on the right-hand side of (2.8) contains contributions at infinity from all atomic states corresponding to open channels, for which $E_\alpha \geq c^2$. The logarithmic phase factors are included to take care of the distortion by the Coulomb field of the target ion and vanish for a neutral target atom. The scattering amplitude for the inelastic collision is given in post form by

$$f_{\alpha i}(\hat{k}_\alpha; \hat{k}_i) = -\frac{4\pi^2 E_\alpha}{c^2} < \phi_\alpha^- J_\alpha M_\alpha |V| \Psi_i^+ >, \tag{2.9}$$

where the superscript "-" indicates the incoming-wave boundary condition

$$|\phi_\alpha^- > \underset{r\rightarrow\infty}{\longrightarrow} e^{iy_\alpha \ln(k_\alpha r - \vec{k}_\alpha \cdot \vec{r})} |\vec{k}_\alpha \mu_\alpha > + \sum_{\mu'_\alpha} \frac{e^{-iy_\alpha \ln 2k_\alpha r}}{r} | - \hat{r} k_\alpha \mu'_\alpha > f^{c^*}_{\mu_\alpha \mu'_\alpha}(\hat{k}_\alpha; -\hat{r}), \tag{2.10}$$

where $f^c_{\mu_\alpha \mu'_\alpha}(\hat{k}_\alpha; \hat{k}'_\alpha)$ is the Coulomb scattering amplitude and contributes for elastic scatterings only. The differential cross section for the scattering is given by

$$\frac{d\sigma_{\alpha i}}{d\Omega_\alpha} = \frac{v_\alpha}{v_i} |f_{\alpha_i}(\hat{k}_\alpha; \hat{k}_i)|^2, \tag{2.11}$$

where $v_i = c^2 k_i / E_i$ is the velocity of the incident electron, and v_α that of the scattered electron. The total cross section is obtained by integrating over all scattering angles.

We now consider the collision process in which a polarized electron with density matrix $\rho_{\mu'_i \mu_i}$ impinges on an unpolarized target atom which has initially a well-defined angular momentum J_0 and is excited to states of angular momentum J_α. If the angular distribution and polarization of the scattered electron are not observed, the density matrices of the incident electron and the excited target are related by

$$\rho_{J'_\alpha M'_\alpha J_\alpha M_\alpha} = \sum_{\mu'_i \mu_i} I(J'_\alpha M'_\alpha J_\alpha M_\alpha; \vec{k}_i \mu'_i \mu_i) \rho_{\mu'_i \mu_i}, \tag{2.12}$$

where the interaction matrix is defined as

$$I(J'_\alpha M'_\alpha J_\alpha M_\alpha; \vec{k}_i \mu'_i \mu_i) = \int d\hat{k}_\alpha \sum_{\mu_\alpha M_0} \frac{1}{2J_0 + 1} f(\vec{k}_\alpha \mu_\alpha J'_\alpha M'_\alpha; \vec{k}_i \mu'_i J_0 M_0) f^*(\vec{k}_\alpha \mu_\alpha J_\alpha M_\alpha; \vec{k}_i \mu_i J_0 M_0). \tag{2.13}$$

Here we have defined the transition amplitude

$$f(\vec{k}_\alpha \mu_\alpha J_\alpha M_\alpha; \vec{k}_i \mu_i J_0 M_0) = (v_\alpha / v_i)^{1/2} f_{\alpha i}(\hat{k}_\alpha; \hat{k}_i). \tag{2.14}$$

It is convenient to describe a polarized atom by state multipoles defined as

$$Q_{lm}(J'J) = \sum_{M'M} [l] \begin{pmatrix} J' & m & M \\ M' & l & J \end{pmatrix} \rho_{J'M'JM}, \tag{2.15}$$

where $[l] = (2l + l)^{1/2}$, and we have made use of the $3 - jm$ coefficients [16,17]. In terms of state multipoles, we may rewrite (2.12) as

$$Q_{lm}(J'_\alpha J_\alpha) = \sum_{\mu'_i \mu_i} I_{lm}(J'_\alpha J_\alpha; \mu'_i \mu_i) \rho_{\mu'_i \mu_i}, \tag{2.16}$$

where we have introduced the coupled interaction matrix

$$I_{lm}(J'_\alpha J_\alpha; \mu'_i \mu_i) = \sum_{M'_\alpha M_\alpha} [l] \begin{pmatrix} J'_\alpha & m & M_\alpha \\ M'_\alpha & l & J_\alpha \end{pmatrix} I(J'_\alpha M'_\alpha J_\alpha M_\alpha; \vec{k}_i \mu'_i \mu_i). \tag{2.17}$$

11-2. Radiative de-excitation

The excited atom will de-excite by radiative or Auger transitions. Here we are interested in the radiative de-excitation

$$A(\alpha) \rightarrow A(a) + 7, \tag{2.18}$$

in which an atom or ion A is initially in the excited state $|J_\alpha M_\alpha >$ and makes the transition to a lower-energy state $|J_a M_a>$ by radiating one photon 7. The angular distribution and polarization of the emitted photon can be measured. The transition amplitude of the spontaneous photon emission at time t after the excitation can be written in the Coulomb gauge as

$$f(\vec{k}q J_a M_a; J_\alpha M_\alpha)$$
$$= -\sqrt{4\pi^2 \alpha \omega} < J_a M_a \left| \sum_{n=1}^{N} \vec{\alpha}_n \cdot \vec{A}^*_{\vec{k}q}(\vec{r}_n) \right| J_\alpha M_\alpha > e^{-i\varepsilon_\alpha t - \gamma_\alpha t/2}, \tag{2.19}$$

where w $= kc$ is the angular frequency of the emitted photon, N is the total number of electrons in the target atom, $\vec{\alpha}_n$ denote the Dirac matrices of the n-th electron, $\vec{A}_{\vec{k}q}$ denotes the linear helicity state [14] of the vector potential, ε_α and γ_α are the energy eigenvalue and decay constant, respectively, of the excited atomic state $|J_\alpha M_\alpha >$. The time-dependent exponential factor characterizes the time evolution of the excited state $|J_\alpha M_\alpha >$. Because we start from a relativistic formulation, all fine structures of the atomic spectrum are built in from the outset. If the excited atomic ensemble has a sufficiently long lifetime such that the electrons can couple with the nucleus through hyperfine interactions, the coupled electronic and nuclear state would be specified by $|(J_\alpha I)FM >$ in place of $|J_\alpha M_\alpha>$, where F is the total angular momentum of the combined electronic and nuclear system. If the de-excited target has a well-defined angular momentum J_a, the density matrix of the emitted photon is then related to that of the excited target by the relation

$$\rho_{q'q} = \sum_{lm} \sum_{J'_\alpha J_\alpha} I_{lm}(q'q; J'_\alpha J_\alpha) Q_{lm}(J'_\alpha J_\alpha), \tag{2.20}$$

where the coupled interaction matrix of the radiative de-excitation is defined as

$$I_{lm}(q'q; J'_\alpha J_\alpha)$$
$$= \sum_{M_a} \sum_{M'_\alpha M_\alpha} [l] \begin{pmatrix} J'_\alpha & m & M_\alpha \\ M'_\alpha & l & J_\alpha \end{pmatrix} f(\vec{k}q' J_a M_a; J'_\alpha M'_\alpha) f^*(\vec{k}q J_a M_a; J_\alpha M_\alpha). \tag{2.21}$$

III. SYMMETRY ANALYSIS

Assume all interactions involved during the collision are invariant under rotations and space inversion. This guarantees that the interaction matrix is invariant under rotations and mirror reflections. The incident direction k_i of the electron and the emitting direction $\hat{k}$ of the photon span the scattering plane, and the angle between them is θ. It suffices to define two coordinate frames (xyz) and $(x'y'z')$ as shown in Fig. 1, where the two z-axes are in the $\hat{k}$ and $\hat{k}_i$ directions, respectively, and their common y-axis is normal to the scattering plane. In the $(x'y'z')$ frame, (2.16) is written explicitly as

$$Q_{lM}(J'_\alpha J_\alpha;\hat{z}') = \sum_{\mu'_i\mu_i} I_{lM}(J'_\alpha J_\alpha;\mu'_i\mu_i)\rho_{\mu'_i\mu_i}, \tag{3.1}$$

where $\hat{z}'$ indicates the direction of the quantization axis. In the (xyz) frame, (2.20) is written as

$$\rho_{q'q} = \sum_{lm}\sum_{J'_\alpha J_\alpha} I_{lm}(q'q;J'_\alpha J_\alpha)Q_{lm}(J'_\alpha J_\alpha;\hat{z}). \tag{3.2}$$

III-l. Rotation symmetry

Under a rotation through angle φ about the z'-axis,

$$\rho_{\mu'_i\mu_i} \to e^{-i(\mu'_i-\mu_i)\varphi}\rho_{\mu'_i\mu_i}, \tag{3.3}$$

$$Q_{lM}(J'_\alpha J_\alpha;\hat{z}') \to e^{-iM\varphi}Q_{lM}(J'_\alpha J_\alpha;\hat{z}'). \tag{3.4}$$

So for interactions invariant under rotations about the Y-axis, we have

$$I_{lM}(J'_\alpha J_\alpha;\mu'_i\mu_i) = \delta_{M(\mu'_i-\mu_i)}I_{l(\mu'_i-\mu_i)}(J'_\alpha J_\alpha;\mu'_i\mu_i). \tag{3.5}$$

Consequently, it follows

$$Q_{l0}(J'_\alpha J_\alpha;\hat{z}') = I_{l0}\left(J'_\alpha J_\alpha;\frac{1}{2}\frac{1}{2}\right)\rho_{\frac{1}{2}\frac{1}{2}} + I_{l0}\left(J'_\alpha J_\alpha;-\frac{1}{2}-\frac{1}{2}\right)\rho_{-\frac{1}{2}-\frac{1}{2}}, \tag{3.6}$$

$$Q_{l\pm1}(J'_\alpha J_\alpha;\hat{z}') = I_{l\pm1}\left(J'_\alpha J_\alpha;\pm\frac{1}{2}\mp\frac{1}{2}\right)\rho_{\pm\frac{1}{2}\mp\frac{1}{2}}. \tag{3.7}$$

Similarly, we have

$$I_{lm}(q'q;J'_\alpha J_\alpha) = \delta_{m(q'-q)}I_{l(q'-q)}(q'q;J'_\alpha J_\alpha), \tag{3.8}$$

and

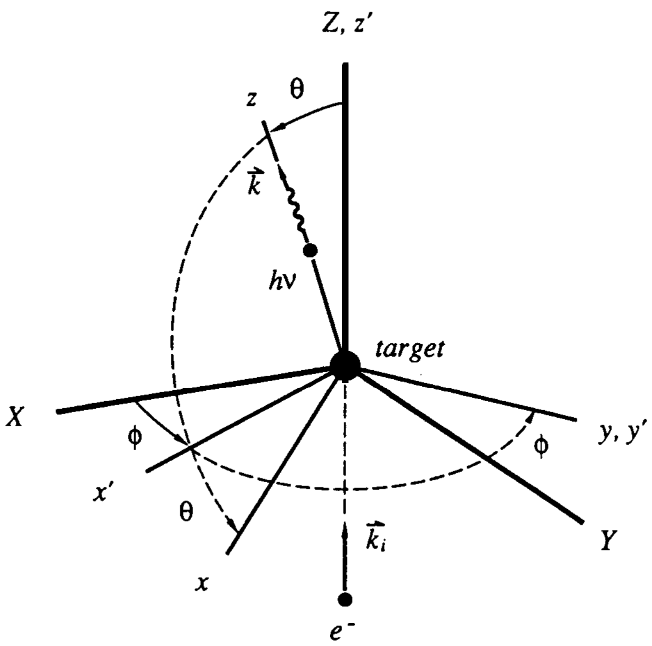

FIG. 1. Geometric relationships between the three coordinate frames $(xyz),(x'y'z')$, and (XYZ).

$$\rho_{\pm1\pm1} = \sum_{l}\sum_{J'_\alpha J_\alpha} l_{l0}(\pm1\pm1; J'_\alpha J_\alpha) Q_{l0}(J'_\alpha J_\alpha;\hat{z}), \tag{3.9}$$

$$\rho_{\pm1\mp1} = \sum_{l}\sum_{J'_\alpha J_\alpha} l_{l\pm2}(\pm1\mp1; J'_\alpha J_\alpha) Q_{l\pm2}(J'_\alpha J_\alpha;\hat{z}). \tag{3.10}$$

The state multipoles in the two frames are related by

$$\begin{aligned} Q_{lm}(J'_\alpha J_\alpha;\hat{z}) = &\sum_{M\geq0}\left(1-\frac{1}{2}\delta_{M0}\right)[Q_{lM}(J'_\alpha J_\alpha;\hat{z}')d^l_{Mm}(\theta) \\ &+(-)^{M+m}Q_{l-M}(J'_\alpha J_\alpha;\hat{z}')d^l_{M-m}(\theta)]. \end{aligned} \tag{3.11}$$

111-2. Rotation and space inversion symmetry

Under the mirror reflection with respect to the $x'z'$-plane, we have

$$|\vec{k}_i\mu_i>_{\hat{z}'}\rightarrow(-)^{\frac{1}{2}-\mu_i}|\vec{k}_i-\mu_i>_{\hat{z}'}, \tag{3.12}$$

$$|J_\alpha M_\alpha>_{\hat{z}'}\rightarrow\pi_\alpha(-)^{J_\alpha-M_\alpha}|J_\alpha-M_\alpha>_{\hat{z}'}, \tag{3.13}$$

where the notation π_α denotes the parity of the atomic state $|J_\alpha M_\alpha>$. *So* for interactions invariant under rotations and space inversion, it can be shown

$$I_{l(\mu_i'-\mu_i)}(J_\alpha' J_\alpha ; \mu_i'\mu_i) = \pi_{\alpha'}\pi_\alpha(-)^l I_{l-(\mu_i'-\mu_i)}(J_\alpha' J_\alpha ; -\mu_i' - \mu_i). \tag{3.14}$$

By adopting the normalization condition

$$\rho_{\frac{1}{2}\frac{1}{2}} + \rho_{-\frac{1}{2}-\frac{1}{2}} = 1, \tag{3.15}$$

we can write the three components of the polarization vector P of the incident electron in the $(x'y'z')$ frame as

$$P_{x'} = \rho_{\frac{1}{2}-\frac{1}{2}} + \rho_{-\frac{1}{2}\frac{1}{2}}, \tag{3.16}$$

$$P_{y'} = i(\rho_{\frac{1}{2}-\frac{1}{2}} - \rho_{-\frac{1}{2}\frac{1}{2}}), \tag{3.17}$$

$$P_{z'} = \rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2}-\frac{1}{2}}. \tag{3.18}$$

Define the spherical components of the polarization vector $\vec{P}$ by

$$P'_{\pm 1} = \mp\frac{1}{\sqrt{2}}(P_{x'} \pm iP_{y'}), \tag{3.19}$$

$$P'_0 = P_{z'}. \tag{3.20}$$

We have therefore

$$P'_{\pm 1} = \mp\sqrt{2}\rho_{\mp\frac{1}{2}\pm\frac{1}{2}}, \tag{3.21}$$

$$P'_0 = \rho_{\frac{1}{2}\frac{1}{2}} - \rho_{-\frac{1}{2}-\frac{1}{2}}. \tag{3.22}$$

Substituting (3.15), (3.21), and (3.22) into (3.6) and (3.7) and making use of (3.14), we obtain

$$Q_{l0}(J_\alpha' J_\alpha ; \hat{z}') = q_{l0}(J_\alpha' J_\alpha ; \hat{z}')(\pi_+ + \pi_- P'_0), \tag{3.23}$$

$$Q_{l1}(J_\alpha' J_\alpha ; \hat{z}') = q_{l1}(J_\alpha' J_\alpha ; \hat{z}')\frac{1}{\sqrt{2}}P'_{-1}, \tag{3.24}$$

$$Q_{l-1}(J_\alpha' J_\alpha ; \hat{z}') = -q_{l1}(J_\alpha' J_\alpha ; \hat{z}')\frac{1}{\sqrt{2}}\pi_{\alpha'}\pi_\alpha(-)^l P'_1, \tag{3.25}$$

where

$$q_{l0}(J_\alpha' J_\alpha ; \hat{z}') = I_{l0}\left(J_\alpha' J_\alpha ; \frac{1}{2}\frac{1}{2}\right), \tag{3.26}$$

$$q_{l1}(J_\alpha' J_\alpha ; \hat{z}') = I_{l1}\left(J_\alpha' J_\alpha ; \frac{1}{2} - \frac{1}{2}\right), \tag{3.27}$$

$$\pi_{\pm} = \frac{1}{2}[1 \pm \pi_{\alpha'}\pi_{\alpha}(-)^{l}]. \tag{3.28}$$

It is also convenient to define the quantities

$$Q_{lM}^{(\pm)\prime} = Q_{lM}(J'_{\alpha}J_{\alpha};\, \hat{z}') \pm \pi_{\alpha'}\pi_{\alpha}(-)^{l-M}Q_{l-M}(J'_{\alpha}J_{\alpha};\, \hat{z}'), \tag{3.29}$$

which is given explicitly by

$$Q_{l0}^{(+)\prime} = q_{l0}(J'_{\alpha}J_{\alpha};\hat{z}')2\pi_{+}, \tag{3.30}$$

$$Q_{l0}^{(-)\prime} = q_{l0}(J'_{\alpha}J_{\alpha};\hat{z}')2\pi_{-}P_{z'}, \tag{3.31}$$

$$Q_{l1}^{(+)\prime} = -q_{l1}(J'_{\alpha}J_{\alpha};\hat{z}')iP_{y'}, \tag{3.32}$$

$$Q_{l1}^{(-)\prime} = q_{l1}(J'_{\alpha}J_{\alpha};\hat{z}')P_{x'}. \tag{3.33}$$

Under the mirror reflection with respect to the $x'z'$ plane, we have

$$P_{x'} \rightarrow -P_{x'}, \tag{3.34}$$

$$P_{y'} \rightarrow P_{y'}, \tag{3.35}$$

$$P_{z'} \rightarrow -P_{z'}, \tag{3.36}$$

$$q_{lM}(J'_{\alpha}J_{\alpha};\hat{z}') \rightarrow q_{lM}(J'_{\alpha}J_{\alpha};\hat{z}'), \tag{3.37}$$

where the last symmetry relation can be seen from (3.14). Consequently we have

$$Q_{lM}^{(\pm)\prime} \rightarrow \pm Q_{lM}^{(\pm)\prime}. \tag{3.38}$$

The differential intensity and Stokes parameters of the emitted photon is defined as

$$\frac{dI}{d\Omega} = \rho_{11} + \rho_{-1-1}, \tag{3.39}$$

$$S_x = (\rho_{1-1} + \rho_{-11})/(\rho_{11} + \rho_{-1-1}), \tag{3.40}$$

$$S_y = i(\rho_{1-1} - \rho_{-11})/(\rho_{11} + \rho_{-1-1}), \tag{3.41}$$

$$S_z = (\rho_{11} - \rho_{-1-1})/(\rho_{11} + \rho_{-1-1}). \tag{3.42}$$

Similar to (3.14), we have

$$I_{l(q'-q)}(q'q;J'_{\alpha}J_{\alpha}) = \pi_{\alpha'}\pi_{\alpha}(-)^{l}I_{l-(q'-q)}(-q'-q;J'_{\alpha}J_{\alpha}). \tag{3.43}$$

By substituting (3.9) and (3.10) into (3.39)-(3.42) and making use of (3.43), we obtain

$$\frac{dI}{d\Omega} = \sum_{l} \sum_{J'_\alpha J_\alpha} K_l(J'_\alpha J_\alpha 1) Q^{(+)}_{l0}, \tag{3.44}$$

$$S_x \frac{dI}{d\Omega} = \sum_{l} \sum_{J'_\alpha J_\alpha} K_l(J'_\alpha J_\alpha - 1) Q^{(+)}_{l2}, \tag{3.45}$$

$$S_y \frac{dI}{d\Omega} = i \sum_{l} \sum_{J'_\alpha J_\alpha} K_l(J'_\alpha J_\alpha - 1) Q^{(-)}_{l2}, \tag{3.46}$$

$$S_z \frac{dI}{d\Omega} = \sum_{l} \sum_{J'_\alpha J_\alpha} K_l(J'_\alpha J_\alpha 1) Q^{(-)}_{l0}, \tag{3.47}$$

where we have defined the short-handed notations

$$Q^{(\pm)}_{lm} = Q_{lm}(J'_\alpha J_\alpha; \hat{z}) \pm \pi_{\alpha'}\pi_\alpha (-)^{l-m} Q_{l-m}(J'_\alpha J_\alpha; \hat{z}), \tag{3.48}$$

and

$$K_l(J'_\alpha J_\alpha q) = I_{l(1-q)}(1q; J'_\alpha J_\alpha). \tag{3.49}$$

The linear combinations of state multipoles in the two coordinate frames are now related by

$$Q^{(\pm)}_{lm} = \sum_{M\geq 0} \left(1 - \frac{1}{2}\delta_{M0}\right) Q^{(\pm)\prime}_{lM} [d^l_{Mm}(\theta) \pm \pi_{\alpha'}\pi_\alpha (-)^{l-m} d^l_{M-m}(\theta)], \tag{3.50}$$

where the unprimed $Q^{(\pm)}_{lm}$ and the primed $Q^{(\pm)\prime}_{lM}$ refer to the (xyz) and $(x'y'z')$ coordinate frames, respectively.

IV. ANGULAR DISTRIBUTION FUNCTION AND STOKES PARAMETERS

It is convenient to introduce the angular distribution function $F(\hat{k})$ as

$$\frac{dI}{d\Omega} = \frac{I}{4\pi} F(\hat{k}), \tag{4.1}$$

where I is the total intensity

$$I = \int d\hat{k} \frac{dI}{d\Omega}. \tag{4.2}$$

Substituting (3.30)-(3.33), (3.50) and (4.1) into (3.44)-(3.47), we have

$$F(\hat{k}) = a + bP_{y'}, \tag{4.3}$$

$$S_x F(\hat{k}) = c + dP_{y'}, \tag{4.4}$$

$$S_y F(\hat{k}) = eP_{z'} + fP_{x'}, \tag{4.5}$$

$$S_z F(\hat{k}) = gP_{z'} + hP_{x'}, \tag{4.6}$$

where the eight functions of the polar angle θ are defined as

$$a = 1 + \sum_{l\geq 1} \beta_{0l} d^l_{00}(\theta), \tag{4.7}$$

$$b = \sum_{l\geq 1} \beta_{2l} d^l_{10}(\theta), \tag{4.8}$$

$$c = \sum_{l\geq 2} \xi_{0l} d^l_{02}(\theta), \tag{4.9}$$

$$d = \sum_{l\geq 2} [\xi_{2l} d^l_{12}(\theta) + \eta_{1l} d^l_{1-2}(\theta)], \tag{4.10}$$

$$e = \sum_{l\geq 2} \eta_{3l} d^l_{02}(\theta), \tag{4.11}$$

$$f = \sum_{l\geq 2} [-\xi_{2l} d^l_{12}(\theta) + \eta_{1l} d^l_{1-2}(\theta)], \tag{4.12}$$

$$g = \sum_{l\geq 0} \zeta_{3l} d^l_{00}(\theta), \tag{4.13}$$

$$h = \sum_{l\geq 1} \zeta_{1l} d^l_{10}(\theta). \tag{4.14}$$

In Eqs. (4.7)-(4.14), the eight classes of angle-independent dynamic parameters are given by

$$\beta_{0l} = \sum_{J'_\alpha J_\alpha}^{(+)} N_{l0}(J'_\alpha J_\alpha 1), \tag{4.15}$$

$$\beta_{2l} = -i \sum_{J'_\alpha J_\alpha}^{(+)} N_{l1}(J'_\alpha J_\alpha 1), \tag{4.16}$$

$$\xi_{0l} = \sum_{J'_\alpha J_\alpha}^{(+)} N_{l0}(J'_\alpha J_\alpha - 1), \tag{4.17}$$

$$\xi_{2l} = -i \sum_{J'_\alpha J_\alpha} \frac{1}{2} N_{l1}(J'_\alpha J_\alpha - 1), \tag{4.18}$$

$$\eta_{1l} = -i \sum_{J'_\alpha J_\alpha} \frac{1}{2} \pi_{\alpha'} \pi_\alpha (-)^l N_{l1}(J'_\alpha J_\alpha - 1), \tag{4.19}$$

$$\eta_{3l} = i \sum_{J'_\alpha J_\alpha}^{(-)} N_{l0}(J'_\alpha J_\alpha - 1), \tag{4.20}$$

$$\zeta_{1l} = \sum_{J'_\alpha J_\alpha}^{(-)} N_{l1}(J'_\alpha J_\alpha 1), \tag{4.21}$$

$$\zeta_{3l} = \sum_{J'_\alpha J_\alpha}^{(-)} N_{l0}(J'_\alpha J_\alpha 1), \tag{4.22}$$

where the superscripts $(\pm)$ in the summation of Eqs. (4.15)-(4.22) denote the parity selection rule

$$(\pm) \equiv \begin{cases} 1, & \text{when } \pi_{\alpha'}\pi_\alpha(-)^l = \pm 1, \\ 0, & \text{when } \pi_{\alpha'}\pi_\alpha(-)^l = \mp 1, \end{cases} \tag{4.23}$$

and the coefficients are

$$N_{lM}(J'_\alpha J_\alpha q) = 2\left(\frac{I}{4\pi}\right)^{-1} K_l(J'_\alpha J_\alpha q) q_{lM}(J'_\alpha J_\alpha; \hat{z}'), \tag{4.24}$$

$$I = 4\pi \sum_{J_\alpha} K_0(J_\alpha J_\alpha 1) q_{00}(J_\alpha J_\alpha; \hat{z}'). \tag{4.25}$$

The explicit expressions of $K_l(J'_\alpha J_\alpha q)$ and $q_{lM}(J'_\alpha J_\alpha; \hat{z}')$ can be evaluated expediently by a graphical method [17] as

$$\begin{aligned} q_{lM}(J'_\alpha J_\alpha; \hat{z}') = & \sum_{J'J} \sum_{k'_i k_i} \sum_{k_\alpha} (-)^{l+J_0+J'+J+J'_\alpha+j_\alpha+1/2} \frac{2\pi^3}{k_i^2} \frac{[l]}{[J_0]^2} [J'J][j'_i j_i] \\ & \times \begin{Bmatrix} J & J' & l \\ j'_i & j_i & J_0 \end{Bmatrix} \begin{Bmatrix} J_\alpha & J'_\alpha & l \\ J' & J & j_\alpha \end{Bmatrix} D_{\alpha'} D^*_\alpha L_{lM}(\alpha'\alpha), \end{aligned} \tag{4.26}$$

$$K_l(J'_\alpha J_\alpha q) = \sum_{j'j}(-)^{J_a+J'_\alpha+1}\frac{\pi e^2\omega}{c}[j'j][l] \cdot \left\{ \begin{array}{ccc} j & j' & l \\ J'_\alpha & J_\alpha & J_a \end{array} \right\} \left(\begin{array}{ccc} j' & j & l \\ -1 & q & l-q \end{array} \right) d^*_{\alpha'} d_\alpha G_l(J'_\alpha J_\alpha; t), \tag{4.27}$$

where

$$L_{l0}(\alpha'\alpha) = (-)^{j'_i+j_i+l'_i+l_i} \left(\begin{array}{ccc} j'_i & j_i & l \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{array} \right) \tag{4.28}$$

$$L_{l1}(\alpha'\alpha) = (-)^{j'_i+l'_i+1/2} \left(\begin{array}{ccc} j'_i & j_i & l \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{array} \right). \tag{4.29}$$

The reduced matrix element for the electron-impact excitation of an atom is defined as

$$D_{\alpha'} = i^{l'_i - l_\alpha} e^{i(\sigma_{\kappa_\alpha} - \sigma_{\kappa'_i})} < (\kappa_\alpha J'_\alpha) J' || V || (\kappa'_i J_0) J'^+ >, \tag{4.30}$$

where $\sigma_{\kappa'_i}$ a nσ_{κ_α} are the relativistic Coulomb phase shifts for the incident and scattered electrons: respectively, and $|(\kappa'_i J_0) J' M'^+ >$ is the coupled total angular-momentum eigen-state of the combined system, which contains incoming spherical Couloumb waves only in the initial channel. The reduced matrix element for photoabsorption of an atom is defined as

$$d_{\alpha'} = -\frac{1}{\sqrt{2}}\left[q d^{(Mj')}_{\alpha'} + i d^{(Ej')}_{\alpha'} \right], \tag{4.31}$$

where the reduced matrix elements for electric and magnetic multipole transitions are

$$d^{(Ej')}_{\alpha'} = -\left\langle J'_\alpha \left\| \sum_{n=1}^{N} \vec{\alpha}_n \cdot \vec{A}^{(Ej')}(\vec{r}_n) \right\| J_a \right\rangle, \tag{4.32}$$

$$d^{(Mj')}_{\alpha'} = -\left\langle J'_\alpha \left\| \sum_{n=1}^{N} \vec{\alpha}_n \cdot \vec{A}^{(Mj')}(\vec{r}_n) \right\| J_a \right\rangle. \tag{4.33}$$

In Eq. (4.27), the time evolution between the excitation and de-excitation is characterized by the time-dependent factor

$$G_l(J'_\alpha J_\alpha; t) = e^{-i\omega_{\alpha'\alpha}t - \gamma_{\alpha'\alpha}t}, \tag{4.34}$$

where $\omega_{\alpha'\alpha}$ and $\gamma_{\alpha'\alpha}$ are given by

$$\omega_{\alpha'\alpha} = \varepsilon'_\alpha - \varepsilon_\alpha, \tag{4.35}$$

$$\gamma_{\alpha'\alpha} = (\gamma'_\alpha + \gamma_\alpha)/2. \tag{4.36}$$

If the hyperfine interaction with the nuclear spin is also important, the time-evolution factor becomes

$$G_l(J'_\alpha J_\alpha; t) = \sum_{F'F} \frac{[F'F]^2}{[I]^2} \left\{ \begin{array}{ccc} F & F' & l \\ J'_\alpha & J_\alpha & I \end{array} \right\} e^{-i\omega_{\alpha'\alpha}t - \gamma_{\alpha'\alpha}t}. \tag{4.37}$$

The polarization vector can be expressed in a fixed frame (XYZ), related to the frame $(x'y'z')$ by a rotation of angle ϕ with respect to the common Z-axis. The geometric relationships among the three coordinate frames, (xyz), $(x'y'z')$, and (XYZ), are shown in Fig. 1; the transformations among their components are given in Appendix A. In terms of (P_X, P_Y, P_Z), we may rewrite (4.3)-(4.6) as

$$F(\theta,\phi) = a + b(P_Y \cos\phi - P_X \sin\phi), \tag{4.38}$$

$$S_x F(\theta,\phi) = c + d(P_Y \cos\phi - P_X \sin\phi), \tag{4.39}$$

$$S_y F(\theta,\phi) = eP_Z + f(P_X \cos\phi + P_Y \sin\phi), \tag{4.40}$$

$$S_z F(\theta,\phi) = gP_Z + h(P_X \cos\phi + P_Y \sin\phi). \tag{4.41}$$

The anglar distribution function $F(\theta,\phi)$ is a function of the angles θ and ϕ which specify the direction of linear momentum $\hat{k}$ of the emitted photon in the (XYZ) coordinate frame as shown in Fig. 1. The eight coefficients a, b, c, d, e, f, g, and h given in (4.7)-(4.14) are functions of the polar angle θ. Their physical meanings are given in Table I, where various asymmetry parameters are defined in Appendix B. From Table I, we see that for measuring the left-right asymmetry parameters (L-R asymm.) of I, S_x, S_y, and S_z it sufficies to choose $\phi = 0$, such that the emitted photon lies on the XZ-plane. When up-down asymmetry parameters (U-D asymm.) are measured, we choose $\phi = \pi/2$, i.e., the emitted photon lies on the YZ-plane. We shall adopt these conventions hereafter, and therefore Table I reduces to Table II. For an unpolarized incident electron beam, only I and S_x exist, and all asymmetry parameters vanish, which can be seen from (4.38)-(4.41). For a longitudinally polarized electron beam, the positive-negative asymmetry parameters (P-N asymm.) of S_y and S_z can be measured, while for a transversely polarized electron beam, either the left-right or up-down asymmetries can be measured.

V. ELECTRIC DIPOLE TRANSITION

The measured intensity of radiation from target atoms or ions after electron-impact excitation is a gross sum of all possible multipole contributions. Nevertheless, the electric

TABLE I. Asymmetry parameters with (θ,ϕ)-dependence for radiation from atoms after electron-impact excitation.

Photon observable	Unpolarized electron Absolute value	Longitudinally polarized electron P-N asymm.	Transversely polarized electron L-R asymm.	Transversely polarized electron U-D asymm.
I	I	0	$(b/a)\cos\phi$	0
S_x	c/a	0	$(d/a)\cos\phi$	0
S_y	0	e/a	0	$(f/a)\sin\phi$
S_z	0	g/a	0	$(h/a)\sin\phi$

TABLE II. Asymmetry parameters with &dependence for radiation from atoms after electron-impact excitation.

Photon observable	Unpolarized electron Absolute value	Longitudinally polarized electron P-N asymm.	Transversely polarized electron L-R asymm.	Transversely polarized electron U-D asymm.
I	I	0	(b/a)	0
S_x	c/a	0	(d/a)	0
S_y	0	e/a	0	(f/a)
S_z	0	g/a	0	(h/a)

dipole amplitude dominates in the long wavelength region, when allowed by selection rules. Besides, it may be the only possible transition under certain circumstances. Consequently, it is desirable to examine the electric dipole transition for which the rank of electromagnetic multipole is $j'=j=1$, and the ranks of state multipoles are $l=0, 1$, and 2. Because the parity of the electric dipole field is odd, the parity of the excited atomic state must be opposite to that of the final atomic state, i.e., $\pi_{\alpha'}=\pi_\alpha=-\pi_a$. Therefore the parity selection rule (4.23) reduces to

$$(\pm) = \begin{cases} 1, \text{ when } l \text{ is even (odd) for } +(-), \\ 0, \text{ when } l \text{ is odd (even) for } +(-). \end{cases} \tag{5.1}$$

By making use of the relation

$$K_2(J'_\alpha J_\alpha - 1) = \sqrt{6} K_2(J'_\alpha J_\alpha 1), \tag{5.2}$$

and writing out the d-functions explicitly, the angular functions in (4.7)-(4.14) reduce in the electric dipole approximation to

$$a = 1 + \frac{1}{2}\beta(3\cos^2\theta - 1), \tag{5.3}$$

$$b = -\xi \sin\theta \cos\theta, \tag{5.4}$$

$$c = \frac{3}{2}\beta \sin^2\theta, \tag{5.5}$$

$$d = \xi \sin\theta \cos\theta, \tag{5.6}$$

$$e = 0, \tag{5.7}$$

$$f = -\xi \sin\theta, \tag{5.8}$$

$$g = \zeta \cos\theta, \tag{5.9}$$

$$h = -\frac{1}{\sqrt{2}}\delta \sin\theta, \tag{5.10}$$

where we have defined the parameters

$$\beta = \beta_{02}, \tag{5.11}$$

$$\xi = \xi_{22}, \tag{5.12}$$

$$\delta = \zeta_{11}, \tag{5.13}$$

$$\zeta = \zeta_{31}. \tag{5.14}$$

Making use of the constraints

$$a \geq 0, \tag{5.15}$$

$$S_x^2 + S_y^2 + S_z^2 \leq 1, \tag{5.16}$$

we can deduce the following kinematic relations:

$$\xi^2 + \delta^2/2 \leq (1 \ t \ \beta)(1 - 2\beta), \tag{5.17}$$

$$\zeta^2 \leq (1 \ t \ \beta)^2. \tag{5.18}$$

When the equality in (5.16) holds, the polarization of the emitted photon is in a pure state. There are three possible situations when this condition is fulfilled. The first is when the equality in (5.17) holds, and the photon is measured in the direction transverse to that of the polarization vector of the incident electron, the second is when the equality in (5.18) holds and the photon is measured in the forward direction, and the last is when $\beta =$ -1, which implies $\xi = \delta = \zeta =$ 0, and in this case the photons emitted in any directions are all in pure states. The physical meanings of these dynamic parameters are given in Table III for photons emitted in the transverse direction $(\theta = \pi/2)$ and in Table IV for the forward direction (θ = 0). This provides a possible set of complete measurement of the five dynamic parameters: the absolute value of S_x, the up-down asymmetry parameter of S_y and S_z in the transverse direction, the positive-negative asymmetry parameter of S_z in the forward direction, and the total intensity I.

It is interesting to note that for the electric dipole transition

$$d = -b, \tag{5.18}$$

$$e = 0\,. \tag{5.19}$$

TABLE III. Asymmetry parameters at $\theta = \pi/2$ in the electric dipole approximation.

Photon observable	Unpolarized electron Absolute value	Longitudinally polarized electron P-N asymm.	Transversely polarized electron L-R asymm.	 U-D asymm.
I	I	0	0	0
S_x	$3\beta/(2-\beta)$	0	0	0
S_y	0	0	0	$-2\xi/(2-\beta)$
S_z	0	0.	0	$-\sqrt{2}\delta/(2-\beta)$

TABLE IV. Asymmetry parameters at $\theta = 0$ in the electric dipole approximation.

Photon observable	Unpolarized electron	Longitudinally polarized electron	Transversely polarized electron	
	Absolute value	P-N asymm.	L-R asymm.	U-D asymm.
I	I	0	0	0
S_x	0	0	0	0
S_y	0	0	0	0
S_z	0	$\zeta/(1+\beta)$	0	0

This offers tests of the validity of the electric dipole approximation. The violation of these conditions implies that the effects of higher multipole transitions are significant. From Table I, this violation occurs either when the left-right asymmetry parameters of I and S_x are not equal in magnitude and opposite in sign or when the positive-negative asymmetry parameter of S_y does not vanish. For a generally polarized target, nine independent parameters are needed to completely specify their polarization states up to the state multipole $l = 2$. For instance, by coincident measurement of the scattered electron with the emitted photon after electron-impact excitation, the nine components can be determined completely. However, when only the emitted photon is observed, the components of the state multipoles can take only the values $M = 0$, 1, or -1, so that the components (lM) = (22) and (2-2) are forbidden. Besides, from Eqs. (3.24) and (3.25), we note that the two components (lM) = (11) and (l-l) are dependent, similarly for (lM) = (21) and (2-l). Consequently, we are left with five independent dynamic parameters: I, ζ, δ, β and ξ; the first results from the state multipole $l = 0$, the next two from 1 = 1, and the last two from 1 = 2 of the excited atomic state:

$$I = 4\pi \sum_{J_\alpha} \sqrt{3} C_\omega \frac{1}{[J_0][J_\alpha]} d_\alpha^* d_\alpha G_0(J_\alpha J_\alpha; t) q_{00}(J_\alpha J_\alpha; \hat{z}'), \tag{5.20}$$

$$\zeta = \left(\frac{I}{4\pi}\right)^{-1} \sum_{J'_\alpha J_\alpha} (-)^{J_a + J'_\alpha} 3\sqrt{2} C_\omega \begin{Bmatrix} 1 & 1 & 1 \\ J'_\alpha & J_\alpha & J_a \end{Bmatrix} \cdot d_{\alpha'}^* d_\alpha G_1(J'_\alpha J_\alpha; t) q_{10}(J'_\alpha J_\alpha; \hat{z}'), \tag{5.21}$$

$$\delta = \left(\frac{I}{4\pi}\right)^{-1} \sum_{J'_\alpha J_\alpha} (-)^{J_a+J'_\alpha} 3\sqrt{2} C_\omega \begin{Bmatrix} 1 & 1 & 1 \\ J'_\alpha & J_\alpha & J_a \end{Bmatrix} \cdot d^*_{\alpha'} d_\alpha G_1(J'_\alpha J_\alpha; t) q_{11}(J'_\alpha J_\alpha; \hat{z}'), \quad (5.22)$$

$$\beta = \left(\frac{I}{4\pi}\right)^{-1} \sum_{J'_\alpha J_\alpha} (-)^{J_a+J'_\alpha} \sqrt{6} C_\omega \begin{Bmatrix} 1 & 1 & 2 \\ J'_\alpha & J_\alpha & J_a \end{Bmatrix} \cdot d^*_{\alpha'} d_\alpha G_2(J'_\alpha J_\alpha; t) q_{20}(J'_\alpha J_\alpha; \hat{z}'), \quad (5.23)$$

$$\xi = \left(\frac{I}{4\pi}\right)^{-1} \sum_{J'_\alpha J_\alpha} (-)^{J_a+J'_\alpha} \sqrt{6} C_\omega \begin{Bmatrix} 1 & 1 & 2 \\ J'_\alpha & J_\alpha & J_a \end{Bmatrix} \cdot d^*_{\alpha'} d_\alpha G_2(J'_\alpha J_\alpha; t) q_{21}(J'_\alpha J_\alpha; \hat{z}'), \quad (5.24)$$

where

$$C_\omega = \pi e^2 \omega / c. \quad (5.25)$$

In order to obtain the five parameters in one experiment, the incident electron must possess both transverse and longitudinal polarization components, and the angular distribution and the degree of circular polarization of the radiation must be measured. If the electron is transversely polarized, we can obtain at most four parameters; while three for longitudinally polarized electron and two for unpolarized electron. When the polarization of the radiation is not measured, we can obtain at most three parameters. So it is clear that polarization analyses enhance our knowledge on the collision process and reveal much valuable information about the collision dynamics.

VI. CONCLUSIONS

We have carried out a kinematic analysis of radiation from targets after excitation by incident spin-l/2 particles. The projectile can be any spin-l/2 particle (e.g., electron, positron, muon, etc.), the target is a bound system composed of any number of particles (e.g., atoms, ions, exotic atoms, etc.), and all the interactions treated here during the collision are invariant under rotations and space inversion. The angular distribution and polarization of the radiation are presented in terms of compact parametrized form. There are eight characteristic angular functions in general, which are expressed in terms of eight classes of angle-independent dynamic parameters. These dynamic parameters are given as linear sums of reduced matrix elements suitable for numerical computation. In the electric dipole approximation, the number of independent dynamic parameters reduces to five. The dynamic parameters can be measured by experiments, and their physical interpretations are given.

The advantages of the present approach are that it includes the polarization, relativistic, and electromagnetic multipole effects. The polarization effects provide additional information about the collision dynamics. Because we start from a relativistic framework, fine structure interaction and other relativistic effects which are important for highly ionized atoms are included automatically. Furthermore, electromagnetic multipole effects are not negligible when the electric dipole transition is forbidden by selection rules or when the photon energy is high.

Another important feature of our treatment is the separation of kinematics and dynamics. These kinematic relations are quite general and may be applied to any kinematically equivalent collision processes. So long as the same kind of species are detected, their angular distribution and polarization will have the same functional dependence on the initial polarization of the incident particle. The present results bear great similarity to those of photoionization processes [14], which can be viewed as a kinematically reversed process as far as the polarization states are concerned.

The dynamic parameters are presented in a form ready for numerical computation in different dynamic theories. This facilitates the comparison between various dynamic theories and with experiment. Applications to ions of the hydrogen isoelectronic sequence in the distorted-wave approximation are in progress.

APPENDIX A: ROTATION TRANSFORMATION OF THE POLARIZATION VECTOR

The right-handed Cartesian coordinate frame (xyz) is related to another one (XYZ) by three successive Euler rotations $(\phi\theta\varphi)$. A polarization vector $\vec{P}$ is expressed in these two frames as (P_x, P_y, P_z) and (P_X, P_Y, P_Z), respectively. Then the two sets of components are related by

$$\begin{pmatrix} P_X \\ P_Y \\ P_Z \end{pmatrix} = R(\phi\theta\varphi) \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix}, \tag{A1}$$

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = R^T(\phi\theta\varphi) \begin{pmatrix} P_X \\ P_Y \\ P_Z \end{pmatrix}, \tag{A2}$$

where

$$R(\phi\theta\varphi) = \begin{pmatrix} \cos\phi\cos\theta\cos\varphi - \sin\phi\sin\varphi & -\cos\phi\cos\theta\sin\varphi - \sin\phi\cos\varphi & \cos\phi\sin\theta \\ \sin\phi\cos\theta\cos\varphi + \cos\phi\sin\varphi & -\sin\phi\cos\theta\sin\varphi + \cos\phi\cos\varphi & \sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{A3}$$

and $R^T(\phi\theta\varphi)$ denotes the transpose of $R(\phi\theta\varphi)$.

APPENDIX B. DEFINITIONS OF ASYMMETRY PARAMETERS

For a transversely polarized incident electron beam, choosing the coordinate frame with $P_X = 0$, the up-down and left-right asymmetry parameters of a physical observable $\omega(\theta,\phi)$ of the photon emitted in the direction (θ,ϕ) are defined as

$$\Omega_{U-D}(\theta,\phi) = \frac{1}{P_Y}\frac{\omega(\theta,\phi)F(\theta,\phi) - \omega(\theta,-\phi)F(\theta,-\phi)}{F(\theta,\phi) + F(\theta,-\phi)}, \tag{B1}$$

$$\Omega_{L-R}(\theta,\phi) = \frac{1}{P_Y}\frac{\omega(\theta,\phi)F(\theta,\phi) - \omega(\theta,\pi-\phi)F(\theta,\pi-\phi)}{F(\theta,\phi) + F(\theta,\pi-\phi)}, \tag{B2}$$

where an example of $F(\theta,\phi)$ is given in (4.38). When the incident electron beam is longitudinally polarized, the positive-negative asymmetry parameter of that observable is defined as

$$\Omega_{P-N}(\theta,\phi) = \frac{1}{P_Z}\frac{\omega(\theta,\phi)F(\theta,\phi) - [\omega(\theta,\phi)]_-[F(\theta,\phi)]_-}{F(\theta,\phi) + [F(\theta,\phi)]_-}, \tag{B3}$$

where the bracket $[]_-$ denotes that the quantity in the bracket is measured by reversing the helicity of the incident electron $(P_Z \to -P_Z)$.

In the present case, where rotation and parity invariance are assumed, the angular distribution function is given explicitly by (4.38) such that

$$F(\theta,\phi) = F(\theta,-\phi) = [F(\theta,\phi)]_-. \tag{B4}$$

Thus (B1) and (B3) become

$$\Omega_{U-D}(\theta,\phi)= \frac{\omega(\theta,\phi) - \omega(\theta,-\phi)}{2P_Y}, \tag{B5}$$

$$\Omega_{P-N}(\theta,\phi) = \frac{\omega(\theta,\phi) - [\omega(\theta,\phi)]_-}{2P_Z}. \tag{B6}$$

From (4.40) and (4.41),

$$S_y(\theta,-\phi) = -[S_y(\theta,\phi)]_-, \tag{B7}$$

$$S_z(\theta,-\phi) = -[S_z(\theta,\phi)]_-. \tag{B8}$$

Thus the positive-negative asymmetry parameters of $S_y S_z$ can also be expressed a

$$(S_y)_{P-N}(\theta,\phi) = \frac{S_y(\theta,\phi)+S_y(\theta,-\phi)}{2P_Z}. \tag{B9}$$

$$(S_z)_{P-N}(\theta,\phi) = \frac{S_z(\theta,\phi)+S_z(\theta,-\phi)}{2P_Z} \tag{B10}$$

We note that (B6),(B9), and (B10) (B3).

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