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Variable Structure Based Nonlinear Missile Guidance and Autopilot Design for a Direct Hit with Thrust Vector Control

Fu-Kuang Yeh [1], Kai-Yuan Cheng [1], and Li-Chen Fu [1,2]
Department of Electrical Engineering [1]
Department of Computer Science and Information Engineering [2]
National Taiwan University, Taipei, Taiwan, R.O.C
Email: lichen@ccms.ntu.edu.tw

Abstract

In this paper, we propose a variable structure based nonlinear missile guidance and autopilot systems for a direct hit with thrust vector control and divert control system inputs for the interception of a theater ballistic missile. The aim of the present work is to achieve the bounded target interception of the missile such that the distance between the missile and the target is close enough to trigger the missile's explosion. First, a 3 degree-of-freedom (DOF) sliding-mode missile guidance law under considering external disturbances and zero-effort-miss (ZEM) are designed to minimize the distance between the missile and the target for the translation motion. Then, we proposed a quaternion-based sliding-mode attitude controller to track the attitude command and to cope with the effects from variations of missile's inertia, aerodynamic force and wind gusts. The exponentially stability of the overall system is analyzed thoroughly via Lyapunov stability theory before entering ZEM phase. Extensive simulation results are conducted to validate the effectiveness of the proposed integrated guidance law and autopilot system by use of the 5 DOF inputs.

1. Introduction

Generally speaking, there are two principal phases for missiles intercepting the ballistic missile. One is the midcourse guidance [1,2,3] which concerns the stage before the missile can lock onto the target by its own sensor, and its task is to deliver the missile to some place near the target with some additional conditions, such as suitable velocity or appropriate attitude. For an upper-tier defender such as the Theater High Altitude Area Defense (THAAD) system [4], the midcourse phase lasts for a long period of time. Thus, the variation of the missile inertia during the traveling period cannot be neglected, and the influence caused by the aerodynamic force and wind gusts have to be compensated to guarantee that the attitude of the missile is stable during its flight.On the other hand, the homing guidance [5,6,7], i.e., the terminal guidance, will be applied when the distance between the missile and the target is less than some pre-specified value, which often depends on the time when the sensor on the missile can lock onto the target. The principal objective of the homing guidance is to insure that the target ballistic missile can be hit by the intercepting missile where the terminal accuracy against some unexpected disturbances can be sufficiently attained.

Based on the concept of the PN guidance law [8], the constant bearing guidance is often employed on the Bank-to-Turn (BTT) missiles [9,10], whereas a different kind of guidance law, namely, zero-sliding guidance law and optimal sliding-mode guidance aim at eliminating the normal velocity perpendicular to line-of-sight (LOS) which is the straight line from the missile to the target [1,5]. Ha and Chong derived a new command to line-of-sight (CLOS) guidance law for short-range surface-to-air missile via feedback linearization [11] and its modified version [12] with improved performance. As for Moon *et al.* [18], they propose the missile guidance law using variable structure control, where the input command is derived under the condition wherein the target acceleration is treated as an uncertainty. In that work, Moon *et al.* take it for granted that the relative velocity between the missile and the target is negative all the time from the launch to the interception of the missile, so that the stability of the sliding surface in the LOS direction is not a critical issue. An adaptive sliding-mode guidance of a homing missile is presented by Zhou *et al.* [21] adaptively to estimate the two parameters and to deal with the robustness for the disturbances based on the linear time-varying system. There, an ideal assumption that the relative velocity between the missile and the target is approaching during the entire interception period is made, such that the proposed guidance law is hardly applicable to practical environment.

Besides guidance, attitude control is another important issue to be addressed for successful missile operation. It is quite often that quaternion representation has been adopted to describe the attitude of a spacecraft [13,14], because it is recognized as a kind of global attitude representation. To account for the non-ideal factors of the spacecraft under attitude control and to strengthen the robustness property of the system, the sliding mode control has been employed by Chen and Lo [15], which is then followed by a smooth version [16] incorporating a boundary layer as has been proposed by [17] to avoid the chattering phenomenon, but at the price of slightly degrading the accuracy of the tracking system. To achieve the same goal, a different approach called adaptive control has been developed by Slotine [19], to deal with the accurate attitude tracking control of rigid spacecraft with large loads of unknown mass. Costic *et al.* [20] also address the attitude-tracking problem without angular velocity measurements based on the quaternion representation and in the presence of unknown inertia matrix. Moreover, Lian [22] conducts a mechanism of parameter estimation so as to solve the problem of orientation control for general nonlinear mechanical systems. All the above research works addressed the issue on attitude control problem is mainly to achieve the goal of attitude tracking.

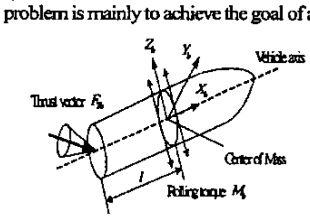

Fig. 1(a) TVC actuator with single nozzle and rolling torque scheme

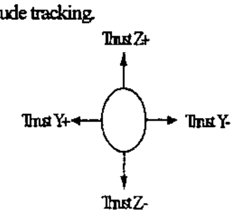

Fig. 1(b) Missile divert control system(bottom view)

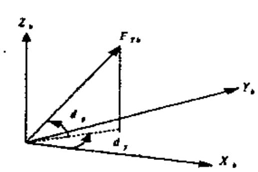

Fig. 1(c) Two angles of TVC in body coordinate frame

2. Preliminaries

2.1 Equations of Motion for Missiles with TVC

The motion of a missile can be described in two parts as follows:

Translation: $\dot{v}_M = a_M + g_M + d_M,\ \dot{r}_M = v_M,$ (1)

Rotation: $J\dot{\omega} = -\dot{J}\omega - \omega \times (J\omega) + T_b + d,$ (2)

where all the variables are defined in the nomenclature listing.

After referring to Fig. 1a to Fig. 1c, the force and torque exerted on the missile can be respectively expressed in the body coordinate frame as

$$F_{Mb}=F_{Tb}+F_{Sb}=N\begin{bmatrix}\cos d_p\cos d_y\\ \cos d_p\sin d_y\\ \sin d_p\end{bmatrix}+\begin{bmatrix}0\\ F_{sy}\\ F_{sz}\end{bmatrix} \quad (3)$$

and

$$T_b=L_b\times F_{Tb}+M_b=lN\begin{bmatrix}M_{bx}/lN\\ \sin d_p\\ -\cos d_p\sin d_y\end{bmatrix}, \quad (4)$$

Let the rotation matrix B_b denote the transformation from the body coordinate frame to the inertial coordinate frame. Thus, the force exerted on the missile observed in the inertial coordinate system is as follows:

$$F_M=B_bF_{Mb} \ . \quad (5)$$

From Eqs. (1) to (5), the motion model of the missile can then be derived as

$$\dot v_M=F_M/m+g_M+d_M=(B_bF_{Mb})/m+g_M+d_M \quad (6)$$

$$J\dot\omega=-\dot J\omega-\omega\times(J\omega)+lN\begin{bmatrix}M_{bx}/lN\\ \sin d_p\\ -\cos d_p\sin d_y\end{bmatrix}+d \quad (7)$$

2.2 Zero-Effort-Miss Analysis

The design of the guidance law is to minimize the relative velocity component v_p which is normal to the relative displacement vector r, or the LOS direction. The singularity will take place only at $r=0$, as will be clear from the equation (11) to be derived subsequently. Because our guidance command only tries to drive the output variable v_p to zero, non-zero error will unfortunately exist in the finite time while r is approaching to zero, or physically while the missile is approaching to the target.

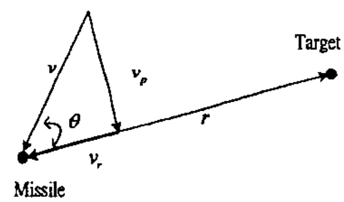

Fig. 2 Relative velocity between the missile and the target

Assume that both the missile and the target are moving only with constant gravitational acceleration, i.e.,

$$r_T=r_{T0}+\frac{1}{2}g(t-t_0)^2,\quad v_T=v_{T0}+g(t-t_0)$$

$$r_M=r_{M0}+\frac{1}{2}g(t-t_0)^2,\quad v_M=v_{M0}+g(t-t_0),\ (8)$$

where $r_{(\cdot)}$ is the position vector, $v_{(\cdot)}$ is the velocity vector, and the variables with subscripts T, T_0 and M, M_0 respectively denote the relevant quantities associated with the target and the missile, and the time t_0 denotes the time instant when the missile stops doing maneuvering, i.e., no acceleration command will be issued to the missile controller ever since. In the sequel, we will call the time period after t_0 ZEM phase. Therefore, the relative position vector r and the relative velocity vector v at $t=t_0$ can be respectively expressed as:

$$r=r_T-r_M=r_{T0}-r_{M0}=r_0$$

$$v=v_T-v_M=v_{T0}-v_{M0}=v_0\ ,$$

where r_0 and v_0 are both denoted as a fixed vector at $t=t_0$ as defined above. Presuming that $r_0\cdot v_0<0$, which means that both target and missile are approaching. Thus, ZEM is equivalent to the minimum norm of $r_0+v_0(t-t_0)$ for $t\ge t_0$, which can be computed as:

$$ZEM=\min_{t\ge t_0}|r_0+v_0(t-t_0)|$$

$$=\sqrt{|r_0|^2-\frac{(r_0^Tv_0)^2}{|v_0|^2}}=|r_0|\sin\theta \quad , \quad (9)$$

where $r_0^Tv_0=|r_0||v_0|\cos(180^\circ-\theta)$. If the convergence of v_p is fast enough so that the missile's velocity will soon almost be lined up with LOS, and hence $\theta\to 0$ and $r^Tv\to|r||v|$. Apparently, from (9) one can then conclude that ZEM $\to 0$ as $t\to\infty$, which is our ideal goal.

3. Guidance System Design

The equations of relative motion in terms of the relative position $r=r_T-r_M$ and the relative velocity $v=v_T-v_M$ are as follows:

$$\dot v(t)=-a_M(t)-d_M(t)\ \text{ and }\ \dot r(t)=v(t), \quad (10)$$

where $\dot v_T=g_M$, $\dot r_T=v_T$.

Here, the sliding mode guidance law is designed based on the component of the relative velocity normal to the LOS, i.e., $v_p=v-(v^T\hat r)\,\hat r$. To proceed, we first derive the equation of the relative motion perpendicular to the LOS as follows:

$$\dot v_p(t)=-a_M-d_M+(a_M+d_M)^T\hat r\ \hat r-\frac{1}{|r|}|v_p|^2\hat r-\frac{(v^T\hat r)}{|r|}v_p$$

$$=-a_{Mp}-d_{Mp}-\frac{1}{|r|}|v_p|^2\hat r-\frac{(v^T\hat r)}{|r|}v_p\ , \quad (11)$$

where $a_{Mp}=a_M-(a_M^T\hat r)\,\hat r$ and $d_{Mp}=d_M-(d_M^T\hat r)\,\hat r$.

Referring to (11), an adequate design of a_{Mp} is the following:

$$a_{Mp}(t)=-\frac{(v^T\hat r)}{|r|}v_p+k_pv_p+\tau_p\ , \quad (12)$$

where $k_p=diag(k_{p1}\ \ k_{p2}\ \ k_{p3})$ is a positive definite diagonal matrix, and τ_p is a switching function of the sliding mode control to be specified later, which readily yields

$$\dot v_p=-k_pv_p-d_{Mp}-\frac{1}{|r|}|v_p|^2\hat r-\tau_p \quad (13)$$

Lemma 1 proposed in the following shows that, under some appropriate condition (to be justified later), the sliding-mode guidance law will always render the missile to eventually enter ZEM phase so that the bounded target interception of the whole system can be achieved, i.e., the distance between the missile and the target is close enough to trigger the missile's explosion.

Lemma 1: Let the equation of the relative motion perpendicular to LOS and the sliding-mode guidance law be given by (11) and (12), respectively. If $v_r^Tr<0$ for $t\ge 0$, with v being bounded away from zero, and the sliding-mode guidance law will switch to the ZEM phase whenever $|r|\le r_{\min}$, then the sliding-mode guidance system will drive the missile to enter the ZEM phase eventually so that the bounded target interception of the whole system can be achieved.

Proof of Lemma 1: Let $V_G = \frac{1}{2} v_p{}^T v_p$ be a Lyapunov function candidate, and evaluate the time derivative of V_G along the trajectories of the system (13) as follows:

$$\dot{V}_G = v_p{}^T(-k_p v_p - d_{Mp} - \frac{1}{|r|}|v_p|^2 \hat{r} - \tau_p)$$
$$= -v_p^T k_p v_p - v_p^T(d_{Mp} + \tau_p) \quad (14)$$

with τ_p now being specified as

$$\tau_p = k_1 \operatorname{sgn}(v_p) \ , \quad (15)$$

where we use the fact $v_p^T \hat{r} = 0$. Assume that the external disturbance d_{Mp} perpendicular to LOS is bounded. It is evident that if we choose $k_1 = diag[k_{11}\ k_{12}\ k_{13}]$, $k_{1i} > d_{Mpi}^{\max}, i = 1,2,3$, and $d_{Mpi}^{\max} \geq |d_{Mpi}|$, then (14) becomes for $|r| > r_{\min}$

$$\dot{V}_G = -v_p{}^T k_p v_p - \sum_{i=1}^{3} |v_{pi}|(k_{1i} + d_{Mpi} \operatorname{sgn}(v_{pi}))$$
$$\leq -\lambda_{\min}(k_p)|v_p|^2 \ , \quad (16)$$

where $\lambda_{\min}(k_p)$ (> 0) is the minimum eigenvalue of k_p. Thus, $\dot{V}_G$ is apparently negative definite before entering the ZEM phase. This implies that, via Lyapunov stability theory, we can conclude that the speed component of v_p will gradually diminish before ZEM phase is reached. Moreover, the diminishing speed of v_p can be arbitrarily increased by adjusting the gain matrix k_p.

On the other hand, in order to verify the intercepting missile will gradually approach to the target and eventually enter the ZEM phase, we take $V_r = \frac{1}{2} r^T r$ as another Lyapunov function candidate, and differentiate it as follows:

$$\dot{V}_r = v^T r = v_r^T r < 0 \ , \quad (17)$$

which can be obtained from the assumption of Lemma 1. Moreover, since $|v_r| = \left(|v|^2 - |v_p|^2\right)^{\frac{1}{2}}$, we can show that the intercepting missile system will eventually enter the ZEM phase using a contraction argument. That is, assuming that the missile system will never reach ZEM phase, but because that $|v_p|$ will diminish to zero gradually so that $|v_r|$ will be bounded away from zero. Then, it is not hard to find that from (17) $|r|$ will diminish to zero in finite time, which contradicts to the assumption that ZEM phase will never be entered. This confirms our claim about ZEM phase convergence.

■

4. Autopilot System Design

The guidance system, as derived in Section 3, receives the information on the kinematic relation between the missile and the target, and via sliding-mode guidance law determines the acceleration command perpendicular to LOS. On the other hand, the autopilot system will generate the torque command to adjust the attitude of the missile based on the desired and actual quaternion and angular velocity. Referring to Fig. 3.

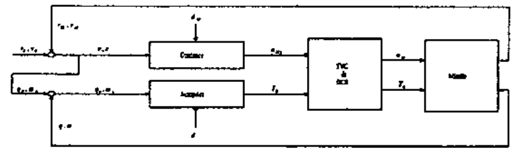

Fig. 3 Block diagram of overall guidance and autopilot systems

Generally speaking, the attitude of a rigid body may be described in various ways, and "quaternion" is one of the means. Thus, for any quaternion, it can be defined as four parameters $q = [q_1 \ \ q_2 \ \ q_3 \ \ q_4]^T = [\bar{q}^T \ \ q_4]^T$ involving n and ϕ, i.e.,

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = n \sin(\phi/2)$$
$$q_4 = \cos(\phi/2) \ . \quad (18)$$

Referring to (18), the unit vector of rotation n of q_d can be easily seen to be the normal unit vector to the plane, containing the x-axis of the inertial coordinate and LOS, i.e.,

$$n = \frac{[1 \ \ 0 \ \ 0]^T \times \hat{r}}{\left|[1 \ \ 0 \ \ 0]^T \times \hat{r}\right|} \quad (19)$$

Continuing this argument, the angle of rotation ϕ is simply the angle between the x-axis of the inertial coordinate and the LOS direction, i.e.,

$$\phi = \cos^{-1}([1 \ \ 0 \ \ 0]^T \hat{r}) \ . \quad (20)$$

Consequently, the desired quaternion q_d can be explicitly computed using (18)-(20).

The dynamic model of a missile, treated as a rigid body, can be derived by differentiation of the associated quaternion as a function of the corresponding angular velocity and the quaternion itself, i.e.,

$$\dot{\bar{q}}_e = \frac{1}{2}\langle \bar{q}_e \times \rangle \omega_e + \frac{1}{2} q_{e4} \omega_e$$
$$\dot{q}_{e4} = -\frac{1}{2} \omega_e^T \bar{q}_e$$
$$J\dot{\omega} = -\dot{J}\omega - \omega \times (J\omega) + T_b + d \ , \quad (21)$$

where $\omega_e = \omega - \omega_d$ is the error between angular velocities at the present attitude and the desired attitude, respectively, and T_b is the torque exerted on the missile due to TVC and the rolling moment.

Equation (4) denotes the torque represented in the body coordinate frame of the missile so that $M_{bx} = T_{b1}$ and the pitch angle and the yaw angle of the TVC hence can be respectively computed as follows:

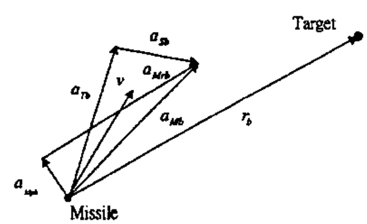

Fig. 4 The relative relation diagram in body coordinate frame

$$d_p = \sin^{-1}(\frac{T_{b2}}{lN}),\quad d_y = \sin^{-1}(\frac{-T_{b3}}{lN\cos d_p})\ , \tag{22}$$

where $T_b = [T_{b1}\quad T_{b2}\quad T_{b3}]^T$ is the torque exerted on the missile.

Accordingly, the desired overall acceleration a_{Mpb} perpendicular to the LOS can be derived due to the result in Section 3, which together with a_{Mrb} in the r direction leads to the desired acceleration a_{Mb} of the missile, namely,

$$a_{Mb} = a_{Tb} + a_{Sb} = a_{Mpb} + a_{Mrb}\ , \tag{23}$$

In order to satisfy Eq. (23), the composed force a_{Mb} of the missile due to TVC and DCS will be lied onto the plane $a_{Mpb} - r_b$ to minimize the rotation motion in the LOS direction such that the translation motion of the missile will always on the plane $a_{Mpb} - r_b$. So the force a_{sb} of DCS in the body coordinate frame should meet the following conditions:

$$\begin{aligned} a_{Sb}^T\hat{a}_{Mpb} &= -a_{Tb}^T\ \hat{a}_{Mpb} + |a_{Mpb}| \\ a_{Sb}^T\hat{a}_{\perp b} &= -a_{Tb}^T\ \hat{a}_{\perp b}\ , \end{aligned} \tag{24}$$

where $\hat{a}_{\perp b} = \frac{a_{Mpb}\times\hat{r}_b}{|a_{Mpb}\times\hat{r}_b|}$, $\hat{r}_b = B_b^T(q)\ \hat{r}$, $\hat{a}_{Mpb} = B_b^T(q)\ \hat{a}_{Mp}$, and $\hat{a}_{Mp} = \frac{a_{Mp}}{|a_{Mp}|}$.

By Cramer's rule, the force a_{Sb} generated by the divert control system denoted as

$$a_{Sb} = \begin{bmatrix} 0 \\ a_{Sby} \\ a_{Sbz} \end{bmatrix} \tag{25}$$

can be derived as:

$$a_{Sby} = \frac{\Delta_y}{\Delta},\quad a_{Sbz} = \frac{\Delta_z}{\Delta},$$

where $\Delta = \hat{a}_{Mpby}\ \hat{a}_{\perp bz} - \hat{a}_{Mpbz}\ \hat{a}_{\perp by}$,

$\Delta_y = (-a_{Tb}^T\hat{a}_{Mpb} + |a_{Mpb}|)\ \hat{a}_{\perp bz} + \hat{a}_{Mpbz}\ a_{Tb}^T\hat{a}_{\perp b}$,

$\Delta_z = -\hat{a}_{Mpby}\ a_{Tb}^T\hat{a}_{\perp b} - (-a_{Tb}^T\hat{a}_{Mpb} + |a_{Mpb}|)\ \hat{a}_{\perp by}$,

$a_{Sb} = [0\quad a_{Sby}\quad a_{Sbz}]^T$, $\hat{a}_{\perp b} = [\hat{a}_{\perp bx}\quad \hat{a}_{\perp by}\quad \hat{a}_{\perp bz}]^T$.

In the following, we will present an attitude tracking controller design to verify the stability and robustness under some appropriate conditions only from the viewpoint of the autopilot system. Lemma 2 proposed in the following shows that the sliding mode control can always make the autopilot system exponentially stable.

Lemma 2: Let the relative rotational motion be given by (21), and the control torque input be proposed as

$$T_b = -k_aS_a + \dot{J}_0\omega - \frac{1}{2}\dot{J}_0S_a - J_0P\left(\frac{1}{2}\langle\bar{q}_e\times\rangle\omega_e + \frac{1}{2}q_{e4}\omega_e\right) + \omega\times(J_0\omega) + J_0\dot{\omega}_d + \tau \tag{26}$$

where

$\tau = [\tau_1\quad \tau_2\quad \tau_3]^T$, $\tau_i = -k_i(q,\omega,q_d,\dot{q}_d,\ddot{q}_d)\cdot\text{sgn}(S_{ai})$, with $\text{sgn}(S_{ai}) = \begin{cases} 1 & S_{ai}>0 \\ 0 & S_{ai}=0 \\ -1 & S_{ai}<0 \end{cases}$, $i = 1,2,3$, and $S_a = [S_{a1}\quad S_{a2}\quad S_{a3}]^T$ is a sliding surface variable defined as

$$S_a = p\bar{q}_e + \omega_e\ , \tag{27}$$

where $P = diag[P_1\quad P_2\quad P_3]$ is a positive definite diagonal matrix. Here, we make an assumption that J is symmetric and positive definite, and let the Lyapunov function candidate be set as

$$V_s = \frac{1}{2}S_a^TJS_a\ . \tag{28}$$

Let the external disturbance d and the induced 2-norm of $\Delta\dot{J}$ and ΔJ are all bounded. If the inequality condition shown below can be guaranteed

$$K_i(q,\omega,q_d,\dot{q}_d,\ddot{q}_d) > \delta_i^{\max}(q,\omega,q_d,\dot{q}_d,\ddot{q}_d) \ge |\delta_i|\ , \tag{29}$$

where

$$\begin{aligned}\delta &= [\delta_1\quad \delta_2\quad \delta_3]^T \\ &= -\Delta\dot{J}\omega - \omega\times(\Delta J\omega) + d - \Delta J\dot{\omega}_d + \Delta JP\left(\frac{1}{2}\langle\bar{q}_e\times\rangle\omega_e + \frac{1}{2}q_{e4}\omega_e\right) + \frac{1}{2}\Delta\dot{J}S_a\ ,\end{aligned}$$

where the bounding functions δ_i, $i = 1,2,3$, are obviously functions of q, ω, q_d, $\dot{q}_d$ and $\ddot{q}_d$, then the exponential stability and robustness of the autopilot system for attitude tracking can be achieved.

Proof of Lemma 2: The principal procedure to verify the stability and robustness of the attitude tracking problem consists of sliding and reaching conditions, and that will be given in detail as follows:

Step 1: Choose the sliding manifold such that the sliding condition will be satisfied and hence the error origin is exponentially stable.

From the sliding-mode theory, once the reaching condition is satisfied, the system is eventually forced to stay on the sliding manifold, i.e., $S_a = P\bar{q}_e + \omega_e = 0$. The system dynamics are then constrained by the following differential equations

$$\begin{aligned}\dot{\bar{q}}_e &= -\frac{1}{2}\langle\bar{q}_e\times\rangle P\bar{q}_e - \frac{1}{2}q_{e4}P\bar{q}_e \\ \dot{q}_{e4} &= \frac{1}{2}\bar{q}_e^TP\bar{q}_e\ .\end{aligned} \tag{30}$$

It has been shown that [5], the system origin $(\bar{q}_e,\omega_e) = (0_{3\times1},0_{3\times1})$ of the ideal system (30) is indeed exponentially stable.

Step 2: Design the control laws such that the reaching condition is satisfied.

Taking the first derivative of V_s, we have

$$\begin{aligned}\dot{V}_s &= S_a^TJ\dot{S}_a + \frac{1}{2}S_a^T\dot{J}S_a \\ &= S_a^T\left[-\dot{J}\omega - \omega\times(J\omega) + T_b + d - J\dot{\omega}_d + JP\left(\frac{1}{2}\langle\bar{q}_e\times\rangle\omega_e + \frac{1}{2}q_{e4}\omega_e\right) + \frac{1}{2}\dot{J}S_a\right]\end{aligned} \tag{31}$$

Due to the assumption and hypothesis, it is evident that Eq. (31) becomes

$$\begin{aligned}\dot{V}_s &= -S_a{}^Tk_aS_a - \sum_{i=1}^{3}|S_{ai}|[k_i - \delta_i\,\text{sgn}(S_{ai})] \\ &\le -\sigma_{\min}(k_a)|S_a|^2 < 0\ ,\end{aligned} \tag{32}$$

for $S_a \ne 0$, where $\sigma_{\min}(k_a)$ is the minimum eigenvalue of k_a, where k_a is a positive definite diagonal matrix. Therefore, the reaching and sliding conditions of the sliding mode $S_a = 0$ are guaranteed. As a result, the exponential stability and robustness of the autopilot system can be achieved as claimed in Lemma 2. ■

5. Integrated Stability Analysis

To verify the stability of the overall system, we define the Lyapunov function candidate of the overall system as

$$V = V_s + V_G\ . \tag{33}$$

The time derivative of the Lyapunov function can be derived as

$$\dot{V} = S_a^T[-\dot{J}\omega - \omega\times(J\omega) + T_b + d - J\dot{\omega}_d + JP\left(\frac{1}{2}(\bar{q}_e\times)\omega_e + \frac{1}{2}q_{e4}\omega_e\right)$$

$$+\frac{1}{2}\dot{J}S_a] + v_p^T(-k_p v_p - d_{Mp} - \frac{1}{|r|}|v_p|^2\hat{r} - \tau_p), \qquad (34)$$

Now, we are ready to state the following theorem which will provide conditions under which the proposed overall sliding mode guidance and autopilot system controlled by TVC, DCS and rolling moment guarantee the stability of the entire system and the target-reaching objective is achieved.

Theorem 1: Let the equation of relative translational motion perpendicular to LOS and the relative rotational motion be described as in Eqs. (11) and (21), and the sliding mode guidance law be proposed as in (12), and the torque input of the autopilot be given as in (26). Now let the missile controller involve TVC, rolling moment, and divert control system, referring to Fig. 1 and Fig. 4, and subject to the guidance law which will switch to ZEM phase when $|r| \le r_{min}$. If v is such that $v^T(t_0)\hat{r}(t_0) < 0$, where t_0 is the starting time, and v is bounded away from zero, then the integrated overall guidance and autopilot systems will drive the missile to enter the ZEM phase eventually so that the bounded target interception of the integrated system can be achieved.

Proof of Theorem 1: From Eq. (34), the expression of $\dot{V}$ can be readily simplified as

$$\dot{V} = -v_p^T k_p v_p - v_p^T(d_{Mp} + \tau_p) - S_a^T k_a S_a - \sum_{i=1}^{3}|S_{ai}|[k_i - \delta_i \operatorname{sgn}(S_{ai})], (35)$$

referring to Eqs. (16) and (32). From Eq. (26), the control input of the autopilot can be computed, and through Eqs. (4) and (22), the rolling moment, the pitch angle and the yaw angle of TVC can also be derived.

From Eqs. (3) and (22)~(25), the desired acceleration a_{Mp} defined in Eq. (12) perpendicular to LOS, for the sake of guidance and the torque input shown in Eq. (26) for attitude tracking can be absolutely computed. Based on the methodology in the aforementioned, Eq. (35) can be expressed as

$$\dot{V} \le -\sigma_{min}(k_p)|v_p|^2 - \sigma_{min}(k_a)|S_a|^2 , \qquad (36)$$

where all the variable definitions are referring to section 3 and 4.

From Eq. (36), it means that $-\dot{V}$ is positive definite, and hence $S_a \to 0$, $v_p \to 0$ as $t \to \infty$ via use of Lyapunov stability theory before entering ZEM phase. In another words, not only the attitude and the component of the relative velocity perpendicular to LOS, v_p, are both stabilized, but also the objectives of attitude tracking and the speed component v_p being diminished gradually are achieved. So we can conclude that the distance $|r|$ between the missile and the target will diminish exponentially before entering the ZEM phase. Thus, this guarantees our claim about the reaching property to enter the ZEM phase in a finite time. Due to Eq. (17) and the derivation of ZEM phase in Lemma 1, the minimum distance between the missile and the target will be less than the pre-specified value r_{min} during ZEM phase, and the target will be destroyed by triggering the missile's explosion when the closest distance between the missile and the target is in the effective interception range through choosing a smaller r_{min}. Therefore, the so-called bounded target interception of the integrated system is achieved.

Therefore, the target-tracking objective during the flight before entering ZEM phase can be completed as derived by the aforementioned proof of theorem 1, and through the ZEM phase, the principal goal of bounded target interception as claimed by the aforementioned theorem can be achieved. ■

6. Simulation

To validate the proposed sliding-mode guidance and autopilot of the missile system presented in Section 3 and Section 4, we provide a realistic computer simulation in this section. We assume the target is launched from somewhere 600 km far away. The missile has a sampling period of 10 ms. The bandwidth of the TVC is 20 Hz and the two angular displacements are both limited to $5°$. Here, we consider the missile's variation of the moment of inertia. Thus, the inertia matrix and the rate of its variation including the nominal part J_0, $\dot{J}_0$ and the uncertain part ΔJ, $\Delta\dot{J}$ used here is as follows:

$$J = J_0 + \Delta J(\text{kg.m}^2)$$
$$\dot{J} = \dot{J}_0 + \Delta\dot{J}(\text{kg.m}^2) ,$$

where

$$J_0 = \begin{bmatrix} 100 & 10 & 300 \\ 10 & 3000 & 300 \\ 300 & 300 & 3000 \end{bmatrix}, \qquad \Delta J = \begin{bmatrix} 10 & 10 & 300 \\ 10 & 300 & 300 \\ 300 & 300 & 300 \end{bmatrix}$$

and the variation of the inertial matrix is as

$$\dot{J} = \begin{bmatrix} -0.4 & -0.1 & -0.2 \\ -0.1 & -12 & -0.2 \\ -0.2 & -0.2 & -12 \end{bmatrix}, \qquad \Delta\dot{J} = \begin{bmatrix} -0.04 & -0.01 & -0.02 \\ -0.01 & -1.2 & -0.02 \\ -0.02 & -0.02 & -1.2 \end{bmatrix} ,$$

where all the components of the inertia matrix and its variation depend on the mass and size of the missile [27], the specific impulse and fuel mass fraction of the propellant [28]. The attitude initial conditions of the missile is set as $q = [0 \ \ -0.707 \ \ 0 \ \ 0.707]^T$ vertical onto the launch pad, and the initial angular velocity is as $\omega(0) = [0 \ \ 0 \ \ 0]^T$, and the variation of missile's mass is as $\dot{m} = -4$ (kg/sec) for the initial mass $m = 1000$ (kg) and the specific impulse $Isp = 250$ (sec). Further, we also consider the aerodynamic force and wind gusts exerted on the missile by $d_i(t) = \sin(t) + 10(u(t-20) - u(t-21))$ (Nt-m) for the rotation motion as in (2) and $d_{Mi}(t) = \sin(t) + 10(u(t-20) - u(t-21))$ (m/s^2) for the translation motion as in (6), for $i = 1,2,3$, where $u(t)$ is the step function. Besides that, we also check the force which is produced by the divert control system equipped on the center of gravity.

In simulation scenario, the feasibility of the presented approach is satisfactorily demonstrated by the results of simulation scenario in Fig. 5. The total simulation time of intercepting phase is 109.46 (sec), switch to ZEM phase, which only spends 0.02 (sec) or so when the distance between the missile and the target is less than 100 meters. Finally, the shortest distance in the intercepting point is less than 1 meter. The effect of the intercepting missile can be shown as in Fig. 5(a), (b) using the lower velocity to intercept the higher velocity of the ballistic missile when it entering the reentry phase. The final velocity of the intercepting missile is 1230.6 (m/sec), which is almost one-third of the velocity, 3168.6 (m/sec), of the target at the final time. On the other hand, the attitude tracking can be verified by the results of Fig. 5(c), (d) which respectively show us the successful tracking effects of the quaternion angle and the perfect approach-to-zero property of the sliding surface for the rotation motion. From Fig. 5(e), (f), which can reveal that we will have a larger value of control torque input in the starting comparing with other times during all the flight phase of the missile, i.e., the less power consumption is enough to complete the attitude tracking for almost the intercepting times. Eventually, the workable divert acceleration of DCS, less than $3\times g$, where g is the gravitational acceleration, is shown in Fig. 5(g). In order to strength the applied practicability of DCS actuator, we propose a low pass filter with bandwidth 5 Hz to limit the response performances of the DCS actuator, so that the desired force of the intercepting missile for the translation motion can be compensated to realistically achieve the superior ballistic missile interception.

Simulation Scenario:

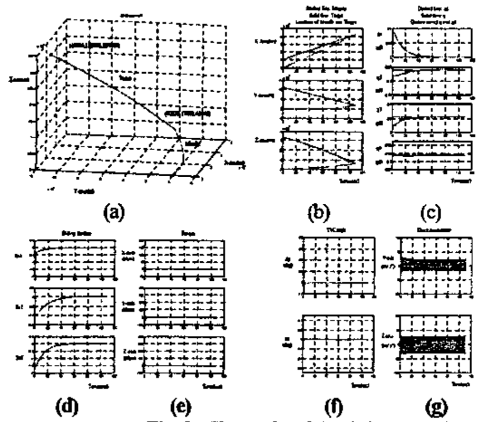

Fig. 5 The results of simulation scenario

7. Conclusions

The overall process of intercepting a ballistic missile generally includes two parts: midcourse and terminal phases. In this paper, we focus on the overall phase composed of the above two phases of the interception, which is a period of time lasting until the ballistic missile can be killed by the intercepting missile. Considering the properties of TVC, DCS and non-ideal conditions during the interception phase, we employed the controller incorporating variable structure based nonlinear missile guidance and autopilot systems, which can robustly adjust not only the missile attitude but also the translation displacement even under the conditions of model uncertainty and disturbances, such as variation of missile's inertia, influence of aerodynamic force and unpredictable wind gusts. We respectively proved the stability of the individual guidance, autopilot and integrated systems via Lyapunov stability theory. Finally, by use of switching to ZEM phase, a bounded target interception can be achieved.

A simulation has been performed to verify the feasibility of the integrated sliding-mode guidance and autopilot systems with 5 DOF inputs. To demonstrate the superior property of the integrated design missile, various unexpected non-ideal phenomena such as external disturbances and internal perturbations are subjected to the interception system. The results are quite satisfactory and encouraging.

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Nomenclature

a	Acceleration vector	$\hat{r}$	The unit vector of r
d	Disturbances vector	$\lvert r \rvert$	The magnitude of r
d_p	Pitch angle of propellant	t	The present time
d_y	Yaw angle of propellant	T	Torque
F	Thrust vector	v	Velocity vector
g	Gravitational acceleration vector	ω	Angular velocity vector
J	Moment of inertial matrix	**Subscripts**	
J_0	Nominal part of J	b	The body coordinate frame
ΔJ	Variation of J	d	Desired
ℓ	Distance between nozzle and center of gravity	e	Error
$L_b = [-\ell \;\; 0 \;\; 0]^T$	Displacement vector	M	Missile
m	Mass of the missile	0	Initial time
N	Magnitude of thrust	p	Perpendicular to LOS
q	Quaternion	S	Divert control
r	Position vector	T	Target/Thrust