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Irreversible Investment, Financing, and Bankruptcy Decisions in an Oligopoly

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Abstract

This paper examines a firm's debt level, investment timing, and investment scale choices in a continuous-time model where the output price of a good that the firm produces depends on a stochastic demand-shift variable and the total industry supply of the good. Using the simple symmetric Cournot–Nash equilibrium assumption that all firms are identical and therefore follow the same financing and investment strategies, we show that competition decreases the output price and hence encourages a firm to wait for a higher demand level before it is profitable to invest. We also demonstrate how uncertainty, bankruptcy costs, and corporate taxation affect the firm's financing and investment decisions.

Keywords: bankruptcy; credit spreads; financing; irreversible investment; leverage; oligopoly; real options

JEL Classification: G13, G32

Irreversible Investment, Financing, and Bankruptcy Decisions in an Oligopoly

I. Introduction

This paper employs a real options model to investigate how competition affects a firm's operating and financing decisions. The real options literature (see, e.g., Dixit and Pindyck, 1994) typically focuses on a firm that is all-equity financed and has monopolistic access to an investment opportunity. This literature finds that the firm will delay investments beyond what is predicted by the net present value (NPV) rule due to the interaction of uncertainty and investment irreversibility. However, given that a firm usually competes for investment opportunities with other firms, it is important to ask whether, in the presence of competition, a firm will return to the simple NPV rule or divert further away from it. Further, if we assume that a firm has financial flexibility, a natural question to ask is how competition affects the firm's financing strategy and the risk premium required by bondholders.

To address these questions, we analyze a firm in an oligopoly and allow the firm to choose both its scale of irreversible investment and its debt level. We assume that the output price of a good that the firm produces depends on a stochastic demand-shift factor and the total industry supply of the good. The value of the firm is a function of the stochastic demand-shift factor and management's optimal operating and financial policies. The optimal operating policy is characterized by an endogenously determined "trigger" level; when the firm reaches this threshold level, it exercises its investment option and simultaneously chooses its scale of operation. The optimal financial policy involves the choice of debt level and an endogenous bankruptcy trigger. The optimal debt level, which is characterized by a trade-off between the tax advantage of debt financing and bankruptcy costs, is determined concurrent with the decision to exercise the investment option. The bankruptcy trigger, in contrast, is determined after the investment option is exercised.

To compare our results with those of the standard real options literature, we first investigate the case in which a firm's investment scale and its debt level are exogenously given. We find that in this context competitive pressure causes a firm to divert farther away from the NPV rule. This finding is inconsistent with the results of Grenadier (2002). Grenadier assumes that an oligopolistic firm has an incremental investment project, which is expandable at any future time, and shows that increased competition will lead a firm to invest sooner because the firm can acquire an option to expand later. By contrast, we assume that a firm has a lumpy investment project, which is non-expandable, and thus we find that the firm will be more cautious when competitive pressure is increased.[1] We therefore demonstrate that the impact of competition on investment timing choices hinges in large part on the assumption regarding the type of investment.

The finding that competitive pressure causes a firm to divert farther away from the NPV rule is also inconsistent with Mauer and Ott (2000). These authors assume, as we do, that investment is lumpy. However, they focus on a firm that has monopolistic access to an investment opportunity and find that competition destroys the value of this opportunity and in turn accelerates investment. By contrast, we assume that the output price decreases as the number of competitors in the market increases and find that as a result, competition encourages a firm to wait for more favorable market conditions before it is profitable to invest. Thus, we also demonstrate that even if the investment is lumpy, the impact of competition on investment timing choices depends largely on whether competition erodes investment value or eliminates investment opportunities.

Next, we examine how competition affects a firm's choice of investment scale by investigating the case in which a firm can freely choose its investment timing and scale. Intuitively, one may conjecture that competition will force a firm to reduce its operating scale because competition decreases the output price and hence an investment's value. This

[1] Throughout, "lumpy investment" means that a firm invests in typically large amounts at discrete points in time, while "incremental investment" means that the firm continually invests small amounts.

conjecture abstracts, however, from the offsetting effect of competition encouraging the firm to wait for more favorable demand conditions, which leads the firm to increase its operating scale. It is therefore possible that a firm will enlarge its investment scale when more competitors are in the market.

Finally, we investigate the most general case, that is, the case in which a firm's operating and financial policies are both endogenously determined. Employing plausible parameter values, we analyze how these policy decisions are affected by competition, uncertainty, bankruptcy costs, and corporation taxation.[2] We find that competition will cause a firm to issue more bonds and thus to declare bankruptcy sooner. Given that the firm is more susceptible to bankruptcy, the firm's bondholders will require a higher premium when buying risky debt. However, the firm's optimal leverage ratio is largely insensitive to changes in the number of competitors.

Before proceeding, it is important to note that our analysis also relates to the optimal debt structure literature. Similar to Kraus and Litzenberger (1973), we assume that a firm's choice of debt level involves a trade-off between the tax shield benefit and bankruptcy costs. However, the bankruptcy costs considered here differ from those in Kraus and Litzenberger (1973) because they view bankruptcy costs as exogenously given.[3] Our article is most closely related to Leland (1994), Mauer and Ott (2000), and Mauer and Sarkar (2005), but differs from these papers in the following respects: Leland assumes that a firm's value follows an exogenously given process, and thus he does not consider the interaction between the firm's operating and financing decisions, whereas the latter two articles do capture this interaction, but they abstract from the decision regarding investment scale.[4]

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[2] Brander and Lewis (1986) is the first paper to investigate the relation between financial structure and product market competition. However, they assume that a firm's choice of debt level involves a trade-off between the strategic benefit of debt financing and the agency cost of the kind conjectured by Myers (1977). Maksimovic (1995) provides a thorough review on the relation between financial structure and product market competition.

[3] Brander and Lewis (1988) also view bankruptcy costs as exogenously given.

[4] The firm we consider is the so-called first-best firm (see, e.g., Leland, 1998 and Mauer and Ott, 2000), which simultaneously makes investment and financing decisions. This first-best firm is distinct from the second-best firm, which issues bonds before making investment decisions, and thus bears the agency cost of the kind conjectured by Jensen and Meckling (1976) and Myers (1977). A rapidly expanding literature investigates

The rest of this paper is organized as follows. In Section II we present the basic assumptions and derive a firm's operating and financing decisions in Cournot–Nash equilibrium. We then investigate the comparative statics of each decision with respect to competitive pressure, uncertainty, bankruptcy costs, and corporate taxation. In Section III we perform a numerical analysis in which the firm's operating and financing decisions are both endogenously determined. In Section IV we conclude.

II. The Model

A. Basic Assumptions

This article extends the model of Grenadier (2002) and investigates the impact of competitive pressure on a firm's operating and financial policies in Cournot–Nash equilibrium.[5] Unlike Grenadier, we allow firms to access debt financing and we focus on lumpy rather than incremental investment. Consider an oligopolistic industry that consists of N identical firms that produce a single, homogeneous good. At date $t=0$, each firm i has no capital assets in place. At any time $t \geq 0$, the firm has the option to invest in a project, in which case it simultaneously chooses the optimal scale of the project and the optimal capital structure. Once the option is exercised, and the project scale and capital structure are determined, these policies are fixed thereafter.

Following Grenadier (2002) and Pindyck (1988), we abstract from the depreciation of capital and assume that each unit of capital allows the firm to produce one unit of output per time period. Denote by q_i the units of output produced by firm i, and by K the unit cost of capital, which is assumed to be constant over time. Output is infinitely divisible and its unit price, $P(t)$, fluctuates stochastically over time so as to clear the market, that is,

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how first- and second-best firms differ in their financing and investment behavior, as well as whether the magnitude of the agency cost of debt is significant. See, for example, Childs, Mauer, and Ott (2005), Ericsson (2000), Jou and Lee (2004a, 2004b), Leland (1998), Mauer and Ott (2000), and Mauer and Sarkar (2005) for further discussion on first-best versus second-best firms.

[5] This article also generalizes Jou (2001) by allowing investment scale decisions as well as strategic interactions between firms.

(1) $P(t) = X(t)Q^{\frac{-1}{\gamma}}, \quad \gamma > 0,$

where $Q = (\sum_{j=1}^{N} q_j)$ is the industry aggregate supply and γ is the constant elasticity of demand. To ensure that marginal profits (for any assumed number of firms, N) are increasing in $X(t)$, we assume that $\gamma > 1/N$. The term $X(t)$ is a multiplicative demand shock that evolves according to a geometric Brownian motion as follows:

(2) $dX(t) = X(t)\mu dt + X(t)\sigma dZ(t),$

where μ and σ are constants and $Z(t)$ is a standard Wiener process. We also assume that cash flows are valued in a risk-neutral framework with ρ as the riskless rate.

While Grenadier (2002) abstracts from variable costs of production, we assume that firm i incurs operating costs equal to $c_0 q_i^{\eta}$, where $c_0 > 0$ and $\eta > 1$.[6] The profit flow for firm i is the product of $P(t)$ and q_i, net of its operating costs, $c_0 q_i^{\eta}$, thus yielding $X(t) q_i Q^{-(1/\gamma)} - c_0 q_i^{\eta}$. After issuing bonds, firm i is obliged to pay the debt holders a fixed coupon payment at each instant, denoted by b, and all bonds are assumed to have no stated maturity.[7] The net earnings to equity holders are thus given by:

(3) $(1-\tau)(X(t) q_i Q^{\frac{-1}{\gamma}} - c_0 q_i^{\eta} - b),$

where τ is the corporate income tax rate. Equation (3) indicates that losses are fully offset, which is an approximation of the current U.S. tax system's allowance for partial offsets of operating losses through both carry-backward and carry-forward provisions. Following Leland (1994) and Mauer and Ott (2000), we assume that the equity holders of firm i can endogenously choose to declare bankruptcy at any time. After bankruptcy, debt holders

[6] The existence of variable costs is necessary since, otherwise, the choice of investment scale becomes trivial; if $\beta_1 > (<)\ \gamma N$, a firm will not invest (will install an infinite capital stock) because the marginal value of capital will be negative (positive). If $\beta_1 = \gamma N$, any level of capacity will satisfy the condition that the marginal value of capital is equal to zero.

[7] Following Leland (1994) and Mauer and Ott (2000), one can easily admit finite average debt maturity by assuming that debt does not have a stated maturity but is retired at par at a constant rate.

receive the residual value of the firm, which is equal to $(1-\lambda)Kq_i$, $0 \le \lambda \le 1$, assuming that λKq_i is lost during the bankruptcy process.[8] We also assume that a firm's equity holders can access unlimited external resources. Absent this last assumption, the firm may be forced to choose a level of debt that is unable to maximize total firm value (Fries, Miller, and Perraudin, 1997).

B. Equity and Debt Values and Bankruptcy Decisions

For ease of exposition, we refer to $X(t)$ as X in what follows. Let firm i's equity value and debt value be given by $V^e(X,Q,q_i,b)$ and $V^d(X,Q,q_i,b)$, respectively. Applying Itô's lemma and using the standard risk-neutral valuation arguments, we obtain the following differential equation for the value of equity, V^e:

$$(4)\quad \frac{1}{2}\sigma^2 X^2 + \mu X\frac{\partial V^e}{\partial X} + (1-\tau)(Xq_iQ^{\frac{-1}{\gamma}} - c_0 q_i^{\eta} - b) - \rho V^e = 0, \quad X > X_b,$$

where X_b is the endogenously determined bankruptcy trigger. The general solution of equation (4) is

$$(5)\quad V^e(X,Q,q_i,b) = (1-\tau)\left[\frac{XqQ^{\frac{-1}{\gamma}}}{(\rho-\mu)} - \frac{(b+c_0q_i^{\eta})}{\rho}\right] + a_1(Xq_iQ^{\frac{-1}{\gamma}})^{\beta_1} + a_2(Xq_iQ^{\frac{-1}{\gamma}})^{\beta_2},\ X > X_b,$$

where a_1 and a_2 are constants to be determined,

$$\beta_1 = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} > 1, \text{ and } \beta_2 = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0.$$[9]

As Leland (1994, 1998) and Mauer and Ott (2000) suggest, if a firm is not constrained by bond covenants, then bankruptcy will occur only when the firm cannot meet the required instantaneous coupon payments by issuing additional equity, i.e., when the value of equity

[8] Our assumption corresponds to the worst outcome for debt holders after bankruptcy, yet it greatly simplifies the analysis. Alternatively, we may assume that debt holders can efficiently operate the firm after bankruptcy, in which case they receive the value of the unlevered firm. This alternative corresponds to the best outcome for debt holders (see, e.g., Mauer and Ott, 2000; Mauer and Sarkar, 2005; Mauer and Triantis, 1994), but it complicates the analysis.

[9] We also assume that $\rho > \mu$ so as to ensure that $V^e(\cdot)$ is finite.

falls to zero. The terms X_b, a_1, and a_2 in equation (5) are jointly solved through the respective limit, value-matching, and smooth-pasting boundary conditions:

$$(6) \quad \lim_{X\to\infty} V^e(X,Q,q_i,b) = (1-\tau)[\frac{X q_i Q^{\frac{-1}{\gamma}}}{(\rho-\mu)} - \frac{(b+c_0 q_i^{\eta})}{\rho}],$$

$$(7) \quad V^e(X_b,Q,q_i,b) = 0,$$

$$(8) \quad \frac{\partial V^e(X_b,Q,q,b)}{\partial X} = 0.$$

Condition (6) holds because default becomes irrelevant when demand conditions are extremely favorable. Condition (7) recognizes that equity has limited liability at the bankruptcy trigger point, X_b. Condition (8) is the first-order condition that requires the demand-shift factor X to be chosen to maximize levered equity value upon bankruptcy. Substituting equation (5) into boundary conditions (6) through (8) yields

$$(9) \quad V^e(X,Q,q_i,b) = (1-\tau)[\frac{X q_i Q^{\frac{-1}{\gamma}}}{(\rho-\mu)} - \frac{(b+c_0 q_i^{\eta})}{\rho}] + \frac{(1-\tau)X_b q_i Q^{\frac{-1}{\gamma}}}{-\beta_2(\rho-\mu)}(\frac{X}{X_b})^{\beta_2}$$

and

$$(10) \quad X_b = \frac{-\beta_2(\rho-\mu)}{(1-\beta_2)\rho}(b+c_0 q_i^{\eta}) q_i^{-1} Q^{\frac{1}{\gamma}}.$$

On the right-hand side of equation (9), the first term is the after-tax expected present value of firm i, assuming that the firm never goes bankrupt, while the second term is the equity holders' value of the option to declare bankruptcy in the future.

Since debt receives a permanent coupon payment of b per period in the absence of bankruptcy, the general solution for the value of debt, is

$$(11) \quad V^d(X,Q,q_i,b) = \frac{b}{\rho} + c_1(X q_i Q^{\frac{-1}{\gamma}})^{\beta_1} + c_2(X q_i Q^{\frac{-1}{\gamma}})^{\beta_2}, \quad X > X_b,$$

where the constants c_1 and c_2 are determined by the requirements that

(12) $$\lim_{X \to \infty} V^d(X, Q, q_i, b) = \frac{b}{\rho}$$

and

(13) $$V^d(X_b, Q, q_i, b) = (1-\lambda)Kq_i.$$

Equation (12) says that debt holders will almost surely receive the coupon value when the demand condition is extremely good. Equation (13) states that, in bankruptcy, debt holders will receive the salvage value of the firm, given that equity holders declare bankruptcy at $X = X_b$. Substituting equations (10) and (11) into equations (12) and (13) gives the value of risky debt as

(14) $$V^d(X, Q, q_i, b) = \frac{b}{\rho} - \left(\frac{b}{\rho} - (1-\lambda)Kq_i\right)\left(\frac{X}{X_b}\right)^{\beta_2}.$$

On the right-hand side of equation (14), the first term is the expected present value of the coupon payment, assuming that firm i never goes bankrupt, while the second term is the loss of debt value that results from the possibility of insolvency in the future. Because a firm can only sell bonds at a fair price equal to V^d, the second term is thus the source of risk premium demanded by bondholders. To see this, we can calculate the interest rate paid by risky debt, R, which is equal to b/V^d, as follows:

(15) $$R = b/V^d = \rho + \left[\rho\left(\frac{b}{\rho} - (1-\lambda)Kq_i\right)\left(\frac{X}{X_b}\right)^{\beta_2} / V^d\right].$$

Equation (15) indicates that the credit spread on risky debt, which is defined as $R-\rho$, will be larger if the second term on the right-hand side, i.e., the ratio of the (flow-equivalent) loss value resulting from the possibility of insolvency in the future to the debt value, increases.

Firm i's total value, denoted by $V(X, Q, q, b)$, is the sum of $V^e(X, Q, q, b)$ in equation (9) and $V^d(X, Q, q, b)$ in equation (14). All of the above equations are derived for an arbitrary coupon level, b. In Section II.D below, we examine the optimal choice of coupon level (and leverage).

Table 1 displays the analytic comparative static properties of the basic model. In particular, we show how V^d, V^e, V, and the credit spread, $R-\rho$, vary as a function of b, σ, λ, τ, X, and q_i. Since most of these results are well known (Leland, 1994, Table 3, p. 1239), we report them without discussion.

Insert Table 1 here

C. Investment Timing and Scale Decisions in Cournot–Nash Equilibrium

Firm i must determine the optimal time to exercise the investment option. Further, at the optimal investment exercise point, firm i must choose the optimal scale and debt level of the firm. The optimal time to invest is characterized by a critical level of demand, X^*, at or above which firm i will install the optimal scale, q^*. Due to the interaction between investment irreversibility and uncertainty, firm i has an option to delay investment. This (delay) option's value is given by

(16) $$F(X) = h_1 (X q_i Q^{\frac{-1}{\gamma}})^{\beta_1} + h_2 (X q_i Q^{\frac{-1}{\gamma}})^{\beta_2}.$$

The constants h_1 and h_2, the investment trigger, X^*, and firm i's choice of investment scale, q^*, must be solved jointly from the limit, value-matching, smooth-pasting, and optimal scale conditions, which are, respectively,

(17) $$\lim_{X \to 0} F(X) = 0,$$

(18) $$F(X^*) = V(X^*, Q, q^*, b) - Kq^*,$$

(19) $$\frac{\partial F(X^*)}{\partial X} = \frac{\partial V(X^*, Q, q^*, b)}{\partial X},$$

(20) $$\frac{\partial V(X^*, Q, q^*, b)}{\partial q_i} - K = 0.$$

Condition (17) states that firm i's value of the option to delay investment becomes worthless as X approaches zero. Condition (18) states that firm i will not invest unless its value of the option to delay investment (the left-hand side) equals its net value of investing immediately (the right-hand side). In addition, condition (19) states that condition (18) must be continuous and smooth at the critical exercise point, X^*. If this were not the case, firm i could do better by exercising investment at a different point in time. Condition (20) is obtained by setting the derivative of the net value of investing immediately, $V(X^*, Q, q_i, b) - Kq_i$, with respect to q_i equal to zero.

We define $W(X^*, Q, q^*, b)$ as firm i's value of the option to delay investment minus its net value from investing immediately, i.e., the left-hand side minus the right-hand side of equation (18). Multiplying equation (19) by $X^*/(-\beta_1)$, imposing condition (17) and the Cournot–Nash equilibrium constraint $q_j = q^*$ $(j = 1, \ldots, N)$, adding the result into equation (18), and rearranging yields

(21) $$W^*(X^*, q^*, b) = \frac{X^*}{\beta_1}\frac{\partial V^*(X^*, q^*, b)}{\partial X} - V^*(X^*, q^*, b) + Kq^* = 0.$$

The explicit form of equation (21) is given by

(22) $$W^*(X^*, q^*, b) = \frac{-(1-\tau)(\beta_1 - 1)}{(\rho - \mu)\beta_1} N^{\frac{-1}{\gamma}} q^{*(1-\frac{1}{\gamma})} X^* + Kq^* - \frac{\tau b}{\rho} + \frac{(1-\tau)c_0 q^{*\eta}}{\rho}$$

$$-(1 - \frac{\beta_2}{\beta_1})(\frac{X^*}{X_*})^{\beta_2}[\frac{(1-\tau)(b + c_0 q^{*\eta})}{\rho(1-\beta_2)} - (\frac{b}{\rho} - (1-\lambda)Kq^*)] = 0,$$

where X_* is derived by substituting the Cournot–Nash equilibrium condition, $q_j = q^*$ ($j = 1, \cdots, N$), into X_b in equation (10), which gives

(23) $$X_* = \frac{-\beta_2(\rho-\mu)}{(1-\beta_2)\rho}(b + c_0 q^{*\eta}) N^{\frac{1}{\gamma}} q^{*(\frac{1}{\gamma}-1)}.$$

Imposing the Cournot–Nash equilibrium constraint $q_j = q^*$ ($j = 1, \cdots, N$) on equation (20) yields[10]

(24) $$\frac{\partial V^*(X^*, q^*, b)}{\partial q_i} - K = (1-\tau)[(1-\frac{1}{\gamma N})\frac{X^* q^{*\frac{-1}{\gamma}} N^{\frac{-1}{\gamma}}}{(\rho-\mu)} - \frac{c_0 \eta q^{*\eta-1}}{\rho}]$$

$$+(\frac{X^*}{X_*})^{\beta_2} q^{*-1}[(1-\lambda)Kq^* + \beta_2(\frac{1}{\gamma N}-1)(\frac{b}{\rho} - (1-\lambda)Kq^* - \frac{(1-\tau)(b + c_0 q^{*\eta})}{\rho(1-\beta_2)})]$$

$$+(\frac{X^*}{X_*})^{\beta_2} q^{*\eta-1} c_0 \eta [\frac{(1-\tau)}{\rho} + \beta_2(\frac{b}{\rho} - (1-\lambda)Kq^*)(b + c_0 q^{*\eta})^{-1}] - K = 0.$$

Equation (22) for W^* implicitly defines X^*, and equation (24) for $\partial V^* / \partial q_i$ implicitly defines q^*. We can use these first-order conditions to compute the comparative statics for X^* and q^*. The results are reported in Rows 1 and 3 of Table 2, respectively. In addition, the comparative statics for X_*, defined in equation (23), are reported in Row 2 of Table 2. Below we discuss the results first for the bankruptcy trigger (Row 2), then for the investment trigger (Row 1), and finally for the optimal scale (Row 3).

The results in Row 2 of Table 2 indicate that the bankruptcy trigger, X_*,

[10] We need to verify that all firms choosing the same timing and the same scale of development will lead to a Nash equilibrium. When one firm invests earlier (later) than the equilibrium timing indicated by X^*, then the option value from waiting will be more (less) than the net value from developing immediately. Consequently, investing earlier (later) will not be better than investing at $X(t) = X^*$. Similarly, when a firm chooses an investment scale that is lower (higher) than q^*, the marginal benefit from investing will be more (less) than the marginal cost. Consequently, a firm that invests at a smaller (larger) scale than $q_i = q^*$ will not be optimal. We therefore establish that the strategy ($X(t) = X^*$, $q_i = q^*$) dominates all other strategies, and hence is a Nash equilibrium.

(a) increases with an increase in the coupon level, b;

(b) decreases with an increase in uncertainty, σ;

(c) is independent of either bankruptcy costs, λ, or the corporate tax rate, τ;

(d) increases with an increase in the number of firms, N; and

(e) is ambiguous in relation to the investment scale, q_i.

Result (a) follows because a larger coupon to debt holders will reduce a firm's cash flow and will thus trigger equity holders to declare bankruptcy earlier. Result (b) follows because greater uncertainty makes a firm equally likely to realize operating profits or losses. This benefits equity holders, since they can avoid downside risks due to limited liability provisions of debt contracts, and thus equity holders will be more reluctant to declare bankruptcy. Result (c) indicates that neither bankruptcy costs nor tax shield benefits directly affect the equity holders' choice of bankruptcy timing. Result (d) follows because, as the number of competitors in the market increases, the output price, and hence a firm's operating cash flow, decrease, which leads the firm to be more susceptible to bankruptcy. Result (e) follows because a firm will be more likely to run into bankruptcy if either its operating revenue is lower or its variable costs are higher. An increase in the scale of operation will increase both the firm's operating revenue and its variable costs, and will thus have an indeterminate impact on the firm's choice of bankruptcy timing.

Row 1 of Table 2 indicates that the investment trigger, X^*,

(a) increases with an increase in the coupon level, b;

(b) is ambiguous in relation to uncertainty, σ;

(c) increases with an increase in bankruptcy costs, λ;

(d) increases with an increase in the corporate tax rate, τ;

(e) is ambiguous in relation to the investment scale, q_i; and

(f) increases with an increase in the number of firms, N.

Result (a) follows because an increase in the coupon will increase the probability of bankruptcy, thus making management more cautious when choosing when to invest. Result (b) follows because greater uncertainty will exhibit an indeterminate impact on a firm's probability of bankruptcy due to the following conflicting effects: (1) greater uncertainty makes hitting any given bankruptcy trigger point more likely, while (2) greater uncertainty decreases the bankruptcy trigger, as shown in Row 2 of Table 2. Result (c) follows because increases in bankruptcy costs will reduce the value that debt holders expect to receive upon bankruptcy, and, anticipating such losses, management would rather delay investment. Result (d) follows because an increase in the tax rate will reduce the value of equity, causing management to be more conservative in exercising the investment option. Result (e) follows because, as mentioned before, an increase in the scale of operation will exhibit an ambiguous impact on the bankruptcy trigger, and thus will have an indeterminate effect on a firm's choice of investment timing.

The rationale behind result (f) is as follows. Competitive pressure delays the expected timing of the investment because of two mutually reinforcing effects. First, competition will induce a firm's equity holders to declare bankruptcy earlier, as indicated by Row 2 of Table 2. Anticipating this, the firm's manager will be more prudent in exercising the investment option. Second, competition will reduce the output price and hence a firm's operating cash flow. To compensate for this, the firm's manager will be more conservative in exercising the investment option.

This result differs from that of Grenadier (2002). Grenadier (2002) assumes an all-equity financed oligopolistic firm that faces the same demand function as ours. However, Grenadier assumes that the firm incurs no variable costs of production, and focuses on incremental investment rather than lumpy investment. The firm will thus acquire an option to expand later, once it has more capital assets in place. As competition becomes more intense, the firm will invest sooner because the fear of preemption diminishes the value of the option to wait. By contrast, we assume that a firm has an investment project that is non-expandable. Because the firm can only exercise the investment project once, the firm will be more cautious when competitive pressure increases. We therefore complement the literature by demonstrating that the results derived from lumpy investment may significantly depart from those derived from incremental investment.[11]

Even if investment is lumpy, however, the impact of competitive pressure on investment timing still depends on whether competition eliminates investment opportunities or erodes investment value. Mauer and Ott (2000) also assume, as we do, that investment is lumpy. However, they focus on a firm that has monopolistic access to an investment opportunity, and faces the threat of entry by potential competitors. In their setting, competition destroys the value of the investment opportunity, which encourages the firm to invest earlier. By contrast, we assume that the output price is decreasing in the total output of the industry, and hence a firm's operating cash flow declines as the number of competitors in the industry increases. Competition encourages the firm to delay investment longer because competition has a negative effect on the output price, thereby forcing the firm to wait for a higher demand level before it can profitably exercise the investment option.

Row 3 of Table 2 indicates that the optimal scale of operation, q^*,

(a) is ambiguous in relation to the coupon, b, uncertainty, σ, and the corporate tax rate, τ;

(b) decreases with an increase in bankruptcy costs, λ, when uncertainty is high;

(c) increases with an increase in the number of firms, N, when uncertainty is low; and

(d) increases when demand conditions are more favorable, i.e., when X increases.

[11] This divergence complements the finding of Bar-Ilan and Strange (1999), who show that the impact of uncertainty on a firm's intensity of investment depends on the type of investment. For lumpy investment, an increase in uncertainty delays the investment project, and increases its size when it occurs. For incremental investment, an increase in uncertainty also delays investment, but decreases the intensity of investment.

Results (a), (b), and (c) indicate indeterminate comparative static results on investment scale. Since we emphasize the role played by competition, we only explain Result (c). One may expect that increased competition forces a firm to invest at a smaller scale because, as the number of competitors in the market increases, the output price and hence the marginal benefit from investing will decrease. However, another subtle effect related to the timing of bankruptcy is also at work. As Row 2 of Table 2 shows, the bankruptcy trigger increases with a decrease in uncertainty, that is, the firm would rather not operate in a less favorable demand environment. In this case the firm's marginal benefit from investing increases, encouraging the firm to expand its scale of operation. Thus, when uncertainty is low, this second effect will be reinforced and the net effect is more likely to be positive.

Result (d) follows because when demand conditions are more favorable, a firm's marginal benefit from investing will increase. Consequently, the firm will expand its scale of operation.

Results (c) and (d) help explain why competition is likely to increase a firm's choice of investment scale when the firm is able to choose both its investment timing and scale. If the firm' s investment timing is fixed, Result (c) provides the condition under which competition encourages a firm to expand its scale of operation. If on the other hand the firm has the flexibility to alter its investment timing, Result (d) shows that competition will encourage the firm to wait for more favorable demand conditions, further encouraging the firm to expand its scale of operation.

D. Optimal Level of Debt

We now turn to the issue of optimal debt financing. The choice of coupon payments, b^*, is obtained by setting the derivative of the net value of investing immediately, $V(X^*, Q, q^*, b) - Kq^*$ (which is the total value of firm i evaluated at $X = X^*$, $q_i = q^*$), with respect to b equal to zero, i.e.,

(25) $$\frac{\partial V(X^*, Q, q^*, b)}{\partial b} = 0.$$

Imposing $b = b^*$ and the Cournot–Nash equilibrium condition, $q_j = q^*$ $(j = 1,..., N)$, on equation (25) yields the explicit form

(26) $$\frac{\partial V^*(X^*, q^*, b^*)}{\partial b} = [\frac{\tau}{\rho}(1-(\frac{X^*}{X_*})^{\beta_2})] - [\frac{-\beta_2}{(b^* + c_0 q^{*\eta})}(\frac{b^*}{\rho} - (1-\lambda)Kq^*)(\frac{X^*}{X_*})^{\beta_2}] = 0,$$

where X_* is as defined in equation (23). Equation (26) states that when a firm issues bonds, it must balance the probability-weighted marginal tax shield benefit from debt financing, the first term, against the discounted marginal bankruptcy costs, the second term.

Equation (26) for $\partial V^* / \partial b$ implicitly defines b^*. Thus, we can use this first-order condition to compute the comparative statics for b^*. The results are shown in Row 4 of Table 2. Observe that the optimal coupon payment, b^*,

(a) is ambiguous in relation to uncertainty, σ;

(b) decreases with an increase in bankruptcy costs, λ;

(c) increases with an increase in the corporate tax rate, τ;

(d) decreases with an increase in the number of firms, N;

(e) increases when the demand conditions are more favorable, i.e., when X increases; and

(f) is ambiguous in relation to the scale of operation, q_i.

Result (a) follows because greater uncertainty will have an ambiguous effect on the firm's probability of bankruptcy, as mentioned before. Results (b) and (c) follow because we assume that the optimal coupon payment balances marginal tax shield benefits against marginal bankruptcy costs. Result (d) follows since, as competition intensifies, a firm is more likely to go into bankruptcy, and therefore its marginal tax shield benefit will decrease while its marginal bankruptcy costs will increase. Result (e) follows because a firm is less likely to run into bankruptcy when the demand conditions that trigger the firm to invest become more favorable. Result (f) follows because, as mentioned before, an increase in the scale of operation will have an indeterminate effect on the bankruptcy trigger, and thus will have an indeterminate effect on the promised coupon level.

Insert Table 2 here

III. Numerical Analysis

In the previous section, we analytically compute comparative static results for each policy decision, holding the other policy decisions constant. In this section, we employ plausible parameter values to perform a numerical analysis that allows all decisions to change in response to a given change in a model parameter (e.g., number of firms in the industry). Table 3 presents the benchmark values, which closely follow those chosen by Grenadier (2002), Leland (1994), and Mauer and Ott (2000). In particular, our benchmark oligopolistic firm is assumed to incur a cost of $1 to install one unit of capital stock (K=1). The firm faces an industrial demand function with a constant elasticity, γ, equal to 1.5, which is just a little above the value estimated by Phillips (1995). We assume initially that there are five firms in the industry, i.e., $N = 5$. To produce the final goods, the firm incurs an operating cost given by $c_0 q_i^{\eta}$, with c_0=1 and η=5. The demand-shift factor, X, is expected to grow 0% per year, μ=0, and the volatility of this growth rate, σ, is equal to 20% per year. The firm suffers a loss of 50% of capital asset value (λ=0.5) when bankruptcy occurs and faces a corporate income tax, τ, equal to 35%. Finally, the risk-less rate, ρ, is equal to 6% per year.[12]

[12] The tax rate, τ, is chosen to reflect the U.S. corporate income tax rate of 35%, while the discount rate is near the average rate on Treasury bills, as reported in Standard & Poor's *The Outlook* in 2001. In addition, the cost elasticity of scale, η, is chosen so as to satisfy the three requirements for an interior solution of the investment scale: $\beta_1 > \gamma N$, $\eta > (1-\frac{1}{\gamma N})/(1-\frac{1}{\beta_1})$, and net firm value being greater than zero. If η is chosen

Table 4 reports the numerical comparative static results where all values are evaluated at the optimal policies, i.e., $X = X^*$, $q_i = q^*$, and $b = b^*$. Given the benchmark values above, Panel A of Table 4 shows that if the current value of X is smaller than 0.193, the level of the investment trigger, X^*, then the firm will wait until this critical level is reached; otherwise, the firm will invest immediately. Equivalently, we can say that the firm will invest immediately if the current output price, P, is greater than or equal to 0.139, which is the level of P^*, i.e., the critical level of P that triggers investment. The instant in which the firm exercises its investment option, the firm will purchase 0.326 units of capital (q^*=0.326) and will issue bonds that promise to pay \$0.0292 per year forever (b^*=0.0292). As a result, the firm's equity holders will not declare bankruptcy until X declines to the level of the bankruptcy trigger, 0.079 (X_*=0.079). In this same instant, the firm's debt value, V^{d*}, will be equal to \$0.385, and its equity value, V^{e*}, will be equal to \$0.184, and thus the optimal leverage ratio, $L^* = V^{d*}/V^*$, will be equal to 67.7%, the yield spread, $R^* - \rho$, will be equal to 157 bps (basis points), and firm's net value, $V^* - Kq^*$, will be equal to 0.230.

Insert Table 3 here

Panels B through E of Table 4 report the effects of competition (N=10, 15, or 20), demand uncertainty (σ=15% or 25%), bankruptcy costs (λ=0 or 1), and corporate taxes (τ=5%, 15%, or 25%), on the firm's policy choices and values. Note that all the other parameters are held at their benchmark values. Consider first the impact of a change in the number of firms, N. In Row 1 of Table 2, we show that when the investment scale and debt level are fixed, competition will divert further away from the NPV rule because the

to be lower, say η=2, then we will be unable to investigate the case for $N \le 2$ and the case for $\sigma < 20\%$. We also find that financial policy variables, such as the optimal leverage ratio and the credit spread, are highly insensitive to the measure of η.

investment trigger, X^*, will increase. Panel B of Table 4 indicates that this delaying effect continues to dominate when we allow a firm to endogenously determine both its scale of operation and debt level. In other words, competition delays investment mainly because competition decreases the output price, and hence decreases the firm's operating cash flow as well as its investment value. Moreover, because the firm invests on the date in which demand becomes more favorable, increased competition will lead the firm to promise to give bondholders a higher optimal coupon; the optimal coupon level is equal to \$0.0292 per year when $N = 5$ and \$0.0326 per year when $N = 20$. This finding differs from that shown in Row 4 of Table 2, which predicts that given a firm's investment timing and scale, the firm's optimal coupon decreases as the firm faces more competitors in the market. Next, in Row 3 of Table 2, we show that the delaying effect resulting from competition leads a firm to expand its scale of operation, q^*, as it may then exercise the investment option when demand is more favorable. Panel B of Table 4 confirms this prediction, indicating that this scale effect dominates all the other effects; the optimal scale of operation is 0.326 units when $N = 5$ and 0.355 units when $N = 20$.

Panel B's remaining results are as follows. Competition forces a firm's equity holders to declare bankruptcy earlier. As a result, bondholders will ask for a larger risk premium; the credit spread of risky debt is 157 bps when $N = 5$, and 164 bps when $N = 20$. However, competition also increases equity value, debt value, and the firm's net value from investing because it encourages the firm to invest when demand is more favorable. Finally, the optimal leverage ratio decreases slightly with an increase in the number of firms: the optimal leverage ratio is 67.7% when $N = 5$ and 67.4% when $N = 20$.

Panel C of Table 4 presents the optimal operating and financial policies for demand volatilities, σ, of 15% and 25% per year. As the panel shows, greater uncertainty will make management more cautious in its investment timing choice, and thus more likely to expand its operation to make use of more favorable future demand. This is in line with the result in Bar-Ilan and Strange (1999), although they focus on an all-equity-financed firm. Given that the firm invests on the date in which demand becomes more favorable, its equity value, debt value, and net value from investing will also increase. Consistent with Leland (1994) and Mauer and Ott (2000), greater uncertainty also increases the promised coupon payment, but leads to a lower leverage ratio and a higher credit spread, as bondholders expect to suffer more losses upon bankruptcy.

Panel D of Table 4 gives the optimal operating and financial policies for bankruptcy costs, λ, of 0% and 100%. A firm will be more cautious in its investment timing choice when it expects to incur larger bankruptcy costs. This seems important in explaining why the firm will expand its scale of operation as bankruptcy costs increase. We also observe that even when bankruptcy costs are zero, i.e., $\lambda = 0$, a firm will not be 100% debt financed because, as Mauer and Ott (2000) suggest, the firm will still suffer a leverage-related cost associated with the loss of tax shields in bankruptcy. The optimal leverage ratio is also decreasing as λ increases; it is 85.4% when $\lambda = 0$ and 55.6% when $\lambda = 100\%$.[13] Surprisingly, both the equity value and the firm's net value increase when bankruptcy costs are increased. This is perhaps due to the fact that the demand level at which a firm decides to invest is more favorable when bankruptcy costs are larger. We also find that the credit spread of risky debt increases as λ increases. The reason is as follows. An increase in λ has two effects: first, debt value will decrease, which increases the credit spread; second, the optimal operating and financial policies will be affected, and the credit spread may therefore decrease. In our specific example, the net effect is positive because the first effect always dominates the second one. This finding contrasts with the results of Leland (1994) and Mauer and Ott (2000), who find that the net effect is negative.

[13] Our result differs from that of Fries et al. (1997). They show that given a firm's residual value is equal to zero at closure, i.e., $\lambda = 1$ in our case, a firm in a competitive industry equilibrium will issue no bonds at all. Note that their result is based upon the assumptions that the number of firms is endogenously determined and that firms can freely enter and exit the output market.

Panel E of Table 4 reports the optimal operating and financial policies for the corporate tax rates of 5%, 15%, and 25%. Naturally, a firm will promise to pay bondholders a higher coupon as the tax rate increases because of the larger interest tax shields. As this promised coupon payment is increased, equity holders will also put the firm to bondholders sooner such that debt value will increase, equity value will decrease, and the firm's optimal leverage ratio will increase. We also find that as the tax rate increases, a firm's after-tax operating cash flow will decrease, such that the firm will delay investment and expand its scale of operation. Finally, as the tax rate increases, the credit spread will increase, as bondholders expect bankruptcy costs to be increasingly imminent.

Insert Table 4 here

IV. Conclusion

This paper examines a firm's debt level, investment timing, and investment scale choices in a continuous-time model in which the output price of a good that the firm produces depends on a stochastic demand-shift variable and the total industry supply of the good. Using the simple symmetric Cournot–Nash equilibrium assumption that all firms are identical and hence follow the same financing and investment strategies, we compute comparative statics regarding how competition, uncertainty, bankruptcy costs, and corporate taxation affect the firm's operating and financial policy decisions. These comparative statics results yield several empirical testable implications which we test numerically using a set of plausibly parameters.

Our results differ significantly from the findings in previous research because we allow a firm to have flexibility over its investment scale, yet competition erodes the value of its investments. For example, while previous research finds that increasing competition accelerates investment, we find that increasing competition delays investment. We also show that increasing competition accelerates bankruptcy, increases the optimal level of debt, and increases the optimal scale of operation. Further, the credit spread on debt increases but the optimal leverage ratio decreases as competition intensifies.

With respect to uncertainty, we find that greater demand uncertainty exhibits the same qualitative effects as increasing competition, and with respect to bankruptcy costs, we find that as expected bankruptcy costs increase, the investment option will be exercised later, the optimal scale of operation will increase, the optimal leverage ratio will decrease, and the credit spread on debt will increase. Finally, we find that an increase in the tax rate delays investment, accelerates bankruptcy, increases both the optimal level of debt and the optimal scale of operation, and increases both the optimal leverage ratio and the credit spread on debt.

Note that our results are based upon several simplifying assumptions. Future studies could test the generality of our results by relaxing these assumptions. For instance, our analysis is based on the assumptions that debt payment is unprotected and that bonds mature indefinitely. One could follow Leland (1994) and Leland and Toft (1996) to relax these two assumptions. We also assume that a firm installs an initial capacity that is non-expandable. Future studies may provide the firm with the option to expand later, which may lead to results that are more comparable to those of Grenadier (2002). Finally, we do not touch upon the issue of asset substitution, which is investigated in Leland (1998) and Mauer and Sarkar (2005). Future studies may extend our framework to analyze this issue.

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Table 1

Comparative Statics of Financial Variables

This table describes properties of the equations describing debt value, V^d, the value of equity, V^e, the total firm value, V, and the yield spread of the risky debt over the risk-free rate ($R-\rho$). The term X is the demand-shift factor, X_b is the bankruptcy trigger, b is the coupon paid on debt, σ and μ are the instantaneous volatility and expected growth of dX/X, respectively, λ is the fraction of asset value lost if bankruptcy occurs, τ is the corporate tax rate, q_i and Q are the operating scale of firm i and that of the industry as a whole, respectively, ρ is the risk-free interest rate, η is the cost elasticity of scale, and γ is the price elasticity of demand.

		Limit as		Sign of Change in Instrument for an Increase in:					
Variable	Shape	$X \to \infty$	$X \to X_b$	b	σ	λ	τ	X	q_i
V^d	Concave in X, b	$\frac{b}{\rho}$	$(1-\lambda)Kq_i$	>0; <0 as $X \to X_b$	<0; >0 as $X \to X_b$	<0	0	>0	$\frac{\geq}{<} 0$
V^e	Convex in X, b	$(1-\tau)(\frac{Xq_iQ^{\frac{-1}{\gamma}}}{\rho-\mu}-\frac{b+c_0q_i^{\eta}}{\rho})$	0	<0	>0	0	<0	>0	$\frac{\geq}{<} 0$
V	Concave in X, b	$\frac{(1-\tau)}{(\rho-\mu)}Xq_iQ^{\frac{-1}{\gamma}}+\frac{\tau b-c_0q_i^{\eta}}{\rho}$	$(1-\lambda)Kq_i$	>0 <0 as $X \to X_b$	<0; >0 as $X \to X_b$	<0	<0	>0	$\frac{\geq}{<} 0$
$R-\rho$	Convex in X, b	0	$\frac{b}{(1-\lambda)Kq_i}-\rho$	>0	>0; <0 as $X \to X_b$	>0	0	<0	$\frac{\geq}{<} 0$

Table 2

Comparative Statics of a Financial or Operating Choice Variable, Holding the Other Variables Constant

This table describes properties of choices of the investment trigger, X^*, the bankruptcy trigger, X_*, operating scale, q^*, and the coupon level, b^*, when the other choices are held constant. The term b is the coupon level, σ is the instantaneous volatility of dX/X, λ is the fraction of asset value lost if bankruptcy occurs, τ is the corporate tax rate, N is the number of firms, X is the shock to demand, q_i is firm $i's$ scale of operation, and β_2 is defined in equation (5).

	Signs of Change in Variable for an Increase in:						
Variable	b	σ	λ	τ	N	X	q_i
X^*	>0	$\gtrless 0$	>0	>0	>0	n.a.	$\gtrless 0$
X_*	>0	<0	0	0	>0	n.a.	$\gtrless 0$
q^*	$\gtrless 0$	$\gtrless 0$	<0, if $\beta_2 \geq -(1-\frac{1}{\gamma N})^{-1}$	$\gtrless 0$	>0, if $\beta_2 < (N-\frac{1}{\gamma})^{-1} - 1$	>0	n.a.
b^*	n.a.	$\gtrless 0$	<0	>0	<0	>0	$\gtrless 0$

Notes: "n.a." means "not available," and β_2 is increased as σ is increased.

Table 3

Base-Case Parameter Values for Numerical Analysis

Parameter	Value
Unit cost of capital	$K = 1$
Price elasticity of demand	$\gamma = 1.5$
Number of firms	$N = 5$
Operating cost multiplier	$c_0 = 1$
Cost elasticity of scale	$\eta = 5$
Expected growth of the demand shock	$\mu = 0\%$ per year
Volatility of the growth rate of the demand shock	$\sigma = 20\%$ per year
Bankruptcy costs	$\lambda = 50\%$
Corporate tax rate	$\tau = 35\%$
Risk-less interest rate	$\rho = 6\%$ annually

Table 4

Optimal Operating and Financial Policies for Different Financial Variables and Market Conditions

This table reports the levels of bankruptcy trigger (X_*), the output price (P^*), the values of debt (V^{d*}) and equity (V^{e*}), net firm values ($V^* - Kq^*$), and the leverage ratio (L^*) as well as the credit spread ($R^* - \rho$), when all the investment trigger (X^*), promised coupon payment (b^*), and scale of operation (q^*) are chosen to maximize net firm values. Panel A reports the results for the base-case with the values of parameters given in Table 3. Panel B reports the results when the number of firms (N) is 10, 15, and 20. Panel C reports the results when demand uncertainty (σ) is 15% and 25%. Panel D reports the results when the cost of bankruptcy (λ) is 0% and 100%. Panel E reports the results when the corporate tax rate (τ) is 5%, 15%, and 25%. Panels B, C, D and E are reported by holding all other parameters at their benchmark values shown in Table 3.

	Investment trigger, X^*	Bankruptcy trigger, X_*	Coupon payment, b^* (cents)	Operating scale, q^*	Price, P^* ($)	Value of debt, V^{d*} ($)	Value of equity, V^{e*} ($)	Net firm value, $V^* - Kq^*$ ($)	Leverage ratio, L^* (%)	Credit spread, $R^* - \rho$ (bps)
A. Benchmark Case: N =5, σ =20%, λ =50%, τ =35%	0.1928	0.0790	2.9180	0.3259	0.1392	0.3852	0.1839	0.2303	67.69	157

Table 4

Optimal Operating and Financial Policies for Different Financial Variables and Market Conditions

	Investment trigger, X^*	Bankruptcy trigger, X_*	Coupon payment , b^* (cents)	Operating scale, q^*	Price, P^* ($)	Value of debt, V^{d*} ($)	Value of equity, V^{e*} ($)	Net firm value, $V^* - Kq^*$ ($)	Leverage ratio, L^* (%)	Credit spread, $R^* - \rho$ (bps)
B. Number of firms:										
N =10	0.3295	0.1364	3.1499	0.3460	0.1440	0.4135	0.1993	0.2522	67.48	162
N =15	0.4422	0.1836	3.2264	0.3523	0.1458	0.4227	0.2044	0.2636	67.40	163
N =20	0.5419	0.2254	3.2633	0.3554	0.1466	0.4271	0.2069	0.2690	67.37	164
C. Demand Uncertainty:										
σ =15%	0.1443	0.0684	2.1800	0.2814	0.1150	0.3079	0.1173	0.0207	72.41	108
σ =25%	0.2498	0.0902	3.8200	0.3650	0.1673	0.4682	0.2684	0.3232	63.57	216
D. Bankruptcy Costs:										
λ =0	0.1298	0.0739	2.7380	0.3097	0.0970	0.3860	0.0662	0.1198	85.35	109
λ =100%	0.2267	0.0747	2.7254	0.3312	0.1619	0.3473	0.2772	0.2746	55.61	185
E. Corporate Tax Rate:										
τ =5%	0.1644	0.0390	1.2900	0.3166	0.1211	0.2063	0.3693	0.2356	35.84	25
τ =15%	0.1738	0.0537	1.8840	0.3197	0.1271	0.2807	0.2909	0.2316	49.10	71
τ =25%	0.1835	0.0668	2.4200	0.3230	0.1333	0.3385	0.2321	0.2306	59.32	115