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A Moment-Based Approach for Deskewing Rotationally Symmetric Shapes

Soo-Chang Pei and Ji-Hwei Horng

***Abstract*—The covariance matrix of a pattern is composed by its second order central moments. For a rotationally symmetric shape, its covariance matrix is a scalar identity matrix. In this work, we apply this property to restore the skewed shape of rotational symmetry. The relations between the skew transformation matrix and the covariance matrices of original and skewed shapes are derived. By computing the covariance matrix of the skewed shape and letting the covariance matrix of the original shape be a scalar identity matrix, the skew transformation matrix can be solved. Then, the rotationally symmetric shape can be recovered by multiplying the inverse transformation matrix with the skewed shape. The method does not rely on continuous contours and is robust to noise, because only the second-order moments of the input shape are required. Experimental results are also presented.**

***Index Terms*—Covariance matrix, reflective symmetry, rotation matrix, rotational symmetry, shear matrix, skewed symmetry.**

I. INTRODUCTION

A rotationally symmetric shape is a shape that repeats itself after being rotated around its centroid through any multiple of a certain angle. To satisfy this property, the minimum repeating angle must be $2\pi/n$, where n is a positive integer. This shape is called an n-fold rotationally symmetric shape. Many researchers on the area of image analysis have pay attention to the importance of rotational symmetry. Lots of methods have been proposed to normalize the rotationally symmetric planar shapes (RSS) [1]–[5]. However, none of them has dealt with the skewed shape of rotational symmetry. In real applications, the view direction is not always perpendicular to the plane containing RSS, it results in a skewed RSS on the image plane. Thus, a complete normalization system must includes the *deskew* procedure.

Many methods have been proposed to recover the skewed shape of reflective symmetry [6]–[8]. But they are not directly applicable to the case of rotational symmetry. Although the set of reflectively symmetric shapes overlaps with the set of rotationally symmetric shapes, they are not identical. The relation of these two sets are shown in Fig. 1, where a typical example is given for each subset.

An alternative procedure for generating a skewed shape is to perform a shear operation with parameter β followed by a rotation in the image plane with parameter α. Friedberg proposed a moment-based method to recover the skewed reflective symmetry [7]. He used the property that the covariance matrix for a reflectively symmetric shape is a diagonal matrix, that is, the moment m_{11} is necessarily equal to zero. For a set of assumed parameters α and β of the skew coordinate transformation matrix, the moment m_{11} of the recovered shape can be written as a function of the set of parameters and the second-order moments of the skewed shape. Let the moment m_{11} of the deskewed shape be equal to zero, α can be described as a function of β. The constraint reduces the search space from

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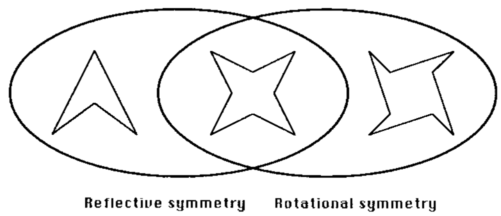


Fig. 1. Relation of reflective symmetry and rotational symmetry.

a two-dimensional (2-D) parameter space to one dimension. The sector symmetry evaluator is applied to search desired solutions along the one-dimensional (1-D) parameter space. This method is computationally expensive, since a search process is required. In addition, it does not guarantee to get exact solutions. In this paper, the Friedberg's method is generalized to recover the skewed RSS by setting an additional constraint according to the second order statistical property of the RSS.

II. Proposed Method

In this section the problem is formulated into a mathematical form, solution is derived, and uniqueness property is discussed. At last, an algorithm is provided to help the programmers in implementing this method.

A. Problem

Let us assume that f and g represent the images containing the original and skewed version of an RSS. The sets of coordinate vectors $S_p = \{p_k | k = 1, 2, \cdots, n.\}$ and $S_q = \{q_k | k = 1, 2, \cdots, n.\}$ represent the pixels occupied by the shapes in f and g respectively. By the definition of the skew transformation [8], $q_k = Tp_k, k = 1, 2, \cdots, n$ where T is the skew transformation matrix and is defined by

$$T = \begin{bmatrix} \cos\alpha & \cos\alpha \cot\beta - \sin\alpha \\ \sin\alpha & \sin\alpha \cot\beta + \cos\alpha \end{bmatrix}, \quad 0 \le \alpha < \pi, \quad 0 < \beta < \pi.$$

The angles α and β are the degrees of rotation and shear, respectively, in the skew generation process. Assuming that the origin of the coordinate system is set to the centroid of each shape. Then, the covariance matrix M and N of S_p and S_q can be represented by their second-order moments m_{ij} and n_{ij} [9] as follows:

$$M = \sum_k p_k p_k^T = \begin{bmatrix} m_{20} & m_{11} \\ m_{11} & m_{02} \end{bmatrix}.$$

$$N = \sum_k q_k q_k^T = \begin{bmatrix} n_{20} & n_{11} \\ n_{11} & n_{02} \end{bmatrix}.$$

Our problem is to solve the skew transformation matrix T based on the second order moments m_{ij} and n_{ij} of the original and the skewed shapes.

B. Solution

It can be proved that the covariance matrix of an RSS is a scalar identity matrix [1]. That is, there are two constraints on the RSS $m_{11} = 0$ and $m_{20} - m_{02} = 0$. In comparison with the method proposed by Friedberg [7], an additional constraint $m_{20} - m_{02} = 0$ is available, since our problem is dealt with the RSS instead of the reflectively symmetric shape. Of course, the solution space to be searched is reduced by the additional constraint.

Based on the skew transformation relation, the covariance matrices of the RSS M and the skewed RSS N are related by

$$N = \sum_k q_k q_k^T = \sum_k (Tp_k)(Tp_k)^T = T\left(\sum_k p_k p_k^T\right)T^T = TMT^T. \tag{1}$$

After manipulations, it can be rewritten in terms of $\tan\alpha$ and $\tan\beta$

$$[n_{20}^2 - n_{20}n_{02} + n_{11}^2]\tan^4\alpha + [-4n_{20}n_{11}]\tan^3\alpha + [6n_{11}^2]\tan^2\alpha + [-4n_{11}n_{02}]\tan\alpha + [-n_{20}n_{02} + n_{11}^2 + n_{02}^2] = 0. \tag{2}$$

$$[-n_{11}\tan^2\alpha - (n_{20} - n_{02})\tan\alpha + n_{11}]\tan\beta + [-n_{20}\tan^2\alpha + 2n_{11}\tan\alpha - n_{02}] = 0. \tag{3}$$

C. Uniqueness

It can be proved that given the covariance matrix N of a skewed RSS, there exists exactly one skew transformation matrix T with $0 < \beta < \pi/2$, such that

$$TMT^T = N,$$

where M is a scalar identity matrix.

D. Algorithm

According to the above discussions, we propose our deskew algorithm as follows.

1) Extract the pixels of the skewed shape from the input image. Translate the centroid of these pixels to the origin of the coordinate system. Let the set $S_q = \{q_k | k = 1, 2, \cdots, n.\}$ represents the resulting coordinate vectors.
2) Compute the covariance matrix N of S_q by the equation

$$N = \sum_{k=1}^{n} q_k q_k^T.$$

3) Compute the two distinct real solutions of the following quartic equation in terms of $\tan\alpha$ by using explicit formula

$$[n_{20}^2 - n_{20}n_{02} + n_{11}^2]\tan^4\alpha + [-4n_{20}n_{11}]\tan^3\alpha + [6n_{11}^2]\tan^2\alpha + [-4n_{11}n_{02}]\tan\alpha + [-n_{20}n_{02} + n_{11}^2 + n_{02}^2] = 0.$$

4) For each resulting $\tan\alpha$, solve its corresponding $\tan\beta$ by substituting into the following equation.

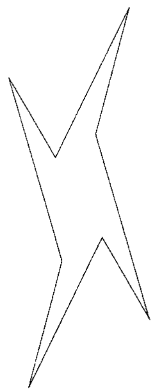
$$[-n_{11}\tan^2\alpha - (n_{20} - n_{02})\tan\alpha + n_{11}]\tan\beta + [-n_{20}\tan^2\alpha + 2n_{11}\tan\alpha - n_{02}] = 0.$$

5) Apply the parameter pair of β within $(0, \pi/2)$, which is unique, to construct the deskew transformation matrix T^{-1}.

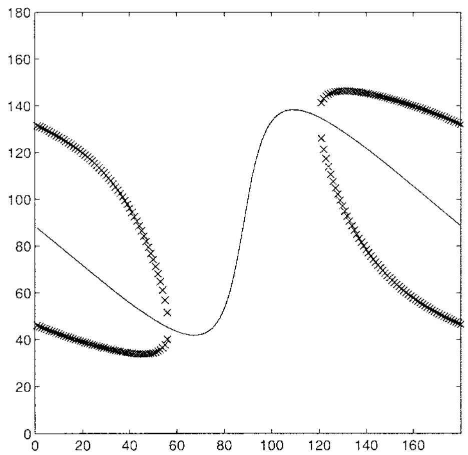
$$T^{-1} = \begin{bmatrix} \cos\alpha + \sin\alpha \cot\beta & \sin\alpha - \cos\alpha \cot\beta \\ -\sin\alpha & \cos\alpha \end{bmatrix}.$$

6) Deskew the given skewed RSS by the equation.

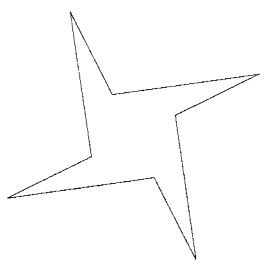
$$p_k = T^{-1}q_k, \qquad k = 1, 2, \cdots, n.$$



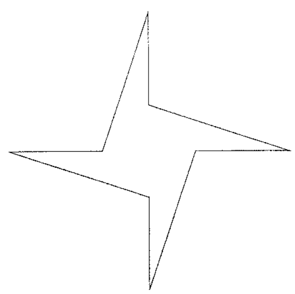
(a)



(b)



(c)



(d)

Fig. 2. (a) Skewed version of a rotationally symmetric shape. (b) Loci of our constraints. The locus corresponding to the constraint $m_{11} = 0$ is represented by a solid line and the locus corresponding to the constraint $m_{20} - M_{02} = 0$ is represented by crosses. (c) Deskewed version of the shape shown in (a). (d) Deskewed version of the shape shown in (a); it is a rotated version of (c).

III. Experimental Results

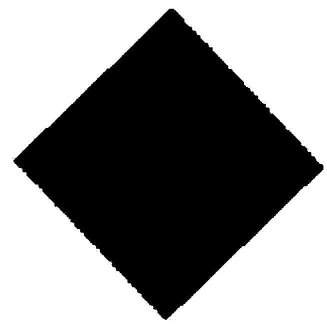
Three experiments are made in this research. In the first experiment, we generate a set of coordinate vectors, which forms the contour of an RSS. The point set is deformed by a skew transformation to serve as the input. In the second experiment, a synthetic image of a parallelogram is used to simulate an orthographic projection of a square from the three-dimensional (3-D) space. In the last experiment, three real images of arbitrarily skewed RSS are examined.

The input skewed RSS of our first experiment is shown in Fig. 2(a). To understand how we get the solution, the loci of parameters corresponding to our two constraints are plotted in Fig. 2(b). Each point on a locus represents a possible solution under its corresponding constraint. The loci are plotted by equally sampling on the α space and solving β for each given value of α. The locus corresponding to the constraint $m_{11} = 0$ is represented by a solid line and the locus corresponding the constraint $m_{20} - m_{02} = 0$ is represented by crosses. Because the constraint $m_{20} - m_{02} = 0$ is a quadratic equation in cot β, there are two β values for each α value or is no solution sometimes(when the two cot β values are complex conjugates). The intersections of the two loci are solutions of the problem. The two deskewed shapes corresponding to the two intersections are shown in Fig. 2(c) and (d). In fact, applying our algorithm gives the version shown in Fig. 2(c). The shape of Fig. 2(d) is a rotated version of Fig. 2(c) and can be ignored.

The synthetic image of a parallelogram is shown in Fig. 3(a). The output image of applying our algorithm is shown in Fig. 3(b). It looks like a perfect square.

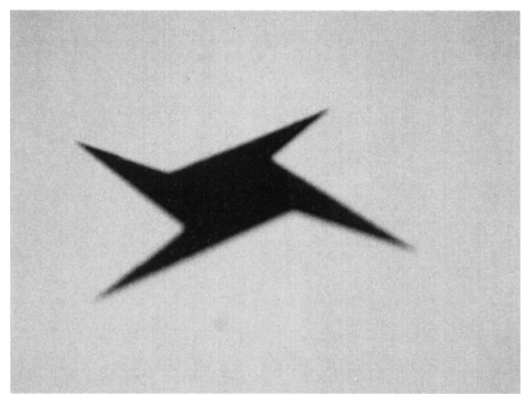
Two real images of skewed RSS are shown in Figs. 4(a) and 5(a). They are taken by digital camera with different space angles to the

(a)

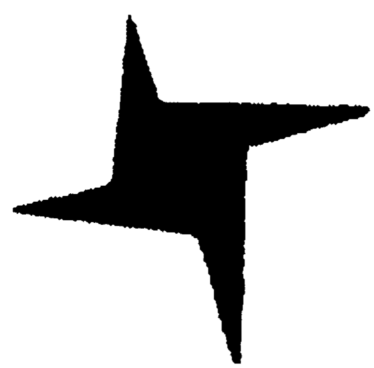


(b)

Fig. 3. (a) Synthetic image of a parallelogram. (b) Recovered pattern by using the proposed method.



(a)



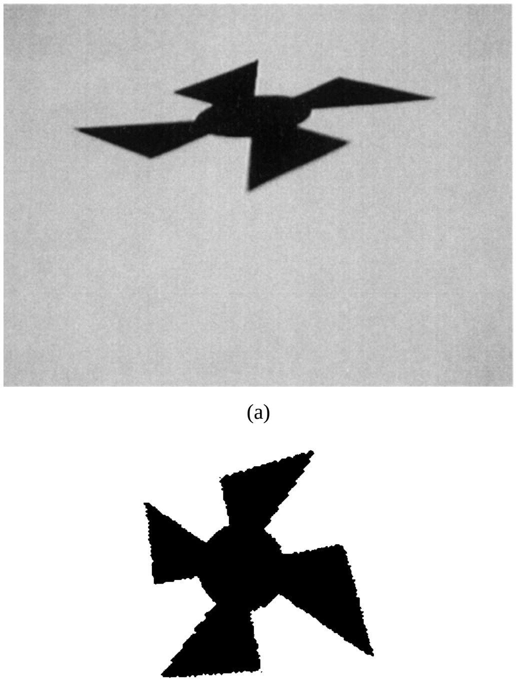
(b)

Fig. 4. (a) Real image of a rotationally symmetric shape. (b) Recovered pattern by using the proposed method.

rotationally symmetric patterns. Experimental results are shown in Figs. 4(b) and 5(b), respectively.

IV. Discussion

The first experiment confirms the theoretical derivations of this research. The second experiment shows the applicability of the proposed method to images. The real images of the third experiment



(a)

(b)

Fig. 5. (a) Real image of a rotationally symmetric shape. (b) Recovered pattern by using the proposed method.

demonstrate the robustness of this method to noise and digitization errors.

The output images of the last experiment are imperfect RSS. This is due to the nonlinear distortion caused by the camera lens. In such situation, orthographic projection assumption is no longer valid. However, fairly well results are provided and this indicates the usability of our method to imperfect rotational symmetry. To achieve better results , a camera calibration procedure may be placed before our system. It is beyond the scope of this research.

V. Conclusion

In this work, we propose an $\mathcal{O}(n)$ algorithm to recover the skewed RSS. Our algorithm needs no numeric method and no information about the number of folds. Since this method is based on the moments, it does not rely on smooth or continuous contours and is robust to noise or digitization errors but assumes there is no occlusion. We do not intend to finding the axis of symmetry, because a given RSS may not have any axis of reflective symmetry. However, it does have axes, several kinds of axes are proposed by previous researchers [1]–[4].

The experimental results confirm our derivations of constraints and show availability of our algorithm. Shapes with and without reflective symmetry are all presented. The algorithm gives accurate estimation of the skew parameters α and β.

After applying our algorithm, any of the algorithms proposed in [1]–[5] can be used to find the axes and normalize the deskewed RSS. Thus, the whole normalization procedure is completed.

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Tri-State Median Filter for Image Denoising

Tao Chen, Kai-Kuang Ma, and Li-Hui Chen

***Abstract*—In this work, a novel nonlinear filter, called *tri-state median* (TSM) filter, is proposed for preserving image details while effectively suppressing impulse noise. We incorporate the standard median (SM) filter and the *center weighted median* (CWM) filter into a noise detection framework to determine whether a pixel is corrupted, before applying filtering unconditionally. Extensive simulation results demonstrate that the proposed filter consistently outperforms other median filters by balancing the tradeoff between noise reduction and detail preservation.**

***Index Terms*—Impulse noise, median filter, noise detection.**

I. Introduction

Digital images are often corrupted by *impulse noise* during the acquisition or transmission through communication channels. Consequently, some pixel intensities are inevitably altered while others remain noise-free. The image model containing impulse noise with probability of occurrence p can be described as follows:

$$X_{ij} = \begin{cases} N_{ij}, & \text{with probability } p; \\ S_{ij}, & \text{with probability } 1-p \end{cases} \tag{1}$$

where S_{ij} denotes the noiseless image pixel and N_{ij} the noise substituting for the original pixel.

In order to remove impulse noise and enhance image quality, the median filter has been extensively studied and presented in the literature (e.g., [1] and [2]). Median filtering being a nonlinear filtering technique, it is generally superior to linear filtering (e.g.,

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