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Globally Fully Adaptive Decentralized Control of Robot Manipulators

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ABSTRAC

In this paper, we develop a globally fully adaptive decentralized controller of robot manipulator for trajectory tracking. With some nonlinear feedback terms, the proposed decentralized control law is guaranteed to drive the tracking error asymptotically zero. Moreover, those nonlinear feedback terms not only improve the closed-loop property from semi-global stability to global stability but also make the position and velocity tracking errors approach to zero, which is a dual-goal hardly attained in the literature. The hereby proposed adaptive law allows all signals in the closed-loop systems to be stabilized while making the tracking error converge to zero, even without any prior knowledge of the robot manipulator or the payload and the bound on the design trajectory so that the law can be claimed to be fully adaptive. Finally, a numerical study is provided to verify the effectiveness of the proposed scheme.

Keywords: Decentralized control, adaptive control, robot manipulators, global stability.

I. INTRODUCTION

The control of a robotic manipulator is especially challenging due to the inherent high non-linearity in its dynamics. In a practical situation, the inevitable uncertainty in the underlying manipulator model, say, payload change, adds additional difficulty into the control task. Although significant achievements, marked by the development of centralized adaptive and robust control schemes [8; 9] have been made to improve the tracking performance of robots, the decentralized controller structure is still adopted by the majority of modern robots in favor of its computation simplicity and low-cost hardware setup. Therefore, how to best improve the tracking performance of robots through decentralized control is still an interesting research topic that attracts great attention from robotic community.

The decentralized robust and adaptive control approaches for linear and linear dominant systems have been well developed, for example, by Ioannou [4], Gavel and Šiljak [3], Shi and Singh [7], and Wen and Soh [12]. Specifically, for a set of linear dominant subsystems whose interconnections are nonlinear but linearly bounded by the norms of the overall system states, the robust and adaptive approaches reported by Ioannou [4], and Gravel and Šilijak [3], etc. guarantee the exponential convergence of the tracking errors and parameter estimation errors to a bounded residual set. In [7], nonlinear feedback was introduced to handle the interconnections bounded by a higher order polynomial of the system-state norms. Moreover, Wen and Soh [12] develop a decentralized model reference adaptive control without restriction on subsystem relative degrees.

For manipulator tracking tasks, decentralized approaches are not that straightforward since the overall system can not be decomposed into subsystems whose states and control inputs are totally decoupled from one another because of the inherent coupling such as moment of inertia, Coriolis force, etc. In the recent researches, several attempts, e.g. by Fu [2], Liu [6], and Tang *et al.* [11], have been made for the adaptive independent joint control (IJC) or the so-called adaptive decentralized control so that a separate actuator taking feedback only from that particular joint is responsible for the joint control. Despite achieving semi-globally asymptotical stability, the approach in [2] however will result in very large PD gains. Although different nonlinear feedback control laws are adopted in [6] and [11] and result in global stability, the resulting tracking errors only converge to a residual set. Besides the above, in all of the former schemes, sufficiently large gains depending on the parameters of the robotic manipulator and the desired trajectory must be provided beforehand so that those schemes cannot be claimed to be fully adaptive. In this study, a globally fully adaptive decentralized trajectory control law is proposed for the robotic manipulators.

II. PROBLEM STATEMENT

For a general n-link rigid manipulator, its dynamic model can be derived by using the Lagrangian-Euler approach and expressed in a symbolic form as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + d(t,q,\dot{q}) \tag{1}$$

where q, $\dot{q} \in \mathrm{R}^n$ are the joint configuration and velocity of the manipulator, respectively, $\tau \in \mathrm{R}^n$ is the control input, M: $\mathrm{R}^n \to \mathrm{R}^{n\times n}$ is the inertia matrix, C: $\mathrm{R}^n \times \mathrm{R}^n \to \mathrm{R}^n$ such that $C(q, \dot{q})\dot{q} \in \mathrm{R}^n$ is the vector of Coriolis and centrifugal force, g: $\mathrm{R}^n \to \mathrm{R}^n$ is the vector due to the gravitational force, and d: $[0, \infty) \times \mathrm{R}^n \times \mathrm{R}^n \to \mathrm{R}^n$ is the input disturbance vector. This dynamic model has the following properties that will be used in the controller design [10]:

Property 1: The matrix M is a symmetric and positive definite matrix, which satisfies $\mu_m I \le M(q) \le \mu_M I$, $\forall q \in \mathrm{R}^n$ for some constants μ_m, $\mu_M > 0$.

Property 2: The matrix C satisfies $\|C(q,\dot{q})\| \le \mu_C \|\dot{q}\|$, $\forall q \in \mathrm{R}^n$, $\dot{q} \in \mathrm{R}^n$ for some constant $\mu_C > 0$.

Property 3: $\dot{M} - 2C$ is skew-symmetric.

Property 4: The vector g satisfies $\|g(q)\| \le \mu_G, \forall q \in \mathrm{R}^n$ for some constant $\mu_G > 0$.

Now let $q_d(t)$ denote the desired position trajectory for tracking, which is generally chosen twice differentiable to guarantee smoothness of the motion. Let $\tau^T = [\tau_1, \ldots, \tau_n]$. If the ith element of the control input for $i = 1, \ldots, n$, is only a function of joint configuration and velocity of the ith joint, the control input τ is called a decentralized control. In this study, a decentralized control scheme is to be developed such that $q(t) \to q_d(t)$ and $\dot{q}(t) \to \dot{q}_d(t)$ as $t \to \infty$.

III. CONTROLLER DESIGN

In this section, two control schemes will be developed for the control of a general n-link rigid time-varying manipulator. The first controller is in a manner of decentralized control scheme. In this model-based control scheme, we have assumed that the model parameters of the rigid manipulator are perfectly known. Under this assumption, the tracking errors of the joint configuration and velocity will be asymptotically vanishing. Based on the design procedure of the first controller, the second one incorporating an adaptive mechanism is then proposed to achieve the former goal in the presence of the parametric uncertainty.

A. Non-Adaptive Decentralized Control

Let $q_d(t)$ denote the desired position trajectory. The following error signals are defined as $e = q - q_d$ and $s \equiv \dot{e} + \Lambda e$ where the feedback gain matrix Λ is a positive definite matrix. Now the dynamics defined by the signals e and s can be derived as

$$\dot{e} = -\Lambda e + s \tag{2a}$$

$$M(q)\dot{s} = -C(q,\dot{q})s + \tau - v(t,q,\dot{q}) \tag{2b}$$

where

$$\begin{aligned} v(t,q,\dot{q}) = {} & M(q)(\ddot{q}_d + \Lambda\dot{e}) + C(q,\dot{q})(\dot{q}_d + \Lambda e) \\ & + g(q) - d(t,q,\dot{q}) \end{aligned} \tag{3}$$

behaves as the interconnection term. In the following, without loss of generality, several technical assumptions are made to pose the problem in a tractable manner.

Assumption 1: The feedback gain matrix Λ are diagonal positive definite matrices; that is, $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n) > 0$ for $\lambda_i > 0$, $i = 1, \cdots, n$.

Assumption 2: The desired position trajectory $q_d(t)$ and the time derivatives of $q_d(t)$ and $\dot{q}_d(t)$ are all bounded time-varying signals.

Assumption 3: The input disturbance $d = [d_1, \ldots, d_n]^T$ is upper bounded as $|d_i(t,q,\dot{q})| \le d_{i\_1} + d_{i\_2}|q_i| + d_{i\_3}|\dot{q}_i|$ $\forall\, t \in [0, \infty)$, $q \in \mathrm{R}^n$, $\dot{q} \in \mathrm{R}^n$, where d_{i_1}, d_{i_2}, d_{i_3} are nonnegative constants for $i = 1, \ldots, n$. For example, the viscous friction is represented as $d = -F\dot{q}$, where $F \in \mathrm{R}^{n\times n}$ is the diagonal coefficient matrix.

Before the claimed decentralized control law is designed, two useful lemmas should be derived first. In this study, we start with adopting the truncated L_∞ norm defined as $\|x\|_{T,\infty} \equiv \max_{1\le i\le n} \sup_{t\in[0,T]} |x_i(t)|$ for all real vector-valued functions $x \in C^n[0, T]$ for some $T > 0$. The following lemma will provide us useful inequalities to develop the control scheme in this study.

Lemma 1: *If there exists a constant $T > 0$ such that $\|s\|_{T,\infty}$ exists, then there are positive constants α_1, α_2, α_3, and α_4 such that for $t \in [0, T]$*

$$\|e(t)\| \le \alpha_1\|e_0\| + \alpha_2\|s\|_{T,\infty} \qquad (4)$$

$$\|\dot{e}(t)\| \le \alpha_3\|e_0\| + \alpha_4\|s\|_{T,\infty} \qquad (5)$$

where $e_0 = e(0)$ is the initial condition of e, and positive constants β_1, β_2, and β_3 are such that for $t \in [0, T]$

$$\|v(t, q, \dot{q})\| \le \beta_1 + \beta_2\|s\|_{T,\infty} + \beta_3\|s\|^2_{T,\infty} \qquad (6)$$

Proof: Consider the linear system defined by $\dot{e} = -\Lambda e + s$, $e(0) = e_0$. Since the matrix $-\Lambda$ is Hurwitz, we obtain (4) for $t \in [0, T]$ where α_1 and α_2 are positive parameters. Furthermore, according to the definition of the linear systems, we then obtain (5) for $t \in [0, T]$ where α_3 and α_4 are positive parameters. Next, according to the definition of the interconnection term v in (3), after applying the aforementioned properties of the robotic manipulator and Assumption 3, we have (6) for $t \in [0, T]$, where β_1, β_2, and β_3 are positive parameters. ■

Now, we consider the a control law $\tau^T = [\tau_1, \ldots, \tau_n]$ given as follows

$$\tau_i = -\theta_{i\_1}\operatorname{sgn}(s_i) - \theta_{i\_2}s_i - \theta_{i\_3}s_i^3 \qquad (7)$$

for $i = 1, \ldots, n$, where the constants θ_{i_j}, $i = 1, \ldots, n$, $j = 1, 2, 3$, are the controller gains to be designed, and $\operatorname{sgn}(\cdot)$ denotes the signum function [5]. With (7), the closed-loop system becomes differential equations with discontinuous right-hand sides [1]. A considerable amount of works has been carried out to deal with situations like this. Here, we base our results for these differential equations on Filippov's concept. Note that the control law (7) is apparently in a decentralized structure. The performance of the control law (7) can be summarized into the following theorem.

Theorem 2 (Non-Adaptive Decentralized Control): *Under Assumptions 1~3, consider the error dynamics of the robotic manipulator with the control law (7) defined above. If the parameters θ_{i_1} are large enough, and $\theta_{i\_j} > 0$ for $i = 1, \ldots, n$, and $j = 2, 3$, then the position tracking error $e(t)$ and the velocity tracking error $\dot{e}(t)$ will converge to zero as $t \to \infty$.*

Proof: The proof proceeds in the following two steps.

Step 1: Prove the signal $s(t)$ is ultimately bounded. Consider the same Lyapunov-like function.

$$V(t) = \tfrac{1}{2}s^T M s \qquad (8)$$

Consider $\| s \|_{T,\infty} < l_1$ for some $T > 0$. Then, after taking the time derivative of (8) along the solution trajectories of the closed-loop system, we obtain for $t \in [0, T]$

$$\begin{aligned}\dot{V} &\le -\theta_{2,\min}\|s\|_2^2 - \tfrac{1}{n}\theta_{3,\min}\|s\|_2^4 + \|s\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) \\ &\le -\theta_{2,\min}\|s\|_2^2\end{aligned} \qquad (9)$$

when $\|s\|_2 \ge [\frac{n(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$, where $\theta_{j,\min} = \min\{\theta_{1\_j}, \ldots, \theta_{n\_j}\}$, for $j = 2, 3$. Note that (9) has adopted the so-called Schwartz inequality so that $\sum_{i=1}^n s_i^4 \ge \frac{1}{n}\|s\|_2^4$. Now, (9) can imply that, there exists $l_2 > 0$ such that for $t \in [0, T]$, $\| s(t) \|_2 \le l_2 < l_1$ if l_1 is sufficiently large. For example, in the inequality above, l_2 can be defined as $l_2 = \sqrt{\frac{\mu_M}{\mu_m}}[\frac{n(\beta_1+\beta_2 l_1+\beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$ via the ultimate boundedness property [5], where l_1 is sufficiently large. Apparently, l_2 growing in the order of $l_2^{1/3}$ can be much smaller than l_1 provided l_1 is large enough. This will consequently lead to the fact $\|s(t)\|$ cannot grow unbounded asymptotically in time t, i.e., there exists finite $l_3 > 0$ such that $l_3 = \sup_{t\in[0,\infty)}\|s(t)\|_2$.

Step 2: Prove the signal $s \to 0$ as $t \to \infty$. After taking the time derivative of (8) along the solution trajectories of the closed-loop system again, because of sufficiently large $\theta_{1,\min}$ where $\theta_{1,\min} = \min\{\theta_{1\_1}, \ldots, \theta_{n\_1}\}$, we obtain

$$\begin{aligned}\dot{V} &\le -\theta_{1,\min}\|s\|_2 - \theta_{2,\min}\|s\|_2^2 + (\beta_1 + \beta_2 l_3 + \beta_3 l_3^2)\|s\|_2 \\ &\le -\theta_{2,\min}\|s\|_2^2\end{aligned} \qquad (10)$$

for $t \in [0, \infty)$, which implies $\|s(t)\|_2 \le \sqrt{\frac{\mu_M}{\mu_m}}\|s(0)\|_2 e^{-\eta t}$ for $t \in [0, \infty)$ with some $\eta > 0$. Hence, $s \to 0$ as $t \to \infty$ so that $e \to 0$ and $\dot{e} \to 0$ as $t \to \infty$. ■

Remark 1: The constants β_1, β_2, β_3 and l_3 in Theorem 2 are related to the parameters of the robotic manipulator, the bounds on the desired position trajectories and the initial conditions. Hence, in order to implement the control law in Theorem 1, the information about the robotic manipulator, the desired position trajectory, and the initial condition should be given *a priori*. Furthermore, we can conclude that since the gains θ_{i_1}, $i = 1, \ldots, n$, are sufficiently large, the decentralized control law (7) will endow the closed-loop system semi-globally uniformly asymptotical stability at $e = 0$ and $\dot{e} = 0$.

B. Adaptive Decentralized Control

As we have mentioned before in Remark 1, the correct estimation of θ_{i_1}, $i = 1, \ldots, n$, is not easy. In this subsection, an adaptive decentralized control scheme is presented to solve this problem. Consider another control law $\tau^T = [\tau_1, \ldots, \tau_n]$ given as follows

$$\tau_i = \begin{cases} -\frac{1}{\varepsilon_i}\hat{\theta}_{i\_1}^2 s_i - \theta_{i\_2} s_i - \theta_{i\_3} s_i^3 & \text{if } \hat{\theta}_{i\_1}|s_i| \le \varepsilon_i \\ -\hat{\theta}_{i\_1}\,\mathrm{sgn}(s_i) - \theta_{i\_2} s_i - \theta_{i\_3} s_i^3 & \text{if } \hat{\theta}_{i\_1}|s_i| > \varepsilon_i \end{cases} \tag{11}$$

for $i = 1, \ldots, n$, where the parameters $\hat{\theta}_{i\_1}$, $i = 1, \ldots, n$, need to be adjusted on line, and $\theta_{i\_j} > 0$ for $i = 1, \ldots, n$, and $j = 2, 3$. Furthermore, the auxiliary signals ε_i, $i = 1, \ldots, n$, satisfy

$$\dot{\varepsilon}_i = -p_i \varepsilon_i, \quad \varepsilon_i(0) > 0 \text{ and } p_i > 0, \quad i = 1, \ldots, n \tag{12}$$

It is worth noting that the time-varying signals ε_i, $i = 1, \ldots, n$, are always positive, which are used as the time-varying boundary layers. In order to account for the parametric uncertainty from the manipulator and the desired position trajectories, we choose the adaptive law as follows

$$\dot{\hat{\theta}}_{i\_1} = \gamma_{i\_1}|s_i| \tag{13}$$

where $\hat{\theta}_{i\_1}(0) \ge 0$, and the adaptive gains $\gamma_{i\_1} > 0$ for $i = 1, \ldots, n$. Note that the adaptive control law (11)-(13) is apparently in a decentralized manner, and its performance can be summarized into the following theorem.

Theorem 3. (Adaptive Decentralized Control Scheme) *Under Assumption* 1~3, *consider the error dynamics of the robotic manipulator with the adaptive control law* (11)-(13) *defined above. Then, all signal are bounded, and, furthermore, the position tracking error $e(t)$ and velocity tracking error $\dot{e}(t)$ will converge to zero as $t \to \infty$.*

Proof: Similarly, the proof proceeds in the following two steps.

Step 1: Prove the signal $s(t)$ is ultimately bounded. Consider the Lyapunve-like function (8). Consider $\|s\|_{T,\infty} < l_1$ for some finite $T > 0$. Then, after taking the time derivative of (8) along $s(t)$ for $t \in [0, T]$, the following two different cases will be obtained:

Case 1: $\hat{\theta}_{i\_1}|s_i| \le \varepsilon_i$ for $t \in [0, T]$.

$$\begin{aligned} \dot{V} \le & -\sum_{i=1}^{n} \frac{1}{\varepsilon_i}\hat{\theta}_{i\_1}^2 s_i^2 - \theta_{2,\min}\|s\|_2^2 - \frac{1}{n}\theta_{3,\min}\|s\|_2^4 \\ & + \|s\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) \end{aligned} \tag{14}$$

Case 2: $\hat{\theta}_{i\_1}|s_i| > \varepsilon_i$ for $t \in [0, T]$

$$\begin{aligned} \dot{V} \le & -\sum_{i=1}^{n} \hat{\theta}_{i\_1}|s_i| - \theta_{2,\min}\|s\|_2^2 - \frac{1}{n}\theta_{3,\min}\|s\|_2^4 \\ & + \|s\|_2(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2) \end{aligned} \tag{15}$$

where $\theta_{j,\min} = \min\{\theta_{1\_j}, \ldots, \theta_{n\_j}\}$, for $j = 2, 3$. From both Case 1 and 2, we can conclude that for $t \in [0, T]$

$$\dot{V} \le -\theta_{2,\min}\|s\|_2^2 \tag{16}$$

when $\|s\|_2 \ge [\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$, which implies that there exists $l_2 > 0$ such that for $t \in [0, T]$, $\|s(t)\|_2 \le l_2 < l_1$ if l_1 is sufficiently large. Similar to the previous argument, l_2 can be defined as $l_2 = \sqrt{\frac{\mu_M}{\mu_m}}[\frac{n(\beta_1 + \beta_2 l_1 + \beta_3 l_1^2)}{\theta_{3,\min}}]^{1/3}$ where l_1 is sufficiently large, and hence one can show that $s(t)$ is actually ultimately bounded so that there exists $l_3 > 0$ such that $l_3 = \sup_{t \in [0,\infty)} \|s(t)\|_2$.

Step 2: Prove all signals are bounded and the signal $s \to 0$ as $t \to \infty$. Now consider the Lyapunov-like function as follows

$$V(t) = \frac{1}{2}s^T M s + \sum_{i=1}^{n}[\frac{1}{2}\gamma_{i\_1}^{-1}(\hat{\theta}_{i\_1} - \theta_{i\_1}^*)^2 + p_i^{-1}\varepsilon_i] \tag{17}$$

where $\theta_{i\_1}^*$, $i = 1, \ldots, n$, are the desirable but unknown parameters as revealed in Theorem 2 for θ_{i_1}, $i = 1, \ldots, n$. After taking the time derivative of (17) along the solution trajectories of the closed-loop system, the following two different cases will be obtained:

Case 1: $\hat{\theta}_{i\_1}|s_i| \le \varepsilon_i$ for $t \in [0, \infty)$.

$$\begin{aligned} \dot{V} \le & -\sum_{i=1}^{n} \frac{1}{\varepsilon_i}\hat{\theta}_{i\_1}^2 s_i^2 - \theta_{2,\min}\|s\|_2^2 - \frac{1}{n}\theta_{3,\min}\|s\|_2^4 \\ & + \|s\|_2(\beta_1 + \beta_2 l_3 + \beta_3 l_3^2) - \theta_{1,\min}^*\|s\|_2 \end{aligned} \tag{18}$$

Case 2: $\hat{\theta}_{i\_1}|s_i| > \varepsilon_i$ for $t \in [0, \infty)$

$$\begin{aligned} \dot{V} \le & -\theta_{2,\min}\|s\|_2^2 - \frac{1}{n}\theta_{3,\min}\|s\|_2^4 + \|s\|_2(\beta_1 + \beta_2 l_3 + \beta_3 l_3^2) \\ & - \theta_{1,\min}^*\|s\|_2 - \sum_{i=1}^{n}\varepsilon_i \end{aligned} \tag{19}$$

where $\theta_{1,\min}^* = \min\{\theta_{1\_1}^*, \ldots, \theta_{n\_1}^*\}$. From both Case 1 and 2, we can conclude that for sufficiently large $\theta_{1,\min}^*$ we obtain

$$\dot{V} \le -\theta_{2,\min}\|s\|_2^2 \tag{20}$$

for $t \in [0, \infty)$, which implies all signals are bounded and $\| s(t) \|_2$ is L_2. Finally, to show the zero convergence of the tracking error $e(t)$ and $\dot{e}(t)$, we need to show $\| s(t) \|_2$ is uniformly continuous and then apply Barbălat's Lemma [5]. Sufficiently, we investigate boundedness of the signal $\dot{s}(t)$ from (2), (6), and (11), and easily we can verify such a condition. As a result, the zero convergence is thus insured. ■

Remark 2: In Theorem 3, we would like to emphasize that here in our proposed scheme, the control gains, θ_{i_j} for $i = 1, \ldots, n, j = 2, 3$, only need to be chosen positive, and the adaptive decentralized control law (11)-(13) will then successfully drive the tracking errors $e(t)$ and $\dot{e}(t)$ of the closed-loop system to converge to zero globally. However, this salient feature is in fact hardly achievable by [2], [6], and [11]. Hence, this adaptive control law can be claimed to be fully adaptive which is more convenient for implementation.

Due to the numerical noise caused by convergence of ε_i, $i = 1, \ldots, n$, to zero, practical implementation of the scheme should be considered, e.g., using addition of σ-modification terms [2]. Now, consider another modified adaptive law as follows:

$$\dot{\hat{\theta}}_{i\_1} = \gamma_{i\_1}(|s_i| - \sigma\hat{\theta}_{i\_1}) \tag{21}$$

where $\hat{\theta}_{i\_1}(0) \geq 0$ and the adaptive gains $\gamma_{i\_1} > 0$, $i = 1, \ldots, n$, and the σ-modification constant $\sigma > 0$. Note that the adaptive law (21) is apparently in a decentralized structure, too. The performance of adaptive control law (11), (12) and (21) can be summarized into the following corollary.

Corollary 4 (Adaptive Decentralized Control Scheme with σ-Modification): *Under Assumption* 1~3, *consider the error dynamics of the robotic manipulator with the adaptive control law* (11), (12) and (21). *Then all signal are bounded, and, furthermore, the position tracking error* $e(t)$ *will converge to a residue set whose size can be reduced by use of larger* $\theta_{2,\min}$, *and* $\gamma_{1,\min}$, *where* $\theta_{2,\min} = \min\{\theta_{2\_1}, \ldots, \theta_{2\_n}\}$, *and* $\gamma_{1,\min} = \min\{\gamma_{1\_1}, \ldots, \gamma_{n\_1}\}$.

Proof: This proof is similar to that in the proof of Theorem 3, and, hence, we neglect it here. ■

Remark 3: Although Corollary 4 proposed a similar conclusion to that in [6] and [11], this statement is guaranteed by means of a different approach. It is worth noting that, in the proof of [6], since the truncated L_∞ vector-valued function norm takes the maximum value of vector norm during some past time interval, it may be not appropriate to take the term $-r^TKr$ in (23) as a negative bounding term for stability analysis in [6]. Hence, we believe the validity of the controller [6] requires further clarification. On the other hand, in [11], the inequalities $\|\dot{q}_i(t)\| \leq k_1\|q_i(t)\| + k_2$ for some $k_1, k_2 \geq 0$ for $i = 1, \ldots, n$, have been used to guarantee the performance of the proposed scheme; however, there is no proof of those inequalities provided in the same paper. Likewise, we believe the validity of the claims in [11] also needs stronger justification.

IV. SIMULATION RESULTS

In order to demonstrate the performance of the proposed adaptive decentralized controller, several numerical results are provided now. A control of two-link planar robot manipulator, shown in Fig. 1, is considered here. The manipulator is assumed to move on a horizontal plane and contact with the environment modeled as a straight line on the plane. The links are of uniform density and have the same length $l = 1$. The masses of the links are $m_1 = 3$ and $m_2 = 1$, respectively. The gravitation is neglected in this study. For practical consideration, the adaptive scheme with σ-Modification is studied. The desired trajectory $q_d(t) = [q_{d1}(t), q_{d2}(t)]^T$ is defined by $q_{d1}(t) = 1 \sin t$, $q_{d2}(t) = 1 \sin t$. The initial conditions used in the numerical study are $q_1(0) = q_2(0) = 0$, $\dot{q}_1(0) = \dot{q}_2(0) = 0$, $\hat{\theta}_{1\_1}(0) = \hat{\theta}_{2\_1}(0) = 0$, $\varepsilon_1(0) = \varepsilon_2(0) = 1$. For practical implementation mentioned in Corollary 4, the adaptive law with the σ-modification term is considered. The control gains, adaptive gain constants, and auxiliary signals are, respectively, defined by use of $\lambda_1 = \lambda_2 = 6$, $\theta_{1\_2} = \theta_{2\_2} = 50$, $\theta_{1\_3} = \theta_{2\_3} = 1$, $\gamma_1 = \gamma_2 = 0.5$, $\sigma = 0.5$, and $p_1 = p_2 = 0.5$. Let $e = [e_1, e_2]^T$. Figures 2 and 3 study the numerical results of Links 1 and 2, respectively. From Figures 2(a), 3(a), 2(b) and 3(b), it is found that the position and velocity tracking errors approach to a residual set as time approaches infinite. Figures 2(c) and 3(c) demonstrate the updated parameters due to the adaptive law. It is represented that the parameters updated are uniformly bounded. Furthermore, Figures 2(d) and 3(d) illustrate that the two control inputs under this adaptive scheme are also bounded.

V. CONCLUSION

In this paper, we develop a globally fully decentralized control scheme of robot manipulator for trajectory tracking that meets in twofold objective: One is non-adaptive one and the other is adaptive one. It shows that, with some nonlinear feedback terms, based on the Lyapunov approach, a decentralized control law is proposed to insure that the position and velocity tracking errors converge to zero. In particular, such nonlinear feedback terms improve the stability property from semi-global stability to global stability in regard to existing results in the literature. Following the same design philosophy, this paper also proposed an adaptive control law, which not only insures all signals inside the closed-loop system are bounded but also drive the position and velocity tracking errors to zero asymptotically, even without both prior knowledge of the manipulator model nor the payload on the reference trajectory such that the control scheme can be claimed to be fully adaptive. For numerical noise caused by the convergence of the time-varying layer, the adaptive law can be also modified with a σ-modification term so that the tracking error will converge to a residual set whose size can be reduced by use of a proper design parameters. Finally, a numerical study is provided to confirm the effectiveness of the proposed scheme.

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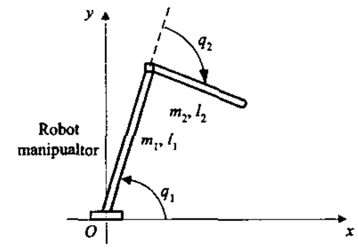
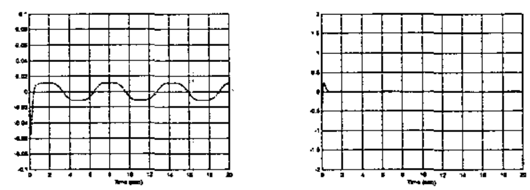
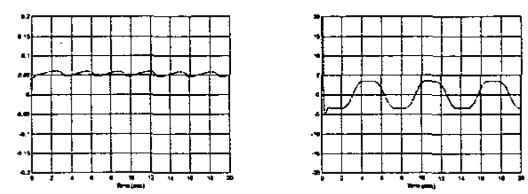


Fig. 1 Two-link planar robotic manipulator

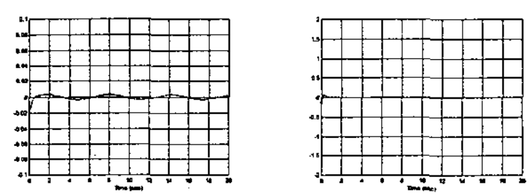


(a) Position tracking error e_1 (b) Velocity tracking error $\dot{e}_1$

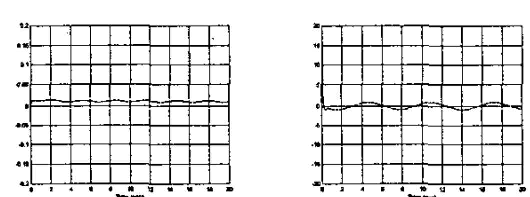


(c) Estimated parameter $\hat{\theta}_{1\_1}$ (d) Control input τ_1

Fig. 2 Numerical results of Link 1



(a) Position tracking error e_2 (b) Velocity tracking error $\dot{e}_2$



(c) Estimated parameters $\hat{\theta}_{2\_1}$ (d) Control input τ_2

Fig. 3 Numerical results of Link 2