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Brief paper

A fully adaptive decentralized control of robot manipulators☆

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Abstract

In this paper, we develop a fully adaptive decentralized controller of robot manipulators for trajectory tracking. With high-order and adaptive variable-structure compensations, the proposed scheme makes both position and velocity tracking errors of robot manipulators globally converge to zero asymptotically while allowing all signals in closed-loop systems to be bounded, even without any prior knowledge of robot manipulators. Thus this control scheme is claimed to be fully adaptive. Even when the proposed scheme is modified to avoid the possible chattering in actual implementations, the overall performance will remain appealing. Finally, numerical results are provided to verify the effectiveness of the proposed schemes at the end.

Keywords: Decentralized control; Adaptive control; Robot manipulators

1. Introduction

The control of robotic manipulators is especially challenging due to the inherent high non-linearity in its dynamics. In a practical situation, the inevitable uncertainty in the underlying manipulator model, say, payload change, adds additional difficulty into the control task. Although significant achievements, marked by the development of adaptive and robust centralized control schemes, have been made to improve the tracking performance of robots (Slotine & Li, 1987; Spong, Thorp, & Kleinwaks, 1987), the decentralized controller structure is still adopted by the majority of modern robots in favor of its computation simplicity and low-cost hardware setup. As a result, how to best improve the tracking performance of robots through decentralized control is still an interesting research topic that attracts great attention from robotic community.

The adaptive decentralized control approaches for linear and linear-dominant systems have been well developed, for example, by Gavel and Šiljak (1989), Ioannou (1986), Shi and Singh (1992) and Wen and Soh (1999). Specifically, for a set of linear-dominant subsystems whose interconnections are nonlinear but linearly bounded by the norms of the overall system states, the approaches proposed by Gavel and Šiljak (1989) and Ioannou (1986) guarantee the exponential convergence of tracking errors and parameter estimation error to a bounded residual set. Shi and Singh (1992) use nonlinear feedback to handle the interconnections bounded by a higher-order polynomial of the system-state norms. Moreover, Wen and Soh (1999) develop a decentralized model reference adaptive control without restriction on subsystem relative degrees.

For manipulator tracking tasks, decentralized approaches are not that straightforward since the overall system cannot be decomposed into subsystems whose states and control inputs are totally decoupled from one another because of the inherent coupling such as moment of inertia and Coriolis force. In recent years, several attempts, e.g. by Fu (1992), Liu (1999), and Tang, Tomizuka, Guerrero, and Montemayor (2000), have been made for the adaptive independent-joint control (IJC) or the so-called adaptive decentralized control such that a separate actuator taking feedback only from that particular joint is responsible for the joint control. Although those schemes result in asymptotical convergence of tracking errors, prior estimation of gains is necessary, that is inconvenient in applying. In order to

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resolve this problem, a novel adaptive decentralized control law is proposed here such that the desired tracking performance is achieved without any prior determination of gains.

This paper is originated as follows: Tracking problem of robot manipulators is introduced in Section 2. And then a novel adaptive decentralized control scheme is proposed in Section 3. In order to demonstrate the performance of the proposed scheme, a numerical study is provided in Section 4. Finally, a conclusion is given in Section 5.

2. Problem statement

For general n-link rigid manipulators, the dynamic model can be derived by using the Euler–Lagrangian approach and expressed in joint space as:

$$M[q(t)]\ddot{q}(t) + C[q(t), \dot{q}(t)]\dot{q}(t) + g[q(t)] = \tau(t) + d[t, q(t), \dot{q}(t)], \tag{1}$$

where $t \geqslant 0$ denotes time; $q(t) = [q_1(t), \ldots, q_n(t)]^{\mathrm{T}} \in R^n$ and $\dot{q}(t) = [\dot{q}_1(t), \ldots, \dot{q}_n(t)]^{\mathrm{T}} \in R^n$ are the vectors of joint position and velocity, respectively; $\tau(t) = [\tau_1(t), \ldots, \tau_n(t)]^{\mathrm{T}} \in R^n$ is the control input; M: $R^n \to R^{n\times n}$ such that $M[q(t)]$ is the inertia matrix; C: $R^n \times R^n \to R^{n\times n}$ such that $C[q(t), \dot{q}(t)]\dot{q}(t)$ is the vector of centrifugal and Coriolis force, g: $R^n \to R^n$ such that $g[q(t)]$ is the vector of gravitational force, and finally d: $[0, \infty) \times R^n \times R^n \to R^n$ such that $d[t, q(t), \dot{q}(t)]$ is the vector of friction input. This dynamic model has the following properties that will be used in controller design (Spong & Vidyasagar, 1989; Fu, 1992):

- (P1) The matrix $M(y)$ is a symmetric and positive-definite matrix and satisfies $\mu_m I \leqslant M(y) \leqslant \mu_M I$ for some constants $\mu_m, \mu_M > 0$, where $y \in R^n$.
- (P2) The matrix $C(y, z)$ satisfies $||C(y, z)||_2 \leqslant \mu_C ||z||_2$ for some constant $\mu_C > 0$, where $y, z \in R^n$.
- (P3) The vector $g(y)$ satisfies $||g(y)||_2 \leqslant \mu_G$ for some constant $\mu_G > 0$, where $y \in R^n$.
- (P4) Time-varying matrix $\mathrm{d}/\mathrm{d}t\, M[y(t)] - 2C[y(t), \dot{y}(t)]$, is always skew-symmetric for all $t \geqslant 0$, where differentiable signals y: $[0, \infty) \to R^n$.

If an adaptive decentralized control scheme achieves the desired tracking performance without any prior knowledge of plants, it is called a fully adaptive decentralized control scheme. In this paper, such a control scheme is proposed for robot manipulators such that all signals of closed-loop systems are bounded as well as both position and velocity tracking errors globally converge to zero asymptotically. Global convergence here means convergence from any initial position and velocity tracking errors in joint space.

3. Controller design

In this section, a novel control scheme is proposed for the tracking control of general n-link rigid manipulators. Let q_d: $[0, \infty) \to R^n$ such that $q_d(t)$ for all $t \geqslant 0$ means the desired position trajectory of robot manipulators and is generally chosen twice differentiable to guarantee smoothness of the motion. Define the position tracing error $e(\cdot)$ as $e(t) = [e_1(t), \ldots, e_n(t)]^{\mathrm{T}} \in R^n$ where $e(t) \equiv q(t) - q_d(t)$ and auxiliary signal $s(\cdot)$ as $s(t) = [s_1(t), \ldots, s_n(t)]^{\mathrm{T}} \in R^n$ where $s(t) \equiv \dot{e}(t) + \Lambda e(t)$ with $\Lambda \in R^{n\times n}$ being a feedback-gain matrix. Now the dynamics defined by the signals $e(\cdot)$ and $s(\cdot)$ is derived as

$$\dot{e}(t) = -\Lambda e(t) + s(t), \tag{2a}$$

$$M[q(t)]\dot{s}(t) = -C[q(t), \dot{q}(t)]s(t) + \tau(t) - v[t, q(t), \dot{q}(t)], \tag{2b}$$

where

$$v[t, q(t), \dot{q}(t)] = M[q(t)][\ddot{q}_d(t) + \Lambda\dot{e}(t)] + C[q(t), \dot{q}(t)][\dot{q}_d(t) + \Lambda e(t)] + g[q(t)] - d[t, q(t), \dot{q}(t)], \tag{3}$$

behaves as the disturbance. Without loss of generality, several technical assumptions are made to pose the problem in a tractable manner.

- (A1) The feedback-gain matrix Λ is constant, diagonal and positive-definite; that is, $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) > 0$ for any constant $\lambda_i > 0$, $i \in \{1, \ldots, n\}$.
- (A2) The desired joint position trajectory $q_d(t)$ and the time derivatives $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are bounded signals.
- (A3) Let d_i: $[0, \infty) \times R \times R \to R$, $i \in \{1, \ldots, n\}$, satisfy that $|d_i(t, y, z)| \leqslant d_{i\_1} + d_{i\_2}|y| + d_{i\_3}|z|$ for all $t \geqslant 0$ for some constant $d_{i\_j} \geqslant 0$, $i \in \{1 \ldots, n\}$ and $j \in \{1, 2, 3\}$, where $y, z \in R$, such that the friction input considered in (1) is assumed as $d[t, q(t), \dot{q}(t)] = [d_1[t, q_1(t), \dot{q}_1(t)], \ldots, d_n[t, q_n(t), \dot{q}_n(t)]]^{\mathrm{T}}$.

Remark 1. From properties (P1)–(P3) and assumptions (A1)–(A3), it is guaranteed that along the trajectory of robot manipulators the disturbance (3) satisfies the following:

$$\|v[t, q(t), \dot{q}(t)]\|_2 \leqslant \eta_1 + \eta_2\|e(t)\|_2 + \eta_3\|\dot{e}(t)\|_2 + \eta_4\|e(t)\|_2\|\dot{e}(t)\|_2, \tag{4}$$

with some positive constants η_1–η_4, which only depend on the desired trajectory and parameters in (1).

Before designing the claimed decentralized control law, one useful lemma should be derived first. Here we start with adopting the norm of vector-valued signals as follows: For vector-valued signals x: $[0, \infty) \to R^n$, $||x||_T$, the norm of $x(\cdot)$ for $T > 0$, is defined as $||x||_T \equiv \sup_{t\in[0,T]} ||x(t)||_2$. The following lemma will obtain another bound estimation of the disturbance (3), which is useful in control design.

Lemma 1. *Under assumptions (A1)–(A3), if there is a constant* $T > 0$ such that $||s||_T$ *exists, then there are positive constants* $\beta 1$, $\beta 2$ *and* $\beta 3$ *such that along the trajectory of robot manipulators it is satisfied that*

$$\|v[t, q(t), \dot{q}(t)]\|_2 \leqslant \beta_1 + \beta_2\|s\|_T + \beta_3(\|s\|_T)^2, \tag{5}$$

for all $t \in [0, T]$. In addition, the positive constants β_1, β_2 and β_3 only depend on the matrix Λ, desired trajectory, initial position condition of robot manipulators and parameters in (1).

Proof. The proof can be referred to Hsu and Fu (2003). □

Remark 2. It should be pointed out that Lemma 1 could be applied for the initial time interval, since the signal $s(\cdot)$ always exists initially.

Now the adaptive decentralized control input $\tau(t)$ is designed. The ith component $\tau_i(t)$, $i \in \{1 \ldots, n\}$, is given as

$$\tau_i(t) = u_i(t) - \theta_{i\_3} s_i^3(t), \tag{6}$$

where the constant $\theta_{i\_3} > 0$, $i \in \{1, \ldots, n\}$, and the component $u_i(t)$, $i \in \{1 \ldots, n\}$, is defined as:

$$u_i(t) = \begin{cases} -\dfrac{1}{\varepsilon_i(t)}\hat{\theta}_{i\_1}^2(t)s_i(t) - \theta_{i\_2}s_i(t) & \text{if } \hat{\theta}_{i\_1}(t)|s_i(t)| \leqslant \varepsilon_i(t) \\ -\hat{\theta}_{i\_1}(t)sgn[s_i(t)] - \theta_{i\_2}s_i(t) & \text{if } \hat{\theta}_{i\_1}(t)|s_i(t)| > \varepsilon_i(t) \end{cases} \tag{7}$$

with $\hat{\theta}_{i\_1}(t)$, $i \in \{1, \ldots, n\}$, being adjusted by the adaptive law on line; θ_{i_2}, $i \in \{1, \ldots, n\}$, being a positive constant; $\mathrm{sgn}(\cdot)$ denoting the signum function (Khalil, 2002), and $|\cdot|$ denoting the absolute value of scalars. In (10), $\varepsilon_i(t)$, $i \in \{1, \ldots, n\}$, is defined by:

$$\dot{\varepsilon}_i(t) = -p_i \varepsilon_i(t), \quad \varepsilon_i(0) > 0, \tag{8}$$

where the constant $p_i > 0$, $i \in \{1, \ldots, n\}$. It is worth noting that $\varepsilon_i(t) > 0$ for all $t \geqslant 0$, $i \in \{1, \ldots, n\}$, and thus the auxiliary signal $\varepsilon_i(\cdot)$ is used as the convergent boundary layer in (6). For accounting for the uncertainty in the tracking task of robot manipulators, we choose the adaptive law as follows:

$$\dot{\hat{\theta}}_{i\_1}(t) = \gamma_{i\_1}|s_i(t)|, \quad \hat{\theta}_{i\_1}(0) \geqslant 0, \tag{9}$$

for $i \in \{1, \ldots, n\}$, where the constant $\gamma_{i\_1} > 0$, $i \in \{1, \ldots, n\}$. Note that the adaptive control law (6)–(9) is apparently in a decentralized manner, and its performance is summarized into the following theorem. Note also that when $\varepsilon_i \to 0$ and $\hat{\theta}_{i\_1}(t)|s_i(t)| \leqslant \varepsilon_i(t)$, the term $(\hat{\theta}_{i\_1}^2(t)s_i(t))/\varepsilon_i$ in control law u_i is bounded with the boundary $-\hat{\theta}_{i\_1}$ and $\hat{\theta}_{i\_1}$. $|s_i(t)|$ will be well defined if the switching law specified in Eq. (7) will not result in infinitely fast switching as time goes on and on. In order to make sure that this is true, we can see from Eq. (29) that $\|s_i(\cdot)\|_2 \in L_2$ which implies that $|s_i(t)|$ converge to zero slower than the exponentially converging $\varepsilon_i(t)$ when t goes long enough. In turn, this implies that the control u_i will eventually switch to the case where $\hat{\theta}_{i\_1}(t)|s_i(t)| > \varepsilon_i(t)$ so that $s_i(t)$ will evolve normally and hence is well defined throughout the whole time.

Theorem 2. *Under assumptions* (A1)–(A3), *consider the error dynamics of robotic manipulators subject to the adaptive decentralized control law* (6)–(9). *If the gain $\theta_{i\_j} > 0$, $i \in \{1, \ldots, n\}$ and $j \in \{2, 3\}$, then all signal are bounded, and the position tracking error $e(t)$ and velocity tracking error $\dot{e}(t)$ will globally converge to zero as $t \to \infty$.*

Proof. The proof proceeds in the following two steps.

Step 1: *Prove the signal $s(\cdot)$ is bounded*. We guarantee this statement by way of contradiction. Let a positive constant l_1 satisfy

$$\|s(0)\|_2 < \left[\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}\right]^{1/3} < \sqrt{\frac{\mu_m}{\mu_M}} l_1, \tag{10}$$

where $\theta_{3,\min} = \min\{\theta_{1\_3}, \ldots, \theta_{n\_3}\} > 0$. Such l_1 always exists. (When l_1 is large enough, (13) can be satisfied.) Now assume the signal $s(\cdot)$ is not bounded. Thus there always is a smallest time $T_1 > 0$ such that $||s(T_1)||_2 = l_1$. Consider a Lyapunov-like function $V_1 : [0, T_1] \times R^n \to R$ as

$$V_1(t, s) = \tfrac{1}{2} s^{\mathrm{T}} M[q(t)]s, \tag{11}$$

where the state $s \in R^n$ (Ioannou & Sun, 1996). The time derivative of (11) along the trajectory of the closed-loop system gives

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}V_1[t, s(t)] &= s^{\mathrm{T}}(t)\{\tau(t) - v[t, q(t), \dot{q}(t)]\} \\ &\leqslant s^{\mathrm{T}}(t)\tau(t) + \|s(t)\|_2 \|v[t, q(t), \dot{q}(t)]\|_2, \end{aligned} \tag{12}$$

for all $t \in [0, T_1]$, where property (P4) has been applied. Hence we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}V_1[t, s(t)] &\leqslant s^{\mathrm{T}}(t)\tau(t) + \|s(t)\|_2 \|v[t, q(t), \dot{q}(t)]\|_2 \\ &\leqslant s^{\mathrm{T}}(t)\tau(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2), \end{aligned} \tag{13}$$

for all $t \in [0, T_1]$, where Lemma 1 has been applied. After taking the second term on the right-hand side of (6) into account, it follows that

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}V_1[t, s(t)] &\leqslant -\sum_{i=1}^{n} \theta_{i\_3} s_i^4(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) \\ &\leqslant -\theta_{3,\min}\tfrac{1}{n}\|s(t)\|_2^4 + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2), \end{aligned} \tag{14}$$

for all $t \in [0, T_1]$. Note that the inequality (14) has adopted the Schwartz inequality (Khalil, 2002) so that

$$\sum_{i=1}^{n} s_i^4(t) \geqslant \tfrac{1}{n}\|s(t)\|_2^4. \tag{15}$$

By the original assumption, there exists a time interval $t_1 > 0$ such that

$$\|s(T_1 - t_1)\|_2 = \left[\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}\right]^{1/3}, \tag{16}$$

and $\mathrm{d}/\mathrm{d}t\, V_1[t, s(t)] \leqslant 0$ for all $t \in [T_1 - t_1, T_1]$. Hence we have

$$V_1[T_1, s(T_1)] \leqslant V_1[T_1 - t_1, s(T_1 - t_1)] \leqslant \frac{1}{2}\mu_M \left[\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}\right]^{2/3}. \quad (17)$$

But, from definition of T_1, it follows that

$$V_1[T_1, s(T_1)] \geqslant \frac{1}{2}\mu_m l_1^{2/3}. \quad (18)$$

Clearly, the inequalities (17) and (18) are in contradiction. This guarantees that the original assumption is false, Thus the signal $s(\cdot)$ is bounded and moreover satisfies $||s(t)||_2 < l_1$ for all $t \geqslant 0$.

Step 2: Prove all signals are bounded and the signal $s(t) \to 0$ globally as $t \to \infty$. Consider the Lyapunov-like function V_2: $[0, \infty) \times R^n \times R \times \cdots \times R \to R$ as

$$V_2(t, s, \hat{\theta}_{1\_1}, \ldots, \hat{\theta}_{n\_1}) = \frac{1}{2}s^{\mathrm{T}} M[q(t)]s + \sum_{i=1}^{n}\left[\frac{1}{2}\gamma_{i\_1}^{-1}(\hat{\theta}_{i\_1} - \theta_{i\_1}^*)^2\right] + \sum_{i=1}^{n} p_i^{-1}\varepsilon_i(t), \quad (19)$$

where the states $s \in R^n$ and $\hat{\theta}_{i\_1} \in R$, $i \in \{1, \ldots, n\}$. In (19), $\theta_{i\_1}^* \in R$, $i \in \{1, \ldots, n\}$, is the desirable value with respect to the updated gain $\hat{\theta}_{i\_1}(\cdot)$, $i \in \{1, \ldots, n\}$, respectively. Here, it is required that

$$\theta_{1,\min}^* \geqslant \beta_1 + \beta_2 l_1 + \beta_3 l_1^2, \quad (20)$$

where $\theta_{1,\min}^* = \min\{\theta_{1\_1}^*, \ldots, \theta_{n\_1}^*\}$. The time derivative of (19) along the trajectory of the closed-loop system gives

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant s^{\mathrm{T}}(t)\tau(t) + \|s(t)\|_2 \|v[t, q(t), \dot{q}(t)]\|_2 + \sum_{i=1}^{n}[\hat{\theta}_{i\_1}(t) - \theta_{i\_1}^*]|s_i(t)| - \sum_{i=1}^{n}\varepsilon_i(t), \quad (21)$$

for all $t \geqslant 0$, where property (P4) has been applied. Hence we have

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant s^{\mathrm{T}}(t)\tau(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) + \sum_{i=1}^{n}[\hat{\theta}_{i\_1}(t) - \theta_{i\_1}^*]|s_i(t)| - \sum_{i=1}^{n}\varepsilon_i(t), \quad (22)$$

for all $t \geqslant 0$, where the conclusion in Step 1 has been applied. Now, without loss of generality, consider two different cases of (22) as:

Case 1: $\hat{\theta}_{i\_1}(t)|s_i(t)| \leqslant \varepsilon_i(t)$ for all $i \in \{1, \ldots, n\}$. Taking the first term on the right-hand side of (6) into account, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n}\frac{1}{\varepsilon_i(t)}\hat{\theta}_{i\_1}^2(t)s_i^2(t) - \sum_{i=1}^{n}\theta_{i\_2}s_i^2(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) + \sum_{i=1}^{n}(\hat{\theta}_{i\_1}(t) - \theta_{i\_1}^*)|s_i(t)| - \sum_{i=1}^{n}\varepsilon_i(t), \quad (23)$$

which implies that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n}\frac{1}{\varepsilon_i(t)}\hat{\theta}_{i\_1}^2(t)s_i^2(t) - \sum_{i=1}^{n}\theta_{i\_2}s_i^2(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) - \sum_{i=1}^{n}\theta_{i\_1}^*|s_i(t)|. \quad (24)$$

From (24), we have

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n}\theta_{i\_2}s_i^2(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) - \sum_{i=1}^{n}\theta_{i\_1}^*|s_i(t)|. \quad (25)$$

Case 2: $\hat{\theta}_{i\_1}(t)|s_i(t)| > \varepsilon_i(t)$ for all $i \in \{1, \ldots, n\}$. Like Case 1, taking the first term on the right-hand side of (6) into account, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n}\hat{\theta}_{i\_1}(t)|s_i(t)| - \sum_{i=1}^{n}\theta_{i\_2}s_i^2(t) - \sum_{i=1}^{n}\varepsilon_i(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) + \sum_{i=1}^{n}(\hat{\theta}_{i\_1}(t) - \theta_{i\_1}^*)|s_i(t)|, \quad (26)$$

which implies that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n}\theta_{i\_2}s_i^2(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) - \sum_{i=1}^{n}\theta_{i\_1}^*|s_i(t)| - \sum_{i=1}^{n}\varepsilon_i(t). \quad (27)$$

From (27), we have

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\sum_{i=1}^{n} \theta_{i\_2} s_i^2(t) + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) - \sum_{i=1}^{n} \theta_{i\_1}^* |s_i(t)|. \quad (28)$$

From the inequalities (25) and (28), it is obtained that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\theta_{2,\min}\|s(t)\|_2^2 + \|s(t)\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) - \theta_{1,\min}^*\|s(t)\|_2, \quad (29)$$

for all $t \geqslant 0$, where $\theta_{2,\min} = \min\{\theta_{1\_2}, \ldots, \theta_{n\_2}\}$. Thus, applying (20), we conclude that

$$\frac{\mathrm{d}}{\mathrm{d}t} V_2[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\theta_{2,\min}\|s(t)\|_2^2 \leqslant 0, \quad (30)$$

for all $t \geqslant 0$, which implies all signals are bounded and furthermore the signal $||s(\cdot)||_2 \in L_2$ (Ioannou & Sun, 1996). Finally, to show the zero convergence of the tracking errors $e(\cdot)$ and $\dot{e}(\cdot)$, we need to guarantee $\|s(\cdot)\|_2$ is uniformly continuous and then apply Barbălat's Lemma (Ioannou & Sun, 1996). Sufficiently, we investigate boundedness of the signal $\dot{s}(\cdot)$ from (2) and (6) and easily verify such a condition. As a result, the zero convergence is insured. □

Remark 3. From Lemma 1 and inequality (10), we known that β_1, β_2, β_3 and l_1 are independent of $\theta_{1,\min}^*$. Thus, inequality (30) is derived from (29) when $\theta_{1,\min}^*$ is large enough.

Remark 4. The scheme in Theorem 3 can be applied without any prior knowledge of robot manipulators such as initial conditions and parameters in (1). Accordingly, the proposed scheme is indeed a fully adaptive decentralized control. This scheme is a modified variable-structure control coupled with a convergent boundary layer. Convergence of the signal $s_i(\cdot)$, $i \in \{1, \ldots, n\}$, into the sliding mode $s_i \equiv 0$ does not correspond to occurrence of infinite gain (Filippov, 1964). When this scheme is implemented via computer controller, what one really concerns is the boundedness of the control input, and it is worth noting that the control input τ in (6) is always bounded. Thus there is no infinite-gain problem in the proposed scheme.

Remark 5. When all knowledge of robot manipulators is provided *a priori*, the constants $\beta1$, $\beta2$, $\beta3$ and the bound l_1 can be estimated in advance, and then the desirable $\theta_{i\_1}^*, i \in \{1, \ldots, n\}$, can be determined by making (20) satisfied. Substituting these determined desirable values into (7) to replace the on-line adjusted gains, the scheme in Theorem 2 becomes a non-adaptive control, which can be guaranteed by the similar procedure in the proof of Theorem 2.

The convergent boundary layer (8) might lead to chattering in actual implementations. As a result, it is modified as follows:

$$\dot{\varepsilon}_i(t) = -p_i\varepsilon_i(t) + w_i, \quad \varepsilon_i(0) > 0, \quad (31)$$

for $i \in \{1, \ldots, n\}$, where the constants p_i, $w_i > 0$, $i \in \{1, \ldots, n\}$, such that the signal $\varepsilon_i(\cdot)$, $i \in \{1, \ldots, n\}$ has a positive lower bound, which can be made smaller by use of design parameters such as smaller w_i, $i \in \{1, \ldots, n\}$. With modification (314), adaptive law (9) should also be made robust such that the parameter drift can be avoided (Fu, 1992). It is achieved by using addition of leakage term as

$$\dot{\hat{\theta}}_{i\_1}(t) = \gamma_{i\_1}|s_i(t)| - \sigma_{i\_1}\hat{\theta}_{i\_1}(t), \quad \hat{\theta}_{i\_1}(0) \geqslant 0 \quad (32)$$

for $i \in \{1, \ldots, n\}$, where the constants γ_{i_1}, $\sigma_{i\_1} > 0, i \in \{1, \ldots, n\}$ (the latter is used as the leakage constant). Note that the modified boundary layer (31) and robust adaptive law (32) is also apparently in a decentralized structure. The performance of the robust adaptive control law (6)–(7) and (31)–(32) is summarized into the following theorem.

Theorem 3. *Under assumptions* (A1)–(A3), *consider the error dynamics of robotic manipulator subjects to the adaptive control law* (6)–(7) *and* (31)–(32). *If the gain* $\theta_{i\_j} > 0$, $i \in \{1, \ldots, n\}$ *and* $j \in \{2, 3\}$, *then all signal are bounded, and the position tracking error* $e(\cdot)$ *and velocity tracking error* $\dot{e}(\cdot)$ *globally "exponentially converge" to a residue set whose size can be reduced by use of smaller* $w_{\max}$ *and* $\sigma_{1,\max}$ *where* $w_{\max} = \max\{w_1, \ldots, w_n\}$ *and* $\sigma_{1,\max} = \max\{\sigma_{1\_1}, \ldots, \sigma_{n\_1}\}$.

Proof. *Step 1*: *Prove the signal* $s(\cdot)$ *is bounded*. This procedure is the same as that in the proof of Theorem 2, and hence is omitted.

Step 2: *Prove all signals are bounded and the signal* $s(\cdot)$ *"globally exponentially converges" to a residual set whose size can be reduced by means of smaller* $w_{\max}$ *and* $\sigma_{1,max}$. Now consider the Lyapunov-like function $V_3 : [0, \infty) \times R^n \times R \times \cdots \times R \to R$ as follows:

$$V_3(t, s, \hat{\theta}_{1\_1}, \ldots, \hat{\theta}_{n\_1}) = \frac{1}{2} s^{\mathrm{T}} M[q(t)]s + \sum_{i=1}^{n} \left[\frac{1}{2}\gamma_{i\_1}^{-1}(\hat{\theta}_{i\_1} - \theta_{i\_1}^*)^2\right], \quad (33)$$

where states $s \in R^n$ and $\hat{\theta}_{i\_1} \in R$, $i \in \{1, \ldots, n\}$. Taking time derivative of (33) along the trajectory of the closed-loop system and then following the similar argument as that in the proof of Theorem 2, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} V_3[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] \leqslant -\delta \cdot V_3[t, s(t), \hat{\theta}_{1\_1}(t), \ldots, \hat{\theta}_{n\_1}(t)] + \sum_{i=1}^{n} \varepsilon_i(t) + \frac{1}{2}\sum_{i=1}^{n} \gamma_i^{-1}\sigma_{i\_1}{\theta_{i\_1}^*}^2, \quad (34)$$

for all $t \geqslant 0$, where the positive constant δ satisfies that

$$\delta \leqslant \min \left\{ \frac{\theta_{2,\min}}{\mu_M}, \sigma_{1\_1}, \ldots, \sigma_{n\_1} \right\}. \tag{35}$$

Following the stability analysis in Ioannou & Kokotović (1984), it is guaranteed that all signals are bounded and the signal s(·) globally converges exponentially to a residual set, which is centered at zero and whose size can be made smaller by use of smaller $w_{\max}$ and $\sigma_{1,\max}$. Thus, the signals $e(\cdot)$ and $\dot{e}(\cdot)$ also globally converge exponentially to a residual set whose size can be reduced by use of smaller $w_{\max}$ and $\sigma_{1,\max}$. □

4. Simulation results and discussions

Numerical results are provided in this section. Tracking control of two-link planar elbow robot manipulator moving on a vertical plane is studied (Spong & Vidyasagar, 1989), where the gravity magnitude is 9.8 (N/kg). Links 1 (lower) and 2 (upper) are assumed uniformly thin cylinders where Link 1 is with mass 3 kg, link length 1 m, and mass center at 0.5 m. And Link 2 is with mass 1 kg, link length 1 m, and mass center at 0.5 m. The friction inputs are assumed zero here. The desired joint trajectory $q_d(t) = [q_{d1}(t), q_{d2}(t)]^{\mathrm{T}} \in R^2$ for all $t \geqslant 0$ is defined

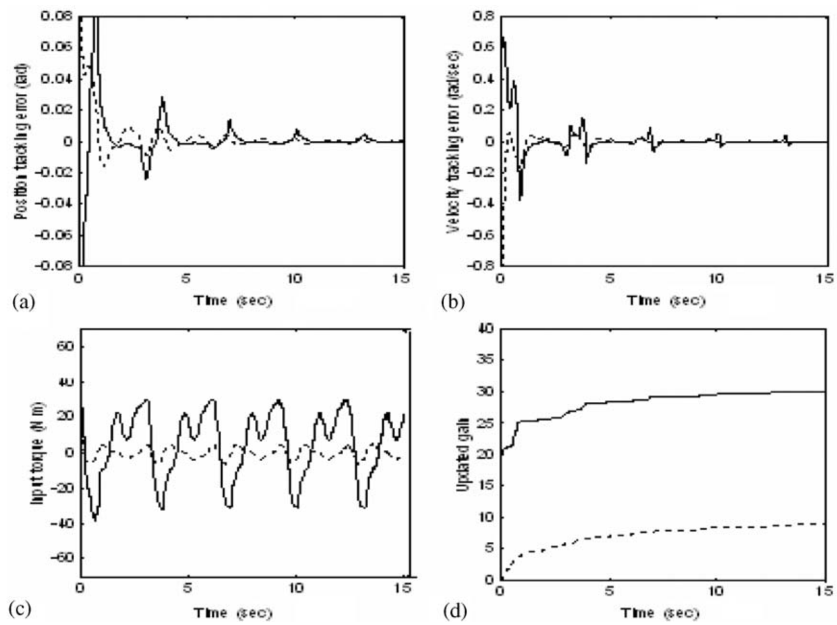

Fig. 1. Numerical results of Joints 1 (solid line) and 2 (dotted line) without boundary-layer modification.

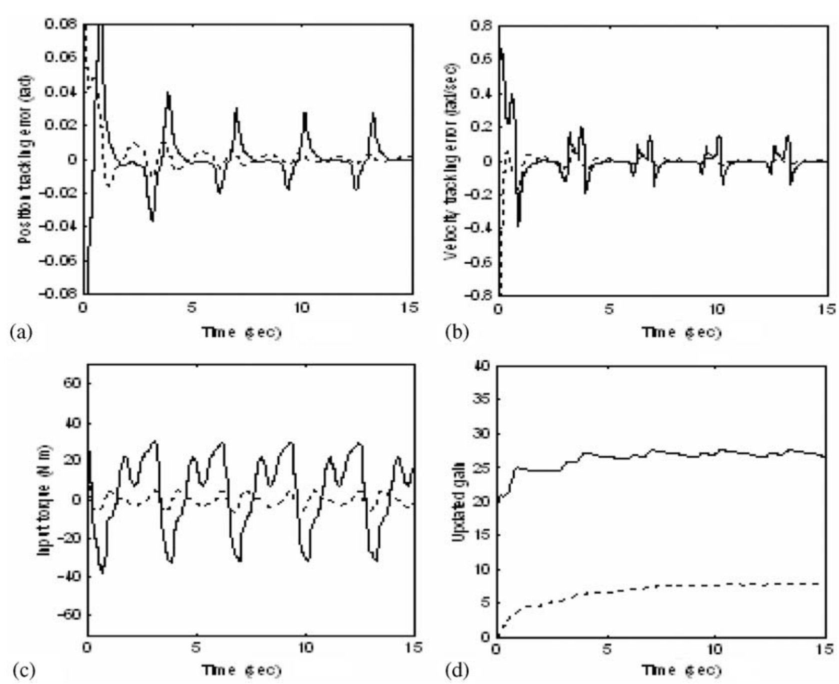

Fig. 2. Numerical results of Joints 1 (solid line) and 2 (dotted line) with boundary-layer modification.

by $q_{d1}(t) = 0.2 + 2\sin 2t$ (rad) and $q_{d2}(t) = -1.7 + 1.8\sin 2t$ (rad). The initial conditions of robot manipulators and adaptive control law are considered as $q_1(0) = 0$ (rad), $q_2(0) = 0.3$ (rad), $\dot{q}_1(0) = 4.25$ (rad/s), $\dot{q}_2(0) = -0.2$ (rad/s), $\hat{\theta}_{1\_1}(0) = 20$, $\hat{\theta}_{2\_1}(0) = 0$, $\varepsilon_1(0) = 1$, and $\varepsilon_2(0) = 1$. The design parameters for Theorem 3 are shown as

$$\begin{aligned} &\lambda_1 = \lambda_2 = 6, \quad \theta_{1\_2} = \theta_{2\_2} = 15, \\ &\theta_{1\_3} = \theta_{2\_3} = 10, \\ &p_1 = p_2 = 0.15, \quad \gamma_{1\_1} = \gamma_{2\_1} = 20. \end{aligned} \tag{36}$$

Besides, the additional design parameters are:

$$w_1 = w_2 = 0.025, \quad \sigma_{1\_1} = \sigma_{2\_1} = 0.025, \tag{37}$$

other design parameters for Theorem 3 are the same as those in (36). All numerical results are obtained by use of Simnon®, where the step time is 1×10^{-3} s and accuracy is 1×10^{-6}. Fig. 1 depicts the numerical results following Theorem 2, in which the boundary layer has no modification (i.e., the boundary layer is always positive and converges to zero exponentially). In this case, both the position and velocity tracking errors converge to zero asymptotically, whereas the input torques are bounded but have the chattering when tracking errors are small (see the input torques after 10 s). On the other hand, numerical results following Theorem 3 are depicted in Fig. 2, where the boundary layer has been modified as (31). It is observed that both the position and velocity tracking errors converge exponentially to a residual set, which is centered at zero. Due to the finite boundary layer, the input torqueses are bounded as well as have no chattering.

5. Conclusion

In this paper, we developed a fully decentralized control scheme of robot manipulator for trajectory tracking. By use of high-order and adaptive variable-structure compensations, the control scheme not only makes tracking errors of robot manipulators globally converge to zero asymptotically but also allows all signals in closed-loop systems to be bounded, even without any prior knowledge of robot manipulators. For avoiding the possible chattering in actual implementations, the proposed scheme can be modified with the finite boundary layers and robust adaptive law such that all signals are bounded, and both position and velocity tracking errors globally converge exponentially to a residual set, which is centered at zero and whose size can be made smaller by the design parameters.

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