Flow Allocation in Multi-hop Wireless Networks: A Cross-Layer Approach

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***Abstract*— This paper addresses the flow allocation problem in multi-hop wireless networks. We define and formulate a new interference model, referred to as the Node-based Interference Model, to better capture the behavior of medium access control protocols and the physical layer interference issues. Based on this model, we formulate the problem as a cross-layer network utility maximization problem that considers the coordination of the transport, MAC and physical layers, and avoid the maximum clique or independent set enumeration approach as adopted in most of the existing work. The objective of the problem is to maximize the aggregate network throughput while maintaining the fairness among flows. We then propose a gradient-based flow allocation algorithm by using the duality approach, and analyze the rate of convergence to the optimum for the proposed algorithm. The simulation results show that the proposed algorithm can rapidly converge to the optimum, and can also rapidly adapt to the changes in network topology and routing paths in different flow scenarios.**

***Index Terms*— flow allocation, interference, multi-hop wireless networks.**

I. Introduction

Research in multi-hop wireless networks has received much attention in recent years. In such a network, packets are forwarded in a hop-by-hop manner without the assistance of a pre-deployed infrastructure. Each flow, in addition to contending for local resource at each intermediate node in its routing path, referred to as *local interference*, must also compete for the shared wireless medium with those flows located within its interference range, referred to as *location-dependent interference*. Local interference is characterized by the half-duplex property of the wireless transceiver, which means that it can either transmit or receive data at any time, but not both simultaneously; location-dependent interference is characterized by the property of the radio signal reception. These unique characteristics spawn many research challenges in resource management for end-to-end sessions in multi-hop wireless networks. Due to resource contention from different layers, traditional single layer design disciplines lead to inefficient performance. This calls for cross-layer design manner [1], [2] to coordinate among the transport, MAC and physical layers so that the resource can be efficiently utilized.

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The two models which are used widely for describing the location-dependent interference among packet transmissions in single channel wireless networks are the *Protocol Model* [13] and the *Physical Model* [13], [14]. Variations to the *Protocol Model* include the static interference model [15], [16] and the flow-dependent conflict graph [17]. Existing work [11], [20], [22] solve resource allocation by building on the concept of the conflict graph. In a conflict graph, each vertex corresponds to a wireless link and an edge between two vertices exists if the transmissions on the two wireless links contend with each other. A complete subgraph of the conflict graph is referred to as a clique. The maximal clique, representing a maximum set of mutually contending wireless links, is a clique that is not contained within any other cliques. The interference problem is then tackled by the enumeration of the maximal cliques in the network. However, clique computation is NP-complete [23], and worse, the clique constraints are insufficient to guarantee the optimality of link utilization [22]. Additional challenges arise when one attempts to implement these ideas in a distributed manner. Previous work [24]–[27] adopts the *Physical Model* for resource management. The idea is to optimize the network capacity while satisfying the power constraint of each node. Once the capacity of each link is determined, the flow allocation problem in wireless multi-hop networks can be treated in a similar way as that in wired networks. However, the calculation of the capacity region based on the *Physical Model* requires selecting the sets of concurrently active communication links. The determination of the sets of concurrently active links is time-consuming because each link in a set will interfere with the other links.

The relationship among the interference caused by wireless communications, the supportable data rate of a node, and the end-to-end flow rate control problem have not been explicitly addressed in all of the existing work. The MAC issues caused by the interference due to simultaneous transmissions have also not been addressed and characterized. This calls for new mechanisms which jointly consider these relations without complicated computation. The following two issues motivate our work: 1) avoiding the enumeration of maximum cliques or the sets of concurrently active links when considering location-dependent interference; 2) providing a general approach which accounts for the interference constraints in MAC protocol designs in arbitrary network topologies.

In this paper, we study the flow allocation problem in wireless multi-hop networks. The objective of this study is to optimize global resource allocation by maximizing the aggregate utilization of wireless resource with coordination between the transport, MAC and physical layers. To achieve this objective, we define and formulate a new interference model, referred to as the *Node-based Interference Model*, which accounts for MAC protocols and captures the behavior of local interference and location-dependent interference for multi-hop wireless networks. This model enables each node to locally identify the interference that occurred at the physical layer and contention behavior at the MAC layer only through signal power measurement. Therefore, the complexity of mutual interference and contentions among neighboring nodes can be reduced while the key factors of physical and MAC layers can be characterized. Compared with the *Protocol Model* and *Physical Model*, this model can simplify the cross-layer design for network planning and wireless resource management, and can characterize the relationship among interference, data rate, and medium access contentions. Based on the *Node-based Interference Model*, we formulate the optimal flow allocation problem as a convex optimization problem such that 1) the behavior of the interference and the supportable data rate at the physical layer, medium contention at the MAC layer, and end-to-end flow issues at the transport layer can be jointly considered, and 2) the clique or the independent set computation can be eliminated with the node-based interference constraints. We then propose a gradient-based flow allocation algorithm with the duality approach, which can be easily extended to a distributed algorithm in a way similar to [3], [4]. The proposed algorithm is shown to be primal-dual optimal and can converge to the optimum within a limited number of iterations. The performance of the proposed algorithm is evaluated via numerical study in different flow scenarios.

The rest of the paper is organized as follows. In Sec. II, the new interference model with respect to local interference and location-dependent interference in multi-hop wireless networks is characterized and formulated. In Sec. III, the flow allocation problem is formulated and solved by the duality approach. In Sec. IV, an optimal flow allocation algorithm based on the gradient projection method is proposed. In Sec. V, the simulation results are provided. Finally, the paper is concluded in Sec. VI.

II. System Model

A. Interference in Wireless Networks

We consider a multi-hop wireless network G=(V,E), where V is the set of nodes and E is the set of links in the network. Let $P_t(i)$ denote the transmission power of node i; $d_{i,j}$ be the distance between nodes i and j; $L(.)$ be the path gain function, and σ be the thermal background noise. Consider a pair of nodes $i, j \in V$, the received power at node j, i.e., $P_r(j) = P_t(i)L(d_{i,j})$, must exceed a threshold to correctly receive a data unit from transmitter i. Hence, we have $SNR_{i,j} \geq \theta$, where $SNR_{i,j} = P_t(i)L(d_{i,j})/\sigma$ is the Signal to Noise Ratio (SNR) of the wireless link (i, j), σ is a constant, and θ is the SNR threshold for a node to correctly decode the signal. The transmission range r_i of transmitter i is the largest distance from i that node i's data packets can be correctly decoded. It is determined once the transmission power $P_t(i)$ of node i and θ are given.

For a multi-hop wireless network, multiple pairs of nodes may transmit data units simultaneously. In addition to the thermal noise, the transmission from node i to node j may be interfered by other concurrent transmitters. Let K denote the set of concurrent transmitters. The Signal to Interference Ratio (SIR) for link (i, j) is defined in [13], [14] by

$$SIR_{i,j} = \frac{P_t(i)L(d_{i,j})}{\sum_{k\in K} P_t(k)L(d_{k,j}) + \sigma}. \tag{1}$$

For node j to receive a data unit from node i correctly, the $SIR_{i,j}$ of link (i, j) must exceed the threshold β. The value of threshold β is determined by the settings of wireless physical layer (PHY). In this paper, we adopt the default setting of IEEE 802.11 PHY, i.e., the collision threshold ($CP\_Threshold$) of IEEE 802.11 PHY is defined as the signal to interference ratio. Thus, we have $CP\_Threshold = \beta \times \sum_{k\in K} P_t(k)L(d_{k,j})$.

B. Node-based Interference Model

We assume that each node $i \in V$ is characterized by an SNR threshold θ_i to receive one data unit from a transmitter. The SIR threshold β_i, provided that $\theta_i > \beta_i$, is also given so as to guarantee correct signal decoding when there are concurrent transmissions contending for the resource. Based on the Shannon theorem, the supportable data rate of any communication link incident to node i is at least $R_i = W \times log_2(1+\beta_i)$, where W is the frequency bandwidth of the communication channel. Only when the SIR of the received signal is smaller than β_i can the supportable data rate of this node be assumed to be zero, and thus the transmission be prohibited from accessing this wireless link.

Let $P_{max}(i)$ denote the maximum transmission power of node i. Suppose each node i can adjust its transmission power $P_t(i)$, $0 \leq P_t(i) \leq P_{max}(i)$ such that the signal power of the receiver node j is slightly larger than $\theta_j \times \sigma$. Then, the maximum supportable data rate of a wireless link connecting node j is given by $R_{j,max} = W \times log_2(1 + \theta_j)$ if there is no interference contributed by the neighboring nodes. The maximum interference budget B_j, which denoted that node j can sustain to correctly decode the signal from a transmitter, is given by $B_j = (\theta_j \times \sigma/\beta_j) - \sigma$. For a particular node k, the ratio of the interference contributed by the concurrent transmission from node i to node j, denoted by $\omega_{i,k,j}$, can be expressed by

$$\omega_{i,k,j} = \frac{P_t(i) \times L(d_{i,k})}{B_k} = \frac{L(d_{i,k})\theta_k\beta_k}{L(d_{i,j})(\theta_k - \beta_k)}. \tag{2}$$

The set of nodes which renders the interference ratio $\omega_{i,k,j} \geq 1$ is called the set of contending nodes for node k. The occurrence of any communication at each contending node will cause the supportable data rate of node k to drop to zero, and therefore, prohibit node k from accessing the wireless medium. Let $\varsigma_{i,k,j}$ denote the interference indicator for the communications performed at the set of contending nodes of node k. $\varsigma_{i,k,j} = 1$ if node k contends with the transmission from node i to node j; otherwise, $\varsigma_{i,k,j} = 0$.

The concept of the *Node-based Interference Model* is illustrated in Fig. 1. Fig. 1(a) shows the relationship between the location-dependent interference contributed by the set of concurrent transmitting nodes and the capacity shared by a

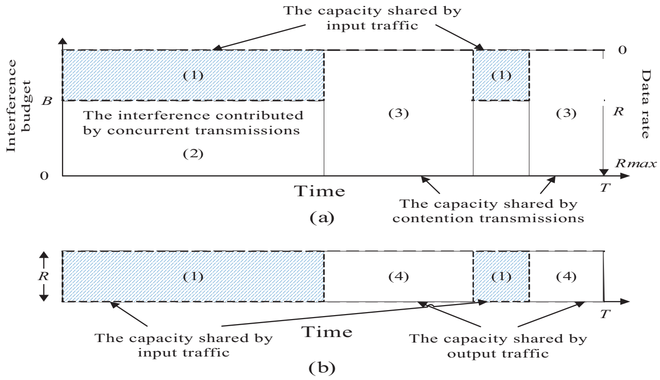


Fig. 1. Node-based interference model: (a) location-dependent interference; (b) local interference.

particular node. During each fixed time period T, the capacity region [1] can be classified into two types of regions: 1) the region which can be shared with the set of concurrent transmitters (i.e., area (1)+(2)), and 2) the entire region which is consumed by the set of contending nodes (i.e., area (3)). During the portion of time that the interference caused by the set of concurrent transmitters (i.e., area (2) in Fig. 1(a)) does not exceed the budget B, the node is allowed to receive input traffic at least data rate R (i.e., area (1) in Fig. 1(a)). During the portion of time for area (3) in Fig. 1(a), the interference contributed by the set of contending nodes is larger than B, and thus the data reception at this node is prohibited.

Consider a set of end-to-end flows, denoted by Γ, in a multi-hop wireless network. Each end-to-end flow, denoted by $f = \{s, d\}$, traverses the system from the source node s through multiple hops to the destination node d. Let $t^f_{i,j}$ denote the portion of time shared by flow f traversing from node i to node j. Based on the location-dependent interference from both sets of concurrent transmitters and contending nodes, we have

$$\sum_{f\in\Gamma}\sum_{j\in V} t^f_{j,i} + \sum_{f\in\Gamma}\sum_{j\in V}\sum_{k\in V(i,j)} \varsigma_{j,i,k} \times t^f_{j,k} \leq T. \quad (3)$$

$$\sum_{f\in\Gamma}\sum_{j\in V}\sum_{k\in V(i,j)} \omega_{j,i,k}(1-\varsigma_{j,i,k})t^f_{j,k} - \sum_{f\in\Gamma}\sum_{j\in V} t^f_{j,i} \leq 0. \quad (4)$$

Fig. 1(b) illustrates the impact of local interference of a node on the channel capacity sharing. The capacity can be shared by the input traffic and the output traffic. This kind of interference is imposed by most MAC protocols, giving rise to the name the *MAC constraint*, and expressed by

$$\sum_{f\in\Gamma}\sum_{j\in V} t^f_{j,i} + \sum_{f\in\Gamma}\sum_{j\in V} t^f_{i,j} \leq T. \quad (5)$$

III. Flow Allocation in Multi-hop Wireless Networks

In this section, we formulate the problem based on the previous discussion with respect to the interference and MAC constraints in multi-hop wireless networks. Note that the formulation below is based on single-path routing. It can also be extended to the multi-path flow allocation problem in a straighforward manner.

[1]Each capacity region is given by a data rate times a period of time.

A. Problem Formulation

Each end-to-end flow $f \in \Gamma$ is associated with a utility function $U_f(x_f)$, which indicates the degree of satisfaction of its end-user. Let $C_i = T \times R_i$ denote the total capacity of node $i \in V$. Assume that the utility function $U_f(.)$ is an increasing, strictly concave, twice continuously differentiable function of x_f over the interval $0 < x_f < max\{C_i | i \in V\}$. The traffic satisfying such a utility function is described as elastic [30]. We further assume that the utilities are additive such that the aggregate utility of flows can be regarded as the network utility. If link (i, j) carries the traffic of flow f, then $r^f_{i,j} = 1$; otherwise, $r^f_{i,j} = 0$. Thus, we have $x_f = \sum_{f\in V} t^f_{s,j} \times R_j$. The objective of this problem is to maximize the total network utility over $\mathbf{x} = (x_f, f \in \Gamma)$ subject to the local and location-dependent constraints along with the medium contention consideration. Such an objective function can achieve the optimal resource utilization and realize the fairness models such as max-min or proportional fairness [30]. Substituting the definition of a flow into (3) and (5), we can formulate the problem of flow allocation in multi-hop wireless networks as a convex optimization problem as follows.

$$\textbf{P: } Maximize\ f(x) = \sum_{f\in\Gamma} U_f(x_f), \quad (6)$$

subject to

$$\sum_{f\in\Gamma}\sum_{j\in V} r^f_{j,i} x_f + \sum_{f\in\Gamma}\sum_{j\in V} r^f_{i,j} x_f \frac{R_i}{R_j} \leq C_i, \quad (7)$$

$$\sum_{f\in\Gamma}\sum_{j\in V} r^f_{j,i} x_f + \sum_{f\in\Gamma}\sum_{j\in V}\sum_{k\in V(i,j)} \varsigma_{j,i,k} r^f_{j,k} x_f \frac{R_i}{R_j} \leq C_i. \quad (8)$$

The objective function in (6) is to maximize the aggregate utility over all flows. By optimizing this objective function, both optimal and fair flow allocation can be achieved. The feasible region of the optimization problem jointly formed by constraints in (7) and (8) is a convex and compact set.

B. Duality

With the assumptions on the utility function, the objective function of the primal problem **P** in (6) is differentiable and concave. In addition, the feasible region of the optimization problem in (7)(8) is convex and compact. Based on the non-linear optimization theory, there exists an optimal value of $\mathbf{x}^*$ for the primal problem **P**. The Lagrangian form of the optimization problem **P** can be expressed as follows.

$$L(x, \lambda, \mu) = \sum_{f=1}^{|\Gamma|} U(x_f) + \sum_{i=1}^{|V|} \lambda_i [C_i - \sum_{f=1}^{|\Gamma|} a_{if} x_f] + \sum_{i=1}^{|V|} \mu_i [C_i - \sum_{f=1}^{|\Gamma|} b_{if} x_f], \quad (9)$$

where $a_{if} = \sum_{j=1}^{|V|} r_{j,i}^f + \sum_{j=1}^{|V|} r_{i,j}^f \frac{R_i}{R_j}$, and $b_{if} = \sum_{j=1}^{|V|} r_{j,i}^f + \sum_{j=1}^{|V|} \sum_{k=1;k\neq i,j}^{|V|} \varsigma_{j,i,k} r_{j,k}^f \frac{R_i}{R_k}$.

In (9), λ_i,, and $\mu_i, i \in V$, are Lagrange multipliers associated with a local interference constraint and a location-dependent interference constraint on node i, respectively. The addition of total network utility and the linearity of constraints lead to a Lagrangian dual decomposition into each individual flow f as follows.

$$L(x,\lambda,\mu) = \sum_{f=1}^{|\Gamma|} L_f(x_f,\lambda^f,\mu^f) + \sum_{i=1}^{|V|} C_i(\lambda_i+\mu_i), \quad (10)$$

where $\lambda^f = \sum_{i=1}^{|V|} \lambda_i a_{if}$ and $\mu_f = \sum_{i=1}^{|V|} \mu_i b_{if}$.

For each flow $f \in \Gamma$, $L_f(x_f,\lambda^f,\mu^f) = U_f(x_f) - (\lambda^f + \mu^f)x_f$ and its value is determined by x_f and flow prices λ^f and μ^f. Considering the expression $\lambda^f + \mu^f$, we obtain

$$\lambda^f + \mu^f = \sum_{i=1, i\neq j}^{|V|} \sum_{j=1}^{|V|} r_{j,i}^f (\lambda_i + \lambda_j \times \frac{R_j}{R_i} + \mu_i + \mu_j + \eta_{j,i}), \quad (11)$$

where $\eta_{j,i} = \sum_{k=1}^{|V|} \mu_k \varsigma_{j,k,i}$ represents the price of link (j,i) that is the aggregate interference price from the neighborhood of link (j,i).

To determine the Lagrange multipliers, we introduce the dual problem **g** of the optimization problem **P**, which can be formulated as follows.

$$\textbf{g:}\ min_{\lambda\geq 0, \mu \geq 0} \quad g(\lambda,\mu), \quad (12)$$

where $g(\lambda,\mu) = \max_x L(x,\lambda,\mu) = \sum_{f=1}^{|\Gamma|} S_f(\lambda,\mu) + V(\lambda,\mu))$, and

$$S_f = \max_{x_f}(U_f(x_f) - \sum_{i=1,i\neq j}^{|V|} \sum_{j=1}^{|V|} r_{j,i}^f (\lambda_i + \frac{\lambda_j R_j}{R_i} + \mu_i + \mu_j + \kappa_{j,i}) x_f),$$

$$V(\lambda,\mu) = max_c \ C_i(\sum_{i=1}^{|V|}(\lambda_i+\mu_i)).$$

The dual approach decomposes the original problem into the rate control problem S_f and the scheduling problem $V(\lambda,\mu)$ given the Lagrange multipliers λ and μ . In (10)-(11), the Lagrange multipliers λ_i can be interpreted as the implied cost of a unit flow accessing node i, and the Lagrange multiplier μ_i can be interpreted as the implied cost of a unit flow contributing interference to node i. From these equations, we observe that each flow f incurs a cost to each node that it traverses and to each node to which it contributes interference.

IV. Gradient-based Flow Allocation Algorithm

To solve the optimization problem presented in the previous section, we propose a de-centralized algorithm based on the node-based pricing framework. The objective is to achieve optimal flow allocation in multi-hop wireless networks. We first design an algorithm to determine the per-node price and to obtain the flow allocation schedule by using the gradient approach. Then, we analyze the properties of the algorithm.

TABLE I

Gradient-Based Flow Allocation Algorithm

Input: A set of nodes V, a set of source-destination pairs Γ, and the routing path of each flow.
Output: Flow assignment x_f for each flow $f \in \Gamma$.
1: Initialize flow $x_f(0) \leftarrow 0, \forall f \in \Gamma$, and node prices $\lambda_i \leftarrow 0, \mu_i \leftarrow 0, \forall i \in V$.
2: Update the price at each node $i \in V$. $\lambda_i(t+1) = [\lambda_i(t) - \alpha(C_i - \sum_{f=1}^{\|\Gamma\|}(\sum_{j=1}^{\|V\|} r_{j,i}^f + r_{i,j}^f \frac{R_i}{R_j})x_f)]^+$. $\mu_i(t+1) = [\mu_i(t) - \alpha(C_i - \sum_{f=1}^{\|\Gamma\|}(\sum_{j=1}^{\|V\|} r_{j,i}^f + \sum_{j=1}^{\|V\|}\sum_{k=1}^{\|V\|} r_{j,k}^f \varsigma_{j,i,k} \frac{R_i}{R_k})x_f)]^+$.
3: For each node $i \in V$, send the prices $\lambda_i(t+1)$ and $\mu_i(t+1)$ to the sender of the flow $f \in \Gamma$, for which $r_{i,j}^f = 1$ or $r_{j,i}^f = 1$ or $r_{j,k}^f \varsigma_{j,i,k} = 1$.
4: For each flow originator, after receiving node prices $\lambda_i(t+1)$ and $\mu_i(t+1)$ from each node $i \in V$, calculate the gradient by $\zeta_f(t+1) = \sum_{i=1,i\neq j}^{\|V\|} \sum_{j=1}^{\|V\|} r_{j,i}^f[\lambda_i(t+1) + \mu_i(t+1)\frac{R_j}{R_i} + \lambda_j(t+1) + \mu_j(t+1) + \sum_{k=1}^{\|V\|} \mu_k(t+1)\varsigma_{j,k,i}]$.
5: The flow allocation is adjusted by $x_f(t+1) = x_f(\zeta_f(t+1))$.

A. Gradient-Based Algorithm

By applying the gradient-based approach to the dual problem **g**, we propose an algorithm to calculate the multipliers of each node iteratively. The net benefit of the flow f is defined as follows.

$$\phi_f(x_f) = U_f(x_f) - \zeta_f x_f. \quad (13)$$

In (17), $\zeta_f = \lambda^f + \mu^f$ is the shadow price of flow f, and $\phi_f(x_f)$is the net benefit of flow f corresponding to the difference between its utility and its cost. Since $U_f(.)$ is an increasing, strictly concave, twice continuously differentiable function of x_f, the maximizer of $\phi_f(x_f)$ exists when

$$\frac{d\phi_f(x_f)}{dx_f} = U_f' - (\lambda^f + \mu^f) = 0, \quad (14)$$

where the maximizer is defined by

$$x_f(\zeta_f) = arg\ max_{x_f \in R}\ \phi_f(x_f). \quad (15)$$

We apply the iterative gradient projection method to solve the dual problem **g**. Let $\varpi = [\lambda,\mu]^T$, where $\lambda = (\lambda_i, i \in V)$ and $\mu = (\mu_i, i \in V)$ are Lagrange multiplier vectors. In this method, ϖ is adjusted in the opposite direction to the gradient $\nabla g(\varpi)$ as follows.

$$\varpi_i(t+1) = \begin{cases} \left[\varpi_i(t) - \alpha(C_i - \sum_{f=1}^{|\Gamma|}(\sum_{j=1}^{|V|}(r_{j,i}^f + r_{i,j}^f \frac{R_i}{R_j}))x_f)\right]^+, & if\ \varpi_i = \lambda_i \\ \left[\varpi_i(t) - \alpha(C_i - \sum_{f=1}^{|\Gamma|}(\sum_{j=1}^{|V|} r_{j,i}^f + \sum_{j=1}^{|V|}\sum_{k=1}^{|V|} r_{j,k}^f \varsigma_{j,i,k}\frac{R_i}{R_k} x_f))\right]^+, & if\ \varpi_i = \mu_i. \end{cases} \quad (16)$$

The iterative algorithm of computing an optimal flow allocation is summarized in Table I.

B. Convergence Analysis

In this section, we analyze the convergence behavior of the algorithm in Table I and characterize the property of its limit points. The result shows that the algorithm can converge to a unique flow allocation schedule such that the summation of all users' utilities is maximized.

To show the properties of the iterative algorithm, we define $Y(f) = \sum_{i=1}^{2|V|} A_{if}$, which gives the definition of $\bar{Y} = \max_{f\in\Gamma} Y(f)$ as, intuitively speaking, the "length" of the "longest" path. We further define $W(n) = \sum_{f=1}^{|\Gamma|} A_{nf}, n = 1, 2, ..., i, ..., 2i$, and $\bar{W} = \max_{n\in 1,...,2i} W(n)$, which gives the number of flows at the most "congested" node. Let $\bar{\kappa} = \max_{f\in\Gamma} \kappa_f$, where κ_f is the upper bound of $U_f(.)$ within the range $[0, \max C_i | i \in V]$.

Since $\varpi = [\lambda, \mu]^T$, we show that $\mathbf{g}$ is the Lipschitz continuity [29] on $\triangledown \mathbf{g}$. For any ϖ , $\varphi(\varpi)$ is defined by

$$\varphi(\varpi) = \begin{cases} \frac{1}{-U_f''(x_f(\varpi))}, & U_f'(M_f) \leq \zeta_f \leq U_f'(m_f), \\ 0, & otherwise \end{cases} \quad (17)$$

where $x_f(\varpi) = x_f(\zeta_f)$, $M_f = \max\{C_i | i \in V\}$, and $m_f = 0$. If U_f is bounded away from zero on $I : -U_f''(x_f) \geq 1/\kappa > 0$, we have $0 < \varphi(\varpi) < (\bar{\kappa})$. We note that the above expression is of the same form as that used in [18]. The sequence $(x(t), \varpi(t))$ generated by the iterative algorithm in TABLE I is primal-dual optimal assuming that $\epsilon \leq \alpha \leq (2-\epsilon)/\bar{\kappa}\bar{W}\bar{Y}$, where ϵ is a fixed positive scalar.

V. Numerical Studies

In this section, we evaluate the proposed algorithm by numerical studies. We assume that message updates are synchronized and communication delays are bounded. The thermal noise σ of each node is assumed to be $-90dBmW$. The maximum transmission power is $0.2mW$. The SNR threshold and SIR threshold of each node are set to 1.8 and 1.5, respectively. We use a simplified path gain function defined as $L(d_{i,j}) = 1/d_{i,j}^4$, where $d_{i,j}$ is the distance between the transmitter and the receiver. If the frequency bandwidth is assumed to be $1.6MHz$, then the wireless channel capacity can be derived as $2Mbps$. The routing path of a flow is determined by the shortest path routing algorithm. The utility function used in our simulation is defined by $U_f(x_f) = ln(x_f)$, because it has been shown in [20] that the proportional fairness can be achieved and the optimal condition can be satisfied with this utility function. We evaluate the convergence rate, the aggregate utility, and the flow throughput in a tandem network in different flow allocation scenarios as shown in Fig. 2. The distance between two adjacent nodes is set to 100 meters. Then, we evaluate the impact of routing configuration on our proposed algorithm in a random network.

A. Tandem Networks

We vary the tandem network size from 6 to 20 nodes and consider four scenarios for flow allocation: 1) a 3-flow scenario, 2) a mutual flow scenario, 3) an aggregate flow scenario, and 4) a reverse flow scenario. Fig. 2 (a) gives the 3-flow scenario, in which flow 1 goes through all hops of the network and flows 2 and 3 each traverse only one hop, *i.e.*, the first hop and last hop, respectively. In this example, flow 1 interferes with both flows 2 and 3, and flows 2 and 3 are forwarded simultaneously. Fig. 2 (b) gives the mutual flow scenario, in which one flow (*i.e.*, flow 1) traverse all hops of the network, and the other five flows are non-overlapped single-hop flows. Fig. 2 (c) gives the aggregate flow scenario with five flows in the network, each with a different source node but destined for the same node, *i.e.*, node 6. This scenario can be used to investigate the optimal flow allocation in wireless mesh networks. In Fig. 2(d), the reverse flow scenario with four flows is given. In this scenario, flows 1 and 2 traverse all hops in the network but in opposite directions. Flows 3 and 4 are both sent from node 3 but each traverses along the path in a different direction.

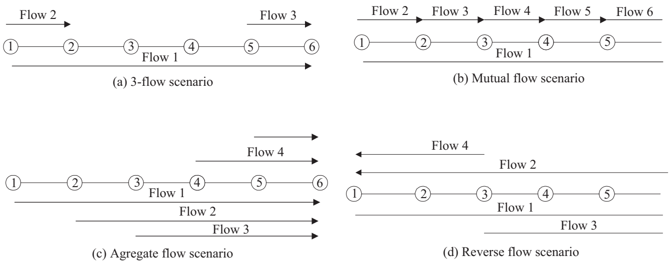


Fig. 2. A 6-node tandem network with three end-to-end flow scenarios.

TABLE II
THE STEP SIZE FOR CONVERGENCE

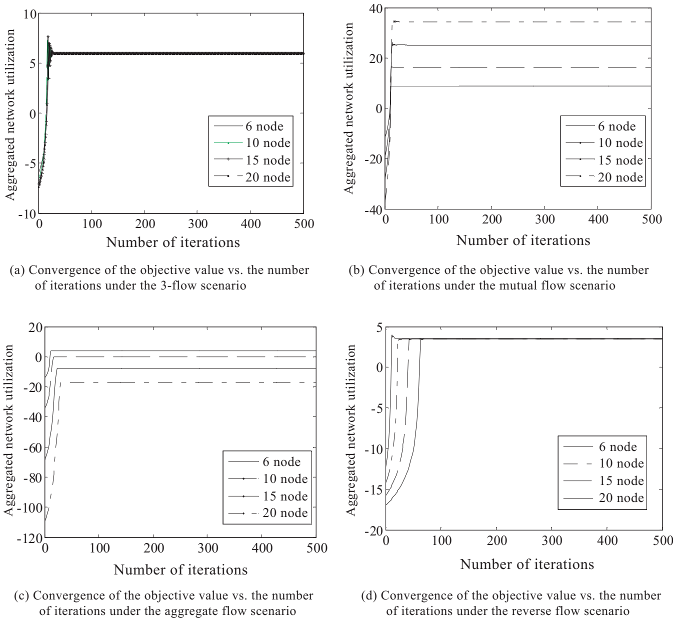
	3-flow	Mutual flow	Aggregate flow	Reverse flow
6-node	3.65	6.08	7.76	7.85
10-node	3.50	5.99	10.6	2.89
15-node	3.28	5.25	16.5	1.39
20-node	3.28	5.00	23.3	0.9

We evaluate the convergence rate of the proposed gradient-based flow allocation algorithm by properly adjusting step sizes. In all simulations, the initial values of all flows are fixed at 0 Kbps and the initial shadow prices are set to 1. Table II and Table III give the relationship between the step size and the number of iterations, respectively, in these four flow scenarios. Given the number of nodes and the flow scenario, we adjust the step size such that convergence to the optimum can be achieved at the least number of iterations. The optimum is achieved if $|x_f(t) - x_f^*| < \epsilon, |\lambda_i(t) - \lambda_i^*| < \epsilon$, and $|\mu_i(t) - \mu_i^*| < \epsilon$ are satisfied for all $f \in \Gamma$ and $i \in V$, where $\epsilon = 10^{-4}$.

From these results, we observe that when the network size grows, smaller step sizes and more number of iterations are required for convergence in the 3-flow, the mutual flow, and the reverse flow scenarios. Among these three scenarios, the reverse flow scenario gives the largest step size variance and the most number of iterations to converge. This is because flows 1 and 2 traverse the same set of nodes but in the opposite directions. Since the neighboring links along the path of a flow will interfere with each other, and the communication in the reverse direction will also contend for the node's bandwidth, a small change in price at each node will result in a higher

TABLE III
NUMBER OF ITERATIONS FOR CONVERGENCE

	3-flow	Mutual flow	Aggregate flow	Reverse flow
6-node	24	17	19	21
10-node	30	40	23	37
15-node	31	63	27	52
20-node	31	75	37	70



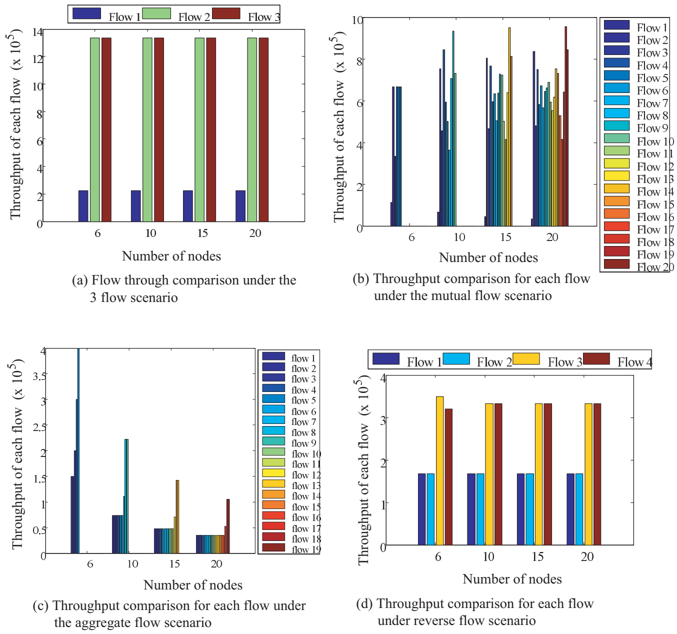
(a) Convergence of the objective value vs. the number of iterations under the 3-flow scenario

(b) Convergence of the objective value vs. the number of iterations under the mutual flow scenario

(c) Convergence of the objective value vs. the number of iterations under the aggregate flow scenario

(d) Convergence of the objective value vs. the number of iterations under the reverse flow scenario

Fig. 3. The convergence of objective value under different flow scenarios.



(a) Flow through comparison under the 3 flow scenario

(b) Throughput comparison for each flow under the mutual flow scenario

(c) Throughput comparison for each flow under the aggregate flow scenario

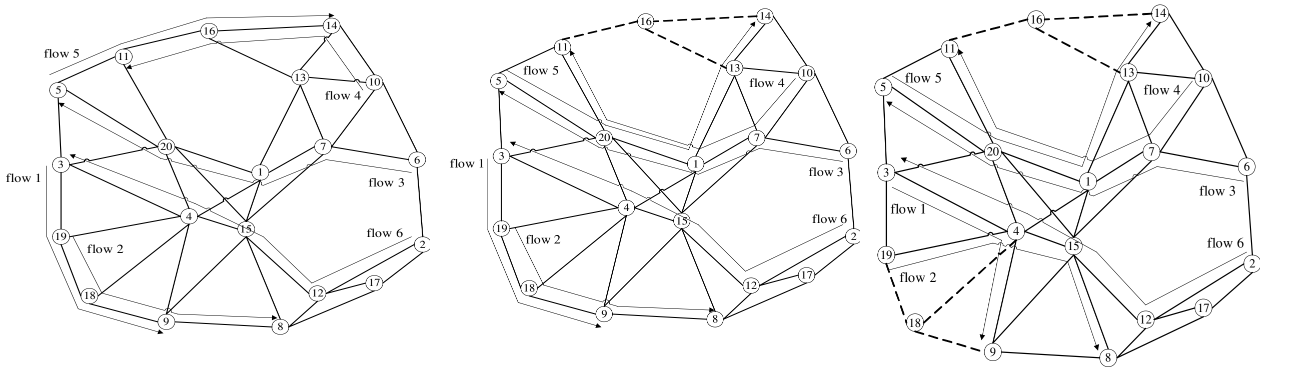
(d) Throughput comparison for each flow under reverse flow scenario

Fig. 4. Throughput comparison under different flow scenarios.

cost to the neighboring nodes and this increased cost will be propagated through the network. Thus, when the network size increases, it needs smaller step sizes and more number of iterations to achieve the optimum. In addition, the variance of step sizes and the number of iterations in the mutual-flow scenario is larger than that in the 3-flow scenario in the steady state for a network with larger than 10 nodes. While there is a long-path flow traversing all nodes in the network, all intermediate nodes of this flow are only interfered by the two neighboring wireless links two hops away. Due to this regularity among all intermediate nodes, the pricing updates at these nodes are consistent and hence the flow converges once other non-mutual interfering flows is determined. In the mutual-flow scenario, flows interfere with each other mutually, and thus more flows are affected when prices are updated at a particular node in the network, compared with the 3-flow scenario, and this may require further flow adjustments. Any change in flow rates may result in further price coordination to approach the optimum. Hence, a higher degree of mutual interference among flows leads to more iterations and a smaller step size for convergence to the optimum. In the aggregate flow scenario, the number of iterations and the step sizes increase with the network size. This is because the degree of mutual interference among flows at a particular node and the number of flows going through the node are un-balanced compared with the mutual flow scenario.

Fig. 3 gives the trajectories of the aggregate network utilization with different network scales in different flow scenarios, and Fig. 4 gives the throughput of each flow in each scenario. Consider Figs. 3 (a) and 4 (a) first, which are the results in the 3-flow scenario. The network utilization remains unchanged even though the network size is larger and the longest flow traverses over more number of nodes. This is because all intermediate nodes for the longest flow are interfered in the same way. Once the longest path flow is determined, the flows traversing the first hop and last hop can use the remaining resource of the destination node optimally since they will not interfere with one another. However, in the mutual-flow scenario as shown in Fig. 3(b), the aggregate network utilization increases with the network size. Due to the spatial reuse property of the wireless channel, the flows not contending mutually may share a node's capacity simultaneously. Fig. 4(b) gives the throughput of each flow in this scenario. The throughput of flow 1 may decrease due to traversing more intermediate nodes and thus incurring more contentions with the other flows.

Next, we move to the aggregate flow scenario. Fig. 3(c) shows that the overall network utilization is decreased as the network size increases. This is because the last hop suffers from the resource competition from all flows, and experiences the highest degree of mutual interference among flows. Therefore, this hop can be regarded as the bottleneck in the network and all other flows going through it will be affected. Fig. 4(c) shows that the two flows with their source nodes closer to the destination node gain more throughputs than the other flows. This is because these two flows experience less wireless resource contentions, and thus are allocated more resources. Finally, we observe the results of the reverse flow scenario. Fig. 3 (d) shows that as the network size increases, the total network utilization stays unchanged but the convergence time is increased. Fig. 4 (d) shows that each flow is allocated the same amount of resources under different network sizes. These results show that the flow change is sensitive to the



(a) Routing paths in the initial network topology (b) Routing paths after node 16 becomes unavailable. (c) Routing paths after node 18 becomes unavailabl

Fig. 5. A 20-node wireless network topology with 6 flows.

network scale, and thus the convergence rate degrades as the network size increase. However, the aggregate network utilization is not sensitive to the network size. When looking into the throughput of each flow in Fig. 4(d), we find that the same throughput distribution can be obtained under different network sizes. This is because the set of nodes that will be interfered by a particular link is limited, and the nodes outside of the interference range of a particular link can transfer data simultaneously thanks to the spatial reuse factor. Thus, irrespective of the increase in the network size, only a finite set of links contend for a node's resource.

B. Random Networks

In this section, we study the impact of sudden changes in the routing path on the convergence rate of our proposed algorithm in a randomly generated network as shown in Fig. 5(a). This network consists of 20 nodes distributed in a $500 \times 500m^2$ region. In this simulation, 6 flows between 6 pairs of source and destination nodes start at the same time instant. The simulation is performed for 1000 iterations. At the 250^{th}iteration, node 16 is assumed to be down, and the set of links incident to it becomes disabled immediately. The routing paths for flows 3 and 4 are changed as shown in Fig. 5(b). Similarly, node 18 becomes unavailable at the 500^{th} iteration and the routing paths for flows 1 and 2 are also changed as shown in Fig. 5(c).

The actual throughput of each flow is plotted in Fig. 6. Initially, each flow is assumed to transmit at $0Mbps$. After about 50 price updates, all flow allocations converge to $500Kbps$ which is a proportional fair value. At the 250^{th} iteration, the routing paths for flows 3 and 4 are changed due to the unavailability of node 16. The sudden change to routing paths causes nodes 1 and 20 to be overloaded. Since nodes 1 and 20 are the bottlenecks for flows 3, 4, and 5, the actual allocated throughput for each of these flows is thus decreased rapidly. Flow 3 suffers more throughput degradation since the number of hops it traverses is more than flows 4 and 5. In addition, the effects of interference and MAC contention make flows 1,2 and 6 contending for the resources of nodes 1 and 20, resulting in a slight throughput degradation for each of these flows even though their routing paths are not affected.

When the routing paths of flows 1 and 2 are changed due to the unavailability of node 18 at the 500 iteration, the rate of

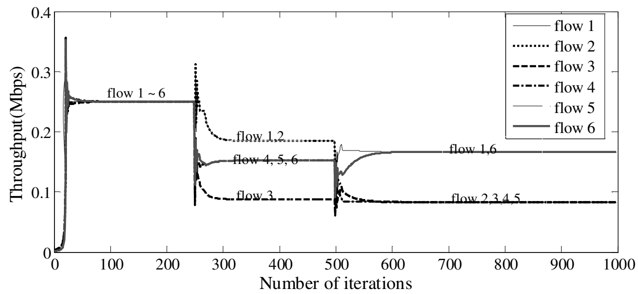


Fig. 6. Throughput trajectory of each flow when the network topology is changed.

each flow still converges within 50 iterations. This time, the flows converge to two groups of rates. The group of the higher rate includes flows 1 and 6. The other flows obtain a lower rate when flow allocation converges. At this stage, flows 1 and 6 obtain a higher throughput since they have lower interference caused by other concurrent flows. We can observe from tracing the trajectory of each flow in Fig. 5 that the throughput of flow 6 decreases when node 16 is down but increases after node 18 becomes unavailable. Therefore, we find that the impact of topology and routing change is not always negative for a particular flow. The flow contending with less number of flows in the routing path and suffering less interference from other concurrent flows will gain higher throughput.

VI. Conclusion

In this paper, we developed the *Node-based Interference Model* and a flexible theoretical framework to consider interference, data rate, and signal reception power at the physical layer, the contention behavior at the MAC layer, and end-to-end flows at the transport layer for multi-hop wireless networks. Based on this framework, we formulate an interference-aware optimal flow allocation problem without clique or independent set enumeration. The objective of the problem is to maximize network utilization and maintains fairness among flows. We then propose a gradient-based flow allocation algorithm by using the duality approach. The convergence of the gradient-based flow allocation algorithm is analyzed. The numerical results show that our proposed algorithm can achieve the optimum within a small number of iterations and can allocate resource to the end-to-end multi-hop flows to maximize optimal network utilization while maintaining fairness among flows. The simulation results show that the proposed algorithm can rapidly adapt to changes in network topology and routing paths. To the best of our knowledge, this is the first work which formulates the interference constraints for the flow allocation problem without any global information in multi-hop wireless networks. The results demonstrate that the proposed solution can also be used in emerging wireless mesh networks.

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