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Applications of Stochastic Media Theory to 1992, 1996, and 2000 National Election Study Panel Data

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We study a class of stochastic models of persuasion that form an application of media theory developed by Falmagne and others. These models describe the evolution of preferences over time. We consider the case where personal preferences are represented by (strict) weak orders and semiorders. Over time, these preferences may change under the influence of ‘‘tokens’’ of information arising in the environment. Successful applications of some weak order implementations of stochastic media theory to 1992 U.S. National Election Study (NES) panel data have been reported by Falmagne and various collaborators. However, past attempts to fit a semiorder model to the same data have failed. We successfully fit four media theoretic models, including two semiorder models based on the ‘‘neighboring’’ response mechanism, to 1992, 1996, and 2000 NES panel data. We compare the fit of these four models, discuss the psychological interpretation of key model parameters and illustrate applications to negative political campaigning.

Keywords: ***Panel Data, Semiorder, Stochastic Media Theory, Thermometer Scores, Weak Order***

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1.Introduction

This study concerns stochastic models of the evolution of preferences over time. While opinions are often expressed in the form of rankings of the choice alternatives (or other forms of responses which are convenient for the interviewer or the statistician), intermediate states of mind of the respondents may be much less structured. The general framework of this research is *stochastic media theory* (Falmagne, 1997; Falmagne & Ovchinnikov, 2002). The concept of a "medium" captures the organization of a collection of states through a set of transformations, or "tokens," each of which maps the set of states into itself. The application in this paper is a special case of stochastic media theory. In our model, each state represents a feasible, yet possibly unobservable, mental preference state of an individual, and the tokens reflect the effects of a persuasion environment on the individuals' states. The theoretical media theory framework provides a variety of models for the evolution of a system that is subjected to a constant probabilistic barrage of not necessarily observable events liable to change the state of the system. Moreover, this stochastic theory is applicable to and statistically testable on various types of data, including opinion polls. For example, the stochastic (strict) weak order model for the evolution of preferences proposed by Falmagne, Regenwetter, and Grofman (1997) and tested by Regenwetter, Falmagne, and Grofman (1999) gives a good statistical account of attitudinal panel data pertaining to the three major candidates in the 1992 U.S. presidential election.

Recently, Hsu, Regenwetter, and Falmagne (2005) showed that the weak order model has the defect of underestimating the degree of response consistency across repeated polls. They presented a generalization of Falmagne et al.'s (1997) model based on the idea that some individuals may become momentarily desinterested in a persuasion campaign and 'tune out.' A respondent in an "active" state behaves as in the original weak order model, except if receiving a "tune-out" token, which effectively "freezes" the respondent's preference state until it is reversed by a "tune-in" token. Similarly to the original weak order model, the tune in-and-out (TIO) extension is a random walk, but on an augmented set of preference states. Hsu et al. (2005) described and successfully fit the "weak order TIO model" to the same 1992 panel data that Regenwetter et al. (1999) studied.

A key new idea of a related paper by Falmagne, Hsu, Leite, and Regenwetter (in press) was to introduce response functions that map latent (possibly unobservable) states in a medium into overt responses. Our paper is a follow-up to the studies by Falmagne et al. (in press) and Hsu et al. (2005). We place special emphasis on situations in which the possible observable responses in an empirical paradigm do not span the entire space of possible latent preference states. In particular, we carefully study a particular response function for semiorder media applied to "thermometer score" data, as this is a case where the interplay between model and data can be complicated. Another emphasis is to expand the application of media theoretic models to additional interesting data sets. We apply a weak order model, a semiorder model, and TIO extensions of both models to 1992, 1996, and 2000 U.S. National Election Study (NES) panel data. We focus on the comparison of the four models in terms of some goodness-of-fit indices, and highlight the improvement of the TIO extensions over the original models. We also discuss the psychological interpretation of key model parameters, namely the "tune-out and tune-in token ratios." In each of the three national surveys, the respondents rated the three major candidates using so called *feeling thermometers* on an integer valued scale from 0 to 100, before and after the election, at two time points, one shortly before the election, the other shortly after the election. A thermometer score of zero expressed "cold feelings," a score of 50 expressed "neutral feelings," and a score of 100 expressed "warm feelings" towards a given candidate.

The paper is organized as follows. In Section

2, we give a brief introduction to weak orders and semiorders, and we review the natural interpretation of such orders as mathematical representations of preferences. In Section 3, we review the core of stochastic media theory (for preference change). Section 4 goes into the details of two special cases: random walks on weak orders and on semiorders. We also describe the TIO extension of each model. We devote Section 5 to the comparison of these four media theoretic models on the three panel data sets.

2.Weak Orders or Semiorders as Mathematical Models of Preferences

In this section, we briefly review the definitions and basic properties of weak orders and semiorders. For an overview of binary relations and their use in the social sciences, see Roberts (1979). We restrict our discussion to the case where the set A of choice alternatives is finite because this is natural for most real world applications and because it allows us to avoid technical issues associated with the infinite case.

Definition 1. We use the abbreviation lk for the ordered pair (l,k). A binary relation R on a finite set A is a collection of ordered pairs of elements of A, i.e., $R \subseteq A^2$. Writing $RS = \{lk \mid \exists\, j, lRj, jSk\}$ for the relative product of the relations R and S, a binary relation R is said to be *transitive* if $RR \subseteq R$. We also use $\overline{R} = (A \times A) \setminus R$ for the *complement* of R (with respect to A). A binary relation R is said to be *negatively transitive* if $\overline{R}\,\overline{R} \subseteq \overline{R}$. Writing $R^{-1} = \{jk \mid kRj\}$ for the *converse* of R, a binary relation R is *asymmetric* if $R \cap R^{-1} = \emptyset$.

Definition 2. Let R be a binary relation on a finite set A, then R is a *strict weak order* if it is asymmetric and negatively transitive. The relation R is a *strict partial order* if it is asymmetric and transitive. Furthermore, R is a *semiorder* if R is a strict partial order with the additional properties that $R\overline{R^{-1}}R \subseteq R$ and $RR\overline{R^{-1}} \subseteq R$.

We leave out the word "strict" in "strict weak order" and "strict partial order" from now on. The relationship between weak orders and semiorders becomes obvious in Theorems 3 and 4 below. For semiorders, we make use of the following representation theorem (see, e.g., Scott & Suppes, 1958).

Theorem 3. *Suppose that R is a binary relation on a finite set A and ε is a positive number. Then, R is a semiorder on A if and only if there exists a real-valued function u on A, such that, for all a, $b \in A$,*

$$aRb \iff u(a) > u(b) + \varepsilon. \tag{1}$$

In psychology and related decision sciences, one frequently interprets u in Equation (1) as a utility function and the number ε as a threshold of utility discrimination. We also use a similar representation theorem for weak orders (see, e.g., Krantz, Luce, Suppes, & Tversky, 1971; Roberts, 1979).

Theorem 4. *Suppose that R is a binary relation on a finite set A. Then, R is a weak order on A if and only if there exists a real-valued function u on A, such that, for all a, $b \in A$,*

$$aRb \iff u(a) > u(b).$$

Weak orders can be thought of as rankings with possible ties. Both weak orders and semiorders have a natural interpretation and a long tradition as mathematical representations of preferences (see, e.g., Fishburn, 1979; Krantz et al., 1971; Luce, 1956, 1959). The key difference between weak orders and semiorders, as representations of individual preferences, is that weak orders force indifference to be transitive, whereas semiorders, first developed by Luce (1956), were designed to capture the notion that indifference need not be transitive.

3.Stochastic Media Theory

Definition 5. A *token system* is an ordered pair ($\mathcal{S}$, $\mathcal{T}$), in which $\mathcal{S}$ is a set of states, and $\mathcal{T}$ a set of "tokens." Each *token* $\tau \in \mathcal{T}$ is a transformation from $\mathcal{S}$ into $\mathcal{S}$ that maps any state S into some state $S\tau$, which may equal S or which may differ from S. Let $\tilde{\tau}$ denote the *reverse* of token τ, if it exists, that is, for any states $S \neq T$, we have $S\tau = T$ iff $T\tilde{\tau} = S$. (In particular, $\tilde{\tilde{\tau}} = \tau$.)

A *(token) medium* is a particular token system specified by some constraining axioms. For instance, it assumes that each token has a unique reverse. Falmagne and Ovchinnikov (2002) provide detailed formal statements and proofs, including the axioms defining a medium. Our application of stochastic media theory concerns random walks on media. The states of each random walk are the same as the states of the medium under consideration. The family of all the semiorders on a finite set can be viewed as an example of a medium, with the transformations consisting in adding a pair to, or removing a pair from, a given semiorder (Doignon & Falmagne, 1997; Falmagne, 1997; Falmagne & Doignon, 1997). While the family of all weak orders on a finite set cannot be seen, in general, as a medium under similar transformations, Ovchinnikov (2005) has developed a fairly natural medium interpretation of the collection of all weak orders on a finite set. How individuals move from one state to another over time is the core of stochastic media theory (in the shape of a random walk on its set of states). This section very briefly summarizes the main relevant results of Falmagne et al. (in press).

We assume the existence of two special probability distributions. The first is a probability distribution on the set of states, namely $\eta : \mathcal{S} \rightarrow [0,1] : S \mapsto \eta_S$. The second is a positive probability distribution on the set of tokens, namely $\theta : \mathcal{T} \rightarrow]0,1] : \tau \mapsto \theta_\tau \neq 0$. We capture the evolution of preferences by three interconnected stochastic processes. To specify these, we consider, for each time $t \in]0, \infty[$, three random variables.

$\mathbf{S}_t$ denotes the state of the random walk at time t,

$\mathbf{N}_t$ is the number of token that have occurred in the half open interval of time $]0,t]$, and

$\mathbf{T}_t$ is the last token to occur before or at time t. For simplicity, we write $\mathbf{T}_t = 0$ if no token has ocurred,that is, if $\mathbf{N}_t = 0$

Thus, each $\mathbf{S}_t$ takes its values in the set $\mathcal{S}$ of states, each $\mathbf{N}_t$ is nonnegative integer valued, and each $\mathbf{T}_t$ takes its values in the set $\mathcal{T} \cup \{0\}$.

Stochastic media theory allows us to model the evolution of preferences over time as a (vector of) stochastic process(es)$(\mathbf{N}_t, \mathbf{T}_t, \mathbf{S}_t)_{t>0}$. This process is uniquely specified up to the parameters $(\eta_S)_{S \in \mathcal{S}}$ and $(\theta_\tau)_{\tau \in \mathcal{T}}$ for the two special probability distributions we have introduced earlier, and a parameter $\lambda > 0$ specifying the intensity of a Poisson process that determines the arrival times of the tokens over time. Let $\mathbf{N}_{t,t+\delta} = \mathbf{N}_{t+\delta} - \mathbf{N}_t$ denote the number of tokens in the half open interval $]t,t+\delta]$, and $\mathcal{E}_t$ denote any arbitrarily chosen history of the process prior to and including time $t > 0$. The stochastic process is completely specified by the following three axioms.

[I] (INITIAL STATE.) Initially, the system is in state S with probability η_S. The system remains in that state until the arrival of the first token. That is, for any $S \in \mathcal{S}$ and $t > 0$, $\mathrm{P}(\mathbf{S}_t = S \mid \mathbf{N}_t = 0) = \eta_S$.

[T] (OCCURRENCE OF THE TOKENS.) The token arrivals are governed by a homogeneous Poisson process of intensity λ. When a Poisson event is realized, then token τ occurs with probability $\theta_\tau > 0$, regardless of past events. Thus, for any nonnegative integer k, any real numbers $t > 0$ and $\delta > 0$, and any history $\mathcal{E}_t$,

$$\mathrm{P}(\mathbf{N}_{t,t+\delta} = k \mid \mathcal{E}_t) = \frac{(\lambda\delta)^k e^{-\lambda\delta}}{k!}, \text{ and } \mathrm{P}(\mathbf{T}_{t+\delta} = \tau \mid \mathbf{N}_{t,t+\delta} = 1, \mathcal{E}_t) = \theta_\tau.$$

[L] (LEARNING.) If R is the state of the system at time t, and a single token τ occurs between times t and $t+\delta$, then the state at time $t+\delta$ will be $R\tau$ regardless of past events before time t. That is,

$$\mathrm{P}(\mathbf{S}_{t+\delta} = S \mid \mathbf{T}_{t+\delta} = \tau, \mathbf{N}_{t,t+\delta} = 1, \mathbf{S}_t = R, \mathcal{E}_t) = \begin{cases} 1 & \text{if } S = R\tau, \\ 0 & \text{otherwise}. \end{cases}$$

Note in passing that the main results of the theory can also be derived from more general assumptions than those stated in Axioms **[L]** and **[T]**.

It is easy to show that $(\mathbf{S}_t)$ is a homogenous Markov process. Accordingly, regardless of the times of occurrence of the tokens, the sequence of states, which we denote by $(\mathbf{M}_n)_{n\in\mathbb{N}}$, is a homogeneous Markov chain. Two distinct states R and S of a medium are called *adjacent* if there exists some token τ such that $R\tau = S$ or, equivalently, $S\tilde{\tau} = R$ This means that the Markov process is a *random walk* in the sense that one-step transitions occur only between adjacent states.

If a persuasion process has been running for a long time, we may be able to fit empirical data using the asymptotic distribution of the process (as time goes to infinity). Such an asymptotic distribution relies on the concept of 'content' of a state that contains all transformations leading 'directly' to that state.

Definition 6. The *content* of a state S is the set $\hat{S} \subset \mathcal{T}$ containing all the tokens τ such that there exists a state $R \neq S$ and a minimal sequence of tokens $\tau_1,\ldots,\tau_k$ satisfying $R\tau_1\ldots\tau_k = S$, with $\tau = \tau_i$ for some $1 \leqq i \leqq k$.

It can be shown that, for any state S and token τ, one of τ and $\tilde{\tau}$ is in $\hat{S}$, but not both. Accordingly, we have $|\hat{S}| = |\mathcal{T}|/2$ (cf. Falmagne, 1997, or Falmagne & Ovchinnikov, 2002). It can also be shown that the asymptotic probabilities of the states exist and are given by

$$\lim_{n\to\infty} \mathrm{P}(\mathbf{M}_n = S) = \lim_{t\to\infty} \mathrm{P}(\mathbf{S}_t = S) = \frac{\prod_{\tau\in\hat{S}} \theta_\tau}{\sum_{R\in\mathrm{S}} \prod_{\zeta\in\hat{R}} \theta_\zeta}. \qquad (2)$$

Note that the asymptotic probabilities can be reparametrized through the so-called "bias ratios." Recall that for any content $\hat{S}$ and any token τ, either τ or $\tilde{\tau}$ is in $\hat{S}$. This means that we can reparametrize the asymptotic model with substantially fewer parameters by dividing the numerator and the denominator of Equation (2) by the product of the token probabilities of all those tokens that are reverses of tokens in the content of S (there are $|\mathcal{T}|/2$ many such reverses). Since the content of any state contains either each token itself or its reverse, this is the same as dividing the numerator and the denominator of Equation (2) by the product of token probabilities of all tokens that are not in the content of S. Our reparametrization uses new parameters which we derive directly from the token probabilities as follows: Let the *bias ratio* of τ be $\mathcal{B}[\tau] = \frac{\theta_\tau}{\theta_{\tilde{\tau}}}$. The asymptotic distribution in Equation (2) can be rewritten in terms of bias ratios by defining

$$\mathcal{B}_S[\tau] = \begin{cases} \mathcal{B}[\tau] & \text{if } \tau \in \hat{S}, \\ 1 & \text{otherwise,} \end{cases}$$

and we have

$$\frac{\prod_{\tau\in\hat{S}} \theta_\tau}{\sum_{R\in\mathrm{S}} \prod_{\zeta\in\hat{R}} \theta_\zeta} = \frac{\prod_{\tau\in\hat{S}} \mathcal{B}_S[\tau]}{\sum_{R\in\mathrm{S}} \prod_{\zeta\in\hat{R}} \mathcal{B}_R[\zeta]}. \qquad (3)$$

For any pair (R,S) of states in the Markov chain $(\mathbf{M}_n)_{n\in\mathbb{N}}$, let $\xi_{R,S}(k)$ denote the k-step transition probabilities. In our application to the U.S. presidential election panel data, voters' responses are recorded at two time points, separated by some amount of time δ. We model such joint preferences using the following asymptotic result.

$$\begin{aligned} \lim_{t\to\infty} \mathrm{P}(\mathbf{S}_t = R, \mathbf{S}_{t+\delta} = S) &= \lim_{t\to\infty} \mathrm{P}(\mathbf{S}_t = R)\mathrm{P}(\mathbf{S}_{t+\delta} = S \mid \mathbf{S}_t = R) \\ &= \frac{\prod_{\tau\in\hat{R}} \theta_\tau}{\sum_{\hat{V}\in\mathrm{S}} \prod_{\zeta\in\hat{V}} \theta_\zeta} \sum_{k=0}^{\infty} \xi_{R,S}(k) \frac{(\lambda\delta)^k e^{-\lambda\delta}}{k!}. \end{aligned}$$

In some cases, we use Equation (3) to replace the ratio in front of the infinite sum.

To deal with situations in which some of the states of the medium might not be directly observable, Falmagne et al. (in press) introduced the concept of *response functions*. To take the opinion poll of the U.S. presidential election as an example, the questions are formulated in a fashion that forces voters to answer using thermometer scales. It may not be straightforward to infer the respondents' states of mind directly from such scores as the voters may have transformed their preference state to accommodate the response format required by the poll. Therefore it is natural to model such a transformation using a response function that maps mental states into observable response patterns. We consider here the case in which each response at time t is a possible state of the medium, yet it need not be identical to the state of the random walk at time t.

Definition 7. Let $\mathbf{R}_t$, $t \in]0,\infty[$ denote the response at time t. The random variable $\mathbf{R}_t$ takes its values in a *response set* $\breve{\mathcal{S}} \subseteq \mathcal{S}$ contained in or equal to the set $\mathcal{S}$ of states. We also define a (*probabilistic*) *response function*

$$\gamma : \mathcal{S} \times \breve{\mathcal{S}} \to [0,1] : (S,R) \mapsto \gamma(S,R)$$

$$\sum_{R \in \breve{\mathcal{S}}} \gamma(S,R) = 1, \qquad (S \in \mathcal{S}).$$

The most natural case, where a response function is necessary, is when the set of allowable responses is a proper subset $\breve{\mathcal{S}} \neq \mathcal{S}$. The response axiom below states that the responses only depend on the states via the function γ, and that, given the states, the responses are independent events.

[R] (RESPONSES.) For any choice of times $t_1 < \ldots < t_n$, and any $R_1,\ldots,R_n$ in $\breve{\mathcal{S}}$ and $S_1,\ldots,S_n$ in $\mathcal{S}$,

$$\mathrm{P}(\mathbf{R}_{t_1} = R_1, \ldots, \mathbf{R}_{t_n} = R_n \mid \mathbf{S}_{t_n} = S_n, \ldots, \mathbf{S}_{t_1} = S_1, \mathcal{E}_{t_n}) = \prod_{i=1}^{n} \gamma(S_i, R_i).$$

Combining the notion of a response function with the results we summarized above, we can predict the joint probability of observing, for large values of t, the responses R and R' at times t and $t+\delta$, with $\delta > 0$.

$$\lim_{t\to\infty} \mathrm{P}(\mathbf{R}_t = R, \mathbf{R}_{t+\delta} = R')$$
$$= \sum_{(S,S') \in \mathcal{S}\times\mathcal{S}} \gamma(S,R)\gamma(S',R') \left(\frac{\prod_{\tau \in \hat{S}} \theta_\tau}{\sum_{V \in \mathcal{S}} \prod_{\varsigma \in \hat{V}} \theta_\varsigma} \sum_{k=0}^{\infty} \xi_{S,S'}(k) \frac{(\lambda\delta)^k e^{-\lambda\delta}}{k!} \right).$$

Here again, in some cases we use Equation (3) to replace the ratio in front of the infinite sum. This response mechanism covers situations where the state space contains the survey response space but where the two spaces need not be identical.

4. Random Walks on Weak Orders or Semiorders

The random walk on the weak order medium has been developed by Falmagne et al. (1997) and successfully applied to a national survey by Regenwetter et al. (1999). As mentioned earlier, Ovchinnikov (2005) has developed a fairly natural medium interpretation of the collection of all weak orders on a finite set. For three choice alternatives (e.g., political candidates) there exist 13 different weak order preference states. The graph of this medium is given in Figure 1.

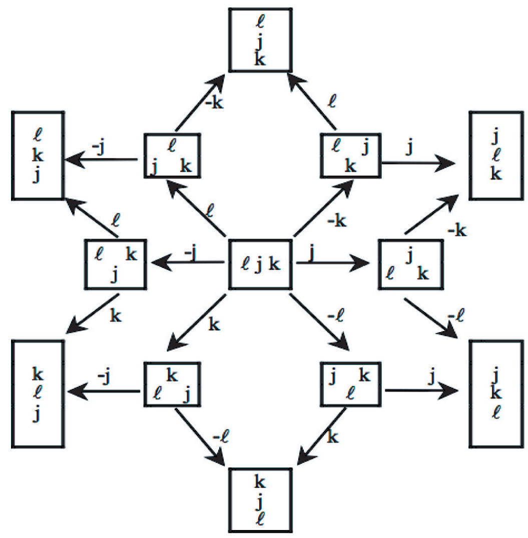

Figure 1. Graph representing the medium whose states are the 13 weak orders on a set of three elements $\{j,k,\ell\}$. The edges represent the positive and the negative tokens, which transform a state into one closer to a linear order. Edges bearing the same label represent the same token. Reversing an edge would yield the reverse token. Note that the labeling of the edges slightly simplifies the notation of the tokens. For example, we write here ℓ rather than τ_ℓ. (Adapted from Falmagne et al., in press, with modifications, with kind permission of authors.)

We also provide in Table 1 an overview of the types of tokens in the weak order medium, their psychological interpretation, as well as the trans-

Table 1
Types of tokens in the weak order medium, their psychological interpretation, and the transformation that each token defines on the collection $\mathcal{WO}$ of weak orders on $\{j,k,\ell\}$. For simplicity in the third column, the weak orders are represented by their Hasse diagrams.

Notation	Psychological Interpretation	Transformation
τ_ℓ	ℓ is the 'best' option	$S\tau_\ell = \begin{cases} \{\ell j, jk\} & \text{if } S = \{\ell k, jk\} \\ \{\ell j, \ell k\} & \text{if } S = \emptyset \\ S & \text{otherwise.} \end{cases}$
$\widetilde{\tau_\ell}$	ℓ is not the 'best' option	$S\widetilde{\tau_\ell} = \begin{cases} \{\ell k, jk\} & \text{if } S = \{\ell j, jk\} \\ \emptyset & \text{if } S = \{\ell j, \ell k\} \\ S & \text{otherwise.} \end{cases}$
τ_{-k}	k is the 'worst' option	$S\tau_{-k} = \begin{cases} \{\ell j, jk\} & \text{if } S = \{\ell j, \ell k\} \\ \{\ell k, jk\} & \text{if } S = \emptyset \\ S & \text{otherwise.} \end{cases}$
$\widetilde{\tau_{-k}}$	k is not the 'worst' option	$S\widetilde{\tau_{-k}} = \begin{cases} \{\ell j, \ell k\} & \text{if } S = \{\ell j, jk\} \\ \emptyset & \text{if } S = \{\ell k, jk\} \\ S & \text{otherwise.} \end{cases}$

formation that each token defines on the collection $\mathcal{WO}$ of weak orders over $\{j,k,\ell\}$. If we think of weak orders as rankings with possible ties, the operation of the tokens can be thought of as moving/removing an option to/from first or last rank and thereby breaking/creating a tie. In our application, we assume that the weak orders are in direct correspondence with the relative magnitudes of the thermometer scores by substituting the thermometer scores for the function u in Theorem 4. In that sense, we treat the weak orders as directly observable without the need for a more complicated response function. The possible transitions are described by the graph of Figure 1 and in Table 1. As we mentioned earlier, Regenwetter et al. (1999) reported a successful application of a weak order model to 1992 NES panel data. To avoid redundancies, we refer the interested reader to Falmagne et al. (1997) and Regenwetter et al. (1999) for details on this model.

The family of all semiorders on a finite set forms a medium (Doignon & Falmagne, 1997; Falmagne, 1997; Falmagne & Doignon, 1997; Falmagne & Ovchinnikov, 2002). A random walk model based on semiorders is different from that based on weak orders. The interest of the semiorder model is that it makes less constraining assumptions concerning the latent mental states of the respondents. In the empirical study described here, the choice set contains only three alternatives. We provide the graph of the semiorder medium in Figure 2.

In Table 2, we provide an overview of the types of tokens in the semiorder medium, their psychological interpretation, as well as the transformation that each token defines on the collection $\mathcal{SO}$ of semiorders over $\{j,k,\ell\}$. It is clear from the table that the tokens of the semiorder medium differ substantially from those of the weak order medium. Unlike the weak order model, here each token provides information about the relative standing of only a pair of candidates in the semiorder. Note that, as indicated in Figure 2 or Table 2, the effect of a token, if any, consists either in the

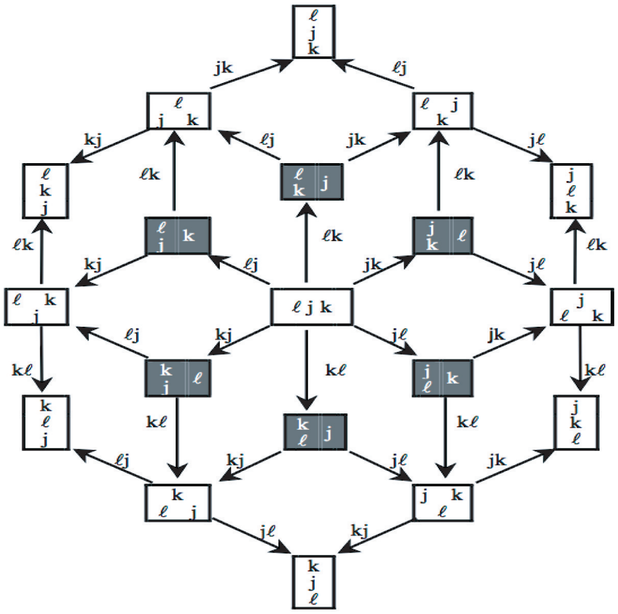

Figure 2. Graph representing the medium whose states are the 19 semiorders on a set of three elements $\{j,k,\ell\}$. The edges represent the tokens transforming a state into one closer to a linear order. Edges bearing the same label represent the same token. Reversing an edge (and transposing the letters of its label) would yield the reverse token. As in Figure 1, our labeling of the edges slightly simplifies the notation of the tokens. For example, we write here $k\ell$ rather than $\tau_{k\ell}$. The six shaded states are those semiorders which are not weak orders. (Adapted from Falmagne et al., in press, with modifications, with kind permission of authors.)

Table 2

Types of tokens in the semiorder medium, their psychological interpretation, and the transformation that each token defines on the collection $\mathcal{SO}$ of all semiorders on $\{j,k,\ell\}$. For simplicity in the third column, the semiorders are represented by their Hasse diagrams.

Notation	Psychological Interpretation	Transformation
τ_{jk}	j is 'preferable' option to k	$S\tau_{jk} = \begin{cases} S \cup \{jk\} & \text{if } S \cup \{jk\} \in \mathcal{SO} \\ S & \text{otherwise.} \end{cases}$
$\widetilde{\tau_{jk}}$	j is not 'preferable' option to k	$S\widetilde{\tau_{jk}} = \begin{cases} S \setminus \{jk\} & \text{if } S \setminus \{jk\} \in \mathcal{SO} \\ S & \text{otherwise.} \end{cases}$

addition of a pair to, or in the removal of such a pair from, the current semiorder state, as long as this leads to a semiorder state again. We do not discuss this model in more detail here, but refer the interested reader to Falmagne and Doignon (1997) and Doignon and Falmagne (1997).

Previous attempts to apply the semiorder model to the 1992 NES panel data have failed. A major reason for the failure has to do with the fact that the NES data were in the form of feeling thermometer ratings. Feeling thermometer ratings have a natural transformation into weak orders via Theorem 4. This is how we could treat the states of the medium as observable in the case of weak order preferences. No such canonical transformation via Theorem 3 of the feeling thermometer ratings is available in the case of semiorders, because each respondent may use a (different) personal threshold $\varepsilon > 0$. Regenwetter et al. (1999) indicated that they tried a semiorder model on the same data but that they encountered a statistically very significant rejection. The transformation underlying their test used Theorem 3 with a fixed utility threshold $\varepsilon > 0$ that was the same for all respondents. Holding the utility threshold constant is an unrealistic feature that may very well lie at the root of the model misfit on these data. Here, we take a different approach when applying the semiorder model to thermometer scores. Instead of assuming a constant threshold across respondents, we bypass the problem by assigning those latent semiorder states, that are not themselves weak orders, to neighboring weak order states via a response function. Then, we apply Theorem 4 to link those weak order responses with the feeling thermometer data. This response function is what we discuss next.

In the semiorder model, the states $\{\ell j\}$ (see the six shaded states in Figure 2) are special, because they are not weak orders and thus can not canonically be matched with thermometer scores via Theorem 3. For these semiorders, there appears to be no way around using Theorem 3 with some positive ε. Nonetheless, one sees from Figure 2 that each of the states $\{\ell j\}$ is linked to three neighboring states that happen to be weak orders. This suggests that using additional (conditional probability) parameters $0 \leqq \alpha_{\ell j, \ell k} \leqq 1$ and $0 \leqq \alpha_{\ell j, kj} \leqq 1$ (with the constraint that $\alpha_{\ell j, \ell k} + \alpha_{\ell j, kj} \leqq 1$), we can define the response function γ in such a way that each state is transformed into one of its three neighboring states $\{\ell j\}$ (in the sense of the combinatorial distance).[1] More precisely, let $\mathcal{S} = \mathcal{SO}$ and $\breve{\mathcal{S}} = \mathcal{WO}$, and we have, for $S \in \mathcal{WO}$,

$$\gamma(S,R) = \begin{cases} 1 & \text{if } R = S, \\ 0 & \text{otherwise.} \end{cases}$$

For the remaining semiorders $\{\ell j\} \in \mathcal{SO}$,

$$\gamma(\{\ell j\},R) = \begin{cases} \alpha_{\ell j,\ell k} & \text{if } R=\{\ell j,\ell k\}, \\ \alpha_{\ell j,kj} & \text{if } R=\{\ell j,kj\}, \\ 1-\alpha_{\ell j,\ell k}-\alpha_{\ell j,kj} & \text{if } R=\varnothing, \\ 0 & \text{otherwise.} \end{cases}$$

In other words, when the respondent's latent preference state is a weak order, then s/he provides thermometer scores that directly reflect that weak order. When the respondent's latent preference state is one of the remaining six semiorders, then s/he translates that semiorder into an adjacent weak order according to some probability distribution, and reports thermometer scores that reflect the newly obtained weak order.

It is reasonable to think of $\alpha_{\ell j, \ell k}$ and $\alpha_{\ell j, kj}$ as closely related to the effective token probabilities that move the states $\{\ell j\}$ to the respective adjacent states. In our application, to keep the model parsi-

[1] Note that for the semiorder medium on a set $n > 3$, it is no longer the case that all those latent semiorder states, that are not themselves weak orders, are directly linked to 'neighboring' weak order states. For an illustration of a generic diagram of $n = 4$, consult p. 138 of Falmagne (1997), from which one sees that the graph of the semiorder medium becomes considerably more complicated. In such a case the 'neighboring' response mechanism will be cumbersome and complicated. We will come back to this point in the Conclusion and Discussion section.

monious, we avoid adding parameters by setting

$$\alpha_{\ell j,\ell k} = \frac{\theta_{\tau_{\ell k}}}{\theta_{\tau_{\ell k}} + \theta_{\tau_{kj}} + \theta_{\widetilde{\tau_{\ell j}}}} \quad \text{and} \quad \alpha_{\ell j,kj} = \frac{\theta_{\tau_{kj}}}{\theta_{\tau_{\ell k}} + \theta_{\tau_{kj}} + \theta_{\widetilde{\tau_{\ell j}}}}. \tag{4}$$

Another application of the response function is for the tune in-and-out case. As indicated by Hsu et al. (2005), the empirical application of the weak order random walk of Regenwetter et al. (1999) suffered from the statistical shortfall of underestimating the number of respondents who provide the same ranking of the candidates at two different time points. Hsu et al. (2005) remedied this shortfall by introducing additional states to the medium, called "frozen states." The main idea is that each of these states is linked only to its "active sibling" (one of the 'old' states), which corresponds to the same weak order, or semiorder, depending on the model. To be more specific, the tokens in the original set $\mathcal{T}$ are redefined so that they are effective only on the active states, whereas they have no effect on the frozen states. In addition, for each active state S, there is a new token o_S called *tune-out* token, which transforms that state into its frozen sibling S^*, but has no effect on any other state. Thus, S^* is the frozen sibling of S, and S is the active sibling of S^*. The reverse of o_S is the tune-in token i_S which transforms S^* into S but is ineffective on any other state. Table 3 states the effect of the tune-in and tune-out tokens algebraically.

The development of the tune in-and-out (TIO) medium is illustrated in Figure 3. The figure describes the extended semiorder medium for three alternatives. Note that the resulting semigroup is also a medium, and thus all the earlier random walk results continue to apply. In the context of a political campaign, tune-out tokens correspond to events that make a given voter lose interest in the campaign, until a special event occurs which triggers the application of a tune-in token specific to the voter's state and makes the voter pay attention to the campaign again. For the tune in-and-out extension of the weak order model, the reader can find more mathematical details in Hsu et al. (2005).

As we mentioned already, for tune in-and-out models, we need response functions, too. In the case of the tune in-and-out version of the weak order model, we have $\mathcal{S} = \mathcal{WO} \cup \{S^* \mid S \in \mathcal{WO}\}$ and $\breve{\mathcal{S}} = \mathcal{WO}$. We define the response function as follows. For any active state $S \in \mathcal{WO}$,

$$\gamma(S,R) = \gamma(S^*,R) = \begin{cases} 1 & \text{if } R = S \\ 0 & \text{otherwise} \end{cases}$$

The tune in-and-out extension of the weak order model was first developed and statistically applied to the 1992 NES panel data by Hsu et al. (2005). Their analysis revealed a dramatic improvement in statistical fit over the original weak order model, as well as over the so-called "Thurstonian" model proposed by Böckenholt (2002) on the same data set. Hsu et al. (2005) also discussed the so-called "Mover-Stayer" extension of the weak order model, as an alternative to the TIO extension. We do not discuss the model here, as the TIO extension is by far more parsimonious.

Combining the above two cases yields the following response function for the tune in-and-out

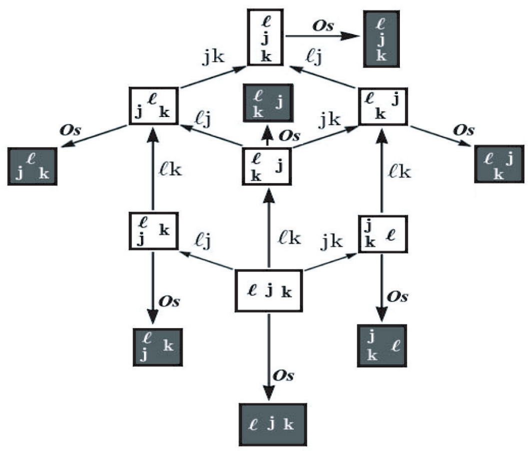

Figure 3. Generic graph of the semiorder tune in-and-out medium. Unshaded states are the active semiorders of the original medium, shaded states are their frozen siblings. Reversing an edge would yield the reverse token. As in previous figures, our labeling of the edges slightly simplifies the notation of the tokens. (Adapted from Falmagne et al., in press, with modifications, with kind permission of authors.)

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Table 3
Tune-in and tune-out tokens in the TIO medium, their psychological interpretation, and the transformation that each token defines on the collection of live and frozen (preference) states on the alternatives in interest.

Notation	Psychological Interpretation	Transformation
τ_{i_S}	Tune-in if in frozen state S^*	$\begin{cases} S^*\tau_{i_S} = S \\ T\tau_{i_S} = T \quad (\text{if } T \neq S^*) \end{cases}$
τ_{O_S}	Tune-in if in live state S	$\begin{cases} S\tau_{O_S} = S^* \\ T\tau_{O_S} = T \quad (\text{if } T \neq S) \end{cases}$

extension of the semiorder model. For any active state $S \in \mathcal{WO}$,

$$\gamma\,(S,R) = \gamma\,(S^*,R) = \begin{cases} 1 & \text{if } R = S, \\ 0 & \text{otherwise.} \end{cases}$$

For any of the remaining active semiorder states $\{\ell j\} \in \mathcal{SO}$,

$$\gamma(\{\ell j\},R) = \gamma(\{\ell j\}^*,R) = \begin{cases} \alpha_{\ell j,\ell k} & \text{if } R=\{\ell j,\ell k\}, \\ \alpha_{\ell j,kj} & \text{if } R=\{\ell j,kj\}, \\ 1-\alpha_{\ell j,\ell k}-\alpha_{\ell j,kj} & \text{if } R=\varnothing, \\ 0 & \text{otherwise.} \end{cases}$$

The forms of $\alpha_{\ell j,\,\ell k}$ and $\alpha_{\ell j,kj}$ are the same as those in the semiorder case (cf. Equation (4)).

5.Empirical Illustration

We analyze panel data from the 1992, 1996, and 2000 National Election Studies (NES) of the Inter-university Consortium for Political and Social Research (ICPSR) at the University of Michigan (Burns, Kinder, Rosenstone, Sapiro, & NES, 2002; Miller, Kinder, Rosenstone, & NES, 1993; Rosenstone, Kinder, Miller, & NES, 1997). We chose these three sets of panel data for empirical illustration because these elections featured three major presidential candidates and because the sample sizes are not too small.

In the case of the weak order model, we treat the states of the medium as observable. We assume that the process was at asymptote when the first poll took place and apply the theoretical results we have reviewed in Sections 3 and 4. Without loss of generality, we treat δ as measuring the interval between the two polls. The situation is slightly more complicated when we apply the semiorder model or one of the tune in-and-out models to thermometer panel data. Here, some states are unobservable, and we need to use a response function γ as introduced in Definition 7. Then, we apply the theoretical results we have reviewed in Section 4 (see Falmagne et al., in press, for more mathematical results.).

In any model of a political campaign, it is important to allow different constituencies to react to the same campaign in different ways. We accomplish this by using the respondents' political self-identification in the poll and by analyzing Democrats, Independents, and Republicans separately for each of the three elections. Regenwetter et al. (1999) provide more details on this approach.

The three major presidential candidates were, respectively: George Bush, Bill Clinton, and Ross Perot in the 1992 campaign; Bob Dole, Bill Clinton, and Ross Perot in the 1996 campaign; and George W. Bush, Al Gore, and Ralph Nader in the 2000 campaign. In our analysis, we leave out those respondents who did not rate themselves on the partisanship scale. Furthermore, we only include participants who rated all three candidates in both polls. Given these constraints, our sample size is 2032 for the 1992 panel, namely 702 Democrats, 784 Independents, and 546 Republicans. For the 1996 panel, the sample size is 1434, namely 562 Democrats, 455 Independents, and 417 Republicans. For the 2000 panel, we use data from a total of 1060 respondents. These break down into 349 Democrats, 403 Independents, and 308 Republicans.

We fit the weak order model, the semiorder model, as well as their TIO extensions. Before we report on the analysis, it is useful to count the number of parameters involved in each of the four models. Because our empirical data come from a panel study in which respondents were polled once before and once after the election, we allow for the possibility that (with the exception of λ) the model parameters may have changed between the two polls.

For each of the three elections, our stochastic processes on the weak orders and on the semiorders is parametrized via

$$\underbrace{3-1+3}_{\text{three constituencies}} \times \left(\underbrace{12-1}_{\substack{\text{pre-election}\\ \text{token}\\ \text{probabilities}}} + \underbrace{12-1}_{\substack{\text{post-election}\\ \text{token}\\ \text{probabilities}}} + \underbrace{1}_{\substack{\text{constant}\\ \text{Poisson}\\ \text{rates}}} \right) = 71$$

independent parameters. In the weak order models we are able to reparametrize the pre-election process through the token bias ratios, thus effectively reducing the number of free parameters for the weak order model (by five parameters for each constituency) to a total of 56 parameters.

For the tune in-and-out extensions, we assume for reasons of parsimony and statistical identifiability[2] that the probabilities of the tune-in tokens i_s and the tune-out tokens o_s do not depend on the states S. In other words, within a given constituency, the probability of a tune-in token is a constant and the probability of a tune-out token is another constant, thus adding only two parameters per constituency. Furthermore, in our illustration here, for purposes of statistical identifiability, we assume that the ratios of the tune-in and tune-out probabilities remain the same before and after the election. This means that we parametrize our stochastic processes on the tune in-and-out extensions of the weak orders and semiorders via

$$\underbrace{3-1+3}_{\text{three constituencies}} \times \left(\underbrace{12-1+2}_{\substack{\text{pre-election}\\ \text{token}\\ \text{probabilities}}} + \underbrace{12-1+2}_{\substack{\text{post-election}\\ \text{token}\\ \text{probabilities}}} - \underbrace{1}_{\substack{\text{constant tune}\\ \text{in-and-out}\\ \text{ratios}}} + \underbrace{1}_{\substack{\text{constant}\\ \text{Poisson}\\ \text{rates}}} \right) = 80$$

independent parameters. Here again, statistical identifiability issues for the pre-election token probabilities effectively reduce that down to 62 parameters for the weak order tune in-and-out model, because for the pre-election process we can only uniquely estimate bias ratios.

As in Böckenholt (2002) and in Hsu et al. (2005), we use the log-likelihood estimate for the goodness-of-fit and likelihood ratio tests. However, the fact that there are many cells with zero (observed) frequency in the pre- vs. post-election contingency tables has created problems with evaluating the goodness-of-fit. Regenwetter et al. (1999) used a specific pooling procedure to group cells with zero frequencies before performing the goodness-of-fit. We have studied the issue of pooling by performing different pooling procedures for some of the goodness-of-fit statistics, including the likelihood-ratio G^2, the Pearson chi-square, as well as the Cressie-Read statistics (see, e.g., Read & Cressie, 1988). We found that pooling could yield biased estimates, in the sense that pooling put more 'weight' on the goodness-of-fit of those cell frequencies with which the zero-frequency cells are pooled. We also discovered that the parameter estimates depended on and varied with the particular pooling procedure we used.

2 By (lack of) statistical identifiability we mean that, for some of the parameters in the model, different estimates would yield an equally good fit. This would make it difficult to distinguish among multiple explanations of the same data using the parameter estimates. By moving to bias ratios and adding various other constraints, we avoid such lack of identifiability.

Therefore, here, as in Hsu et al. (2005), we estimate without pooling.

Writing $\hat{p}_i$ for the (statistically estimated) probability of drawing an observation in cell i according to the model in any randomly drawn observation, and N_i for the number of actual observations in cell i, the likelihood Lik(data) of the data is given by

$$Lik(\text{data}) = \prod_i \hat{p}_i^{N_i}. \qquad (5)$$

For each of the three elections, we fit each of the four models to the same data by maximum likelihood estimation without pooling. In other words, our best fitting parameter estimates are those $\hat{p}_i$ that maximize the likelihood Lik(data) of the data.

The empty cells in the data matrices complicate the assessment of the model fit. For sparse contingency tables, some researchers have warned that the Pearson chi-square and the likelihood-ratio G^2 goodness-of-fit statistics cannot be expected to follow χ^2 distributions with known degrees of freedom (Fienberg, 1979; Koehler & Larntz, 1980). Some researchers, such as von Davier (1997) suggest to solve this problem by performing a parametric bootstrap (Efron, 1982).

We have conducted a small scale parametric bootstrap simulation of the test statistics. However, because of the large number of parameters involved in the parametric bootstrap procedure for each of the four models, it is difficult to assure that in each run of the simulation, the final parameter values really came from a global optimum. Needless to say that one needs a considerable load of computer time to run a simulation of reasonable size for each of the four models. Thus, while our simulation study suggests that none of the four models are rejected statistically by a parametric bootstrap, we omit the details here.

Instead, to assess the quality of fit for each of the three constituencies in all four models, we have followed an alternative (post-hoc) procedure used previously by Böckenholt (2002). After estimating the parameter values obtained by maximizing the likelihood function of Equation (5), we collapsed cells of each data matrix so that the expected frequencies exceed one under that model, and then we computed the Pearson chi-square values. For a fixed set of parameter estimates, we collapsed the cells 1,000 times. Across different runs, the number of collapsed cells tended to vary. We summarize the average final chi-square values for each constituency rounded to the nearest integer in Table 4. We also report the average number of remaining cells (with each cell having an expected frequency of at least 1) in the collapsed data matrix in parentheses. From Table 4, it appears that the overall statistical fit of the four models for each of the three campaigns is acceptable, given that we are accounting for very complex data with comparatively very simple models.[3] Qualitatively speaking, the TIO extension seems to outperform the original model in all cases. To make this observation more precise and to quantify the improvement in fit, we also computed the discrepancy between the TIO extension and the original model for each of the four models based on the log-likelihood ratio. We discuss this next.

First, note that maximizing both sides of Equation (5) is the same as minimizing the discrepancy between data and model, given by

$$-2\left(\sum_i N_i \ln(\hat{p}_i) - c\right), \qquad (6)$$

where c is a constant based on the data alone. The results are reported in Table 5. The second column indicates the model under consideration and the third column provides the number of free parameters that this model uses. The fourth column provides the discrepancy between the model predictions and the data, as defined in Equation (6). The weak order and the semiorder models are both

[3] More specifically, we are not aware of any comparably parsimonious model in the literature that can account this well or better for these data.

Table 4
(Post-hoc) Pearson chi-square measure of fits for the weak order model (WO), the weak order tune in-and-out model (WO-TIO), the semiorder model (SO), and the semiorder tune in-and-out model (SO-TIO) to the three constituencies of respondents in the 1992, 1996, and 2000 campaigns. The average number of cells of the collapsed data matrix for each constituency is given in parentheses.

Year	Model	Democrats	Independents	Republicans	Total
1992					
	WO	123 (96)	168 (137)	81 (87)	372 (320)
	WO-TIO	103 (94)	145 (141)	78 (88)	326 (323)
	SO	121 (77)	175 (122)	86 (79)	382 (278)
	SO-TIO	103 (77)	158 (128)	82 (79)	343 (284)
1996					
	WO	71 (54)	105 (88)	65 (61)	241 (203)
	WO-TIO	52 (53)	95 (88)	52 (62)	199 (203)
	SO	57 (41)	124 (86)	66 (56)	247 (183)
	SO-TIO	40 (44)	118 (87)	50 (57)	208 (188)
2000					
	WO	49 (58)	133 (94)	37 (42)	219 (194)
	WO-TIO	44 (58)	110 (93)	33 (42)	187 (193)
	SO	50 (54)	130 (88)	43 (41)	223 (183)
	SO-TIO	39 (55)	129 (92)	34 (40)	202 (187)

clearly outperformed by their TIO extensions. This can easily be seen by taking the log-likelihood ratio of two models, as follows. Writing *Lik*(data) for the likelihood function of the model without TIO extension and Lik_{TIO}(data) for the likelihood function of the TIO extension of that model, the log-likelihood ratio statistic, G^2, is given by

$$G^2 = -2\ln\left(\frac{Lik(\text{data})}{Lik_{TIO}(\text{data})}\right). \tag{7}$$

This is a standard statistic for comparing nested models against each other. Its asymptotic distribution is a χ^2 with degrees of freedom equalling the difference in the number of parameters that the two models employ. A nonsignificant G^2 indicates that the more parsimonious nested model fits equally well as the more general model, whereas a significant G^2 indicates that the more general model (with more free parameters) fits substantially better (and thus the extra parameters are warranted).

As Table 5 shows, adding the TIO mechanism improves the fits significantly. For the 1992 campaign the G^2 of Equation (7) is 37.6 (with 6 degrees of freedom) for the weak order case and 42.8 (with 9 degrees of freedom) for the semiorder case. Both of these are extremely statistically significant, indicating a dramatic improvement in statistical fit of the TIO extensions over the original (nested) models. The improvement is even more impressive for the 1996 campaign, in which the G^2 values are 48.8 (with 6 degrees of freedom) for the weak order case, and 65.2 (with 9 degrees of

Table 5
Goodness-of-fit (discrepancy between model predictions and data) of the weak order model (WO), the weak order tune in-and-out model (WO-TIO), the semiorder model (SO), and the semiorder tune in-and-out model (SO-TIO) to the 1992, 1996, and 2000 campaigns. We also report the Akaike Information Criterion (AIC) values for all four models.

Year	Model	Number of parameters	Discrepancy Model vs. Data	AIC
1992				
	WO	56	612.4	724.4
	WO-TIO	62	574.8	698.8
	SO	71	657.6	799.6
	SO-TIO	80	614.8	774.8
1996				
	WO	56	494.4	606.4
	WO-TIO	62	445.6	569.6
	SO	71	548.2	690.2
	SO-TIO	80	483.0	643.0
2000				
	WO	56	466.4	578.4
	WO-TIO	62	442.4	566.4
	SO	71	543.8	685.8
	SO-TIO	80	486.4	646.4

freedom) for the semiorder case. The improvement in the 2000 campaign is similar.

Next, we turn to the goodness-of-fit comparison between the four models. The log-likelihood ratio only allows us to compare models that are nested within each other. The Akaike Information Criterion (AIC = Discrepancy $+2 \times$ number of parameters), provided in the last column of Table 5, is a standard model selection criterion that penalizes each model for the number of parameters it uses and that permits comparisons among all models (Akaike, 1974). A standard rule of thumb for model selection is to favor models with low AIC values over models with high AIC values. We see from Table 5 that by the AIC criterion, the weak order TIO model performs best, followed by the weak order model, then the semiorder TIO model, with the semiorder model (without TIO) performing the worst. The same pattern applies to all three campaigns.

Note in passing that a closer examination of the fit to those three election panel data can also be done by summing up the individual chi-square values across all cells for each of the three constituencies. The results (not shown here) also show a statistically significant improvement of the TIO extension against the (nested) original model in both the weak order case and the semiorder case. Furthermore, when comparing the four models for each of the three election panel data, the order of the total chi-square values among those models replicates the order obtained with the AIC criterion. We feel that this multi-method consistency reinforces our conclusion that the weak order TIO model consistently performs best, whereas the semiorder model (without TIO) consistently performs worst among the four models on these three data sets.

Since the psychological interpretation of the tokens, as well as the transformation mechanisms, for the weak order model and for the semiorder model are quite different, it is difficult to compare the estimated values of token probabilities between the two model types. Thus, we do not report the detailed parameter estimates here. However, we briefly discuss some interesting findings about the bias ratios. From here on, we report the remaining results only in qualitative terms. We leave it for later to work out quantitative tests.

Recall that a bias ratio of token is defined as the ratio of that token probability divided by the probability of its reverse: $\mathcal{B}[\tau] = \frac{\theta_{\tau}}{\theta_{\tilde{\tau}}}$. We found that for the TIO extension of the weak order model, although the estimated parameter values of the token probabilities (not shown here) are, themselves, not the same as those in the original weak order model, the estimated bias ratios of token probabilities are similar in both cases. A parallel remark applies to the bias ratios of token probabilities in the semiorder model and its TIO extension. This suggests that the TIO extensions, while adding the tune in-and-out mechanism to the original models, are able to retain some of the main characteristics (here, the bias ratios of token probabilities) of the original models.

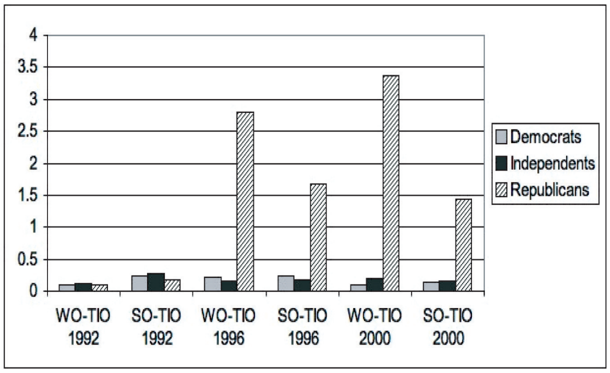

Figure 4. Estimates of the tune-out/tune-in (token) ratio for the weak order tune in-and-out model (WO-TIO) and the semiorder tune in-and-out model (SO-TIO) for the three constituencies of respondents in the 1992, 1996, and 2000 campaigns.

Another interesting feature of the tune-out/tune-in (token) ratios in both the weak order TIO model and the semiorder TIO model is as follows. As we show in Figure 4, even though the two models are based on different primitives in terms of tokens and transitions, the tune-out/tune-in (token) ratios among the three constituencies of respondents satisfy a very similar order (from smallest to largest) in the two cases. More precisely, Figure 4 suggests that, for the 1996 and 2000 campaigns, Republicans had the strongest tendency to lose interest and tune out of the campaign. All in all it appears that there was a nonnegligible proportion of respondents who had a higher tendency to tune out of the campaign than to tune into the campaign. This is especially apparent for Republicans in the 1996 and 2000 campaigns. It is well known that negative campaigning played a major role in the 1996 and 2000 campaigns. Our findings are consistent with the interpretation that negative campaigning may encourage voters to tune out of a campaign.

This leads us to consider the *boomerang effect* of negative campaigning. This term is commonly used in political science to describe the notion that negative campaigning may backfire against the sponsor of the attacks. Our analysis is consistent with a boomerang effect for Dole's 1996 negative campaign against Clinton. The models in this paper are capable of developing a picture of such an effect. As an illustration, we report in Figure 5 the estimated positive bias ratios and in Figure 6 the estimated negative bias ratios from the application of the weak order TIO model to the 1996 NES panel data.[4] From these two figures one sees an increase of the negative bias by Democrats against Clinton (from 0.59 to 1.46) and a decrease of the positive bias by Independents toward Clinton (from

4 Notice that the vertical axes of Figures 5 and 6 are scaled differently.

4.35 to 2.97), suggesting that, on the one hand, Dole's negative campaign strategy might have worked as intended. On the other hand, the negative bias by Republicans against Clinton only increased slightly (from 3.74 to 4.14). The positive bias of Democrats towards Clinton was attenuated but remained strong (from 13.50 to 10.28), and whatever little positive bias Democrats displayed towards Dole did not increase much (from 0.50 to 0.88). Furthermore, the positive bias by Republicans towards Dole diminished considerably (from 8.17 to 3.58), suggesting that some Republicans were annoyed or embarrassed by Dole's negative campaign strategy. Our findings are consistent with the interpretation that the boomerang effect may have shifted support away from Dole to Perot.

6.Conclusion and Discussion

In this paper, we have tested four media theoretic models, namely the weak order model, the semiorder model (based on the neighboring response mechanism), as well as their tune in-and-out extensions, to 1992, 1996, and 2000 U.S. presidential election panel data. The converging evidence that both TIO extensions dramatically outperform their predecessor models (that had no TIO mechanism) on several statistical indices suggests that the TIO mechanism plays an important role in modeling persuasion successfully. We have also compared the fit between the four models based on the Akaike Information Criterion, and found that the weak order TIO model performs best, followed by the weak order model, then the semiorder TIO model, with the semiorder model performing the worst in every campaign. We have briefly discussed the role of the TIO mechanism in the persuasion process, especially negative campaigning and the apparent occurrence of a boomerang effect in Dole's 1996 attacks on Clinton.

We mention in passing that when we lumped together Democrats, Independents, and Republicans, i.e., we did not allow for different parameters across constituencies, then all the models studied in this paper fit extremely poorly. This indicates that we are able to accommodate important differences between constituencies by carrying out separate analyses.

Finally, a specific feature of this paper is the

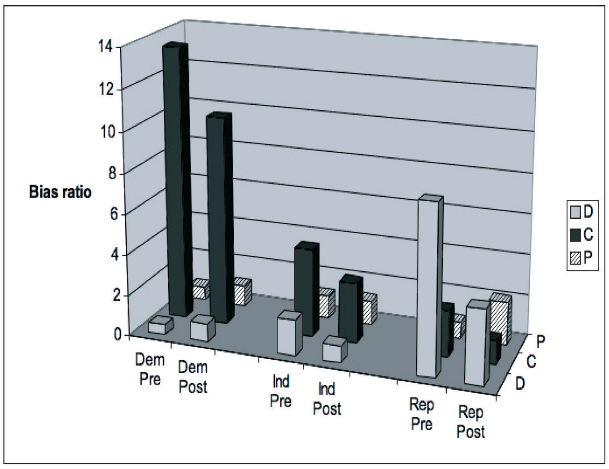

Figure 5. Estimated positive bias ratios $\mathcal{B}[\tau_\ell]=\frac{\theta_{\tau_\ell}}{\theta_{\tilde{\tau}_\ell}}$ of the tokens based on the weak order TIO model for the three constituencies of respondents in the 1996 campaign. *D* : Dole, *C*: Clinton, *P*: Perot. Note that the labeling in the figure slightly simplifies the notation of the tokens. For example, we write here *D* rather than $\mathcal{B}[\tau_D]$.

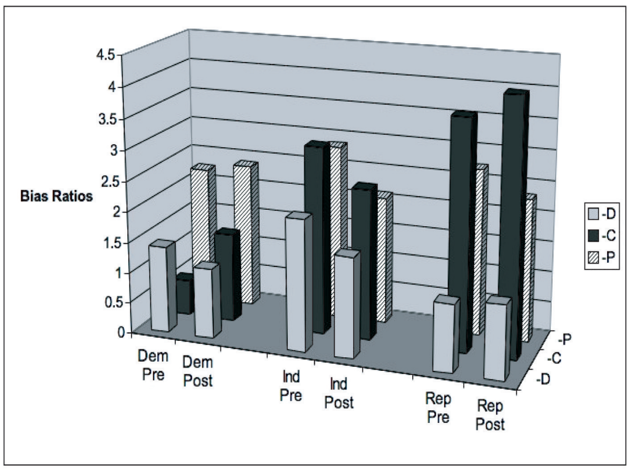

Figure 6. Estimated negative bias ratios $\mathcal{B}[\tau_{-\ell}]=\frac{\theta_{\tau_{-\ell}}}{\theta_{\tilde{\tau}_{-\ell}}}$ of the tokens based on the weak order TIO model for the three constituencies of respondents in the 1996 campaign. *D* : Dole, *C*: Clinton, *P* : Perot. Note that the labeling in the figure slightly simplifies the notation of the tokens. For example, we write here *-D* rather than $\mathcal{B}[\tau_{-D}]$.

response function for the semiorder model originally introduced theoretically by Falmagne et al. (in press). This response function takes a specific functional form of Equation (4) and assigns each of the six special semiorder states to its three neighboring weak orders. However, as mentioned in Footnote 1, no similar straightforward procedure of the neighboring response mechanism is available for the semiorder medium on a set of $n>3$, because in these cases those semiorder states, that are not themselves weak orders, are not necessarily directly linked to neighboring weak order states. Indeed, there are other plausible response mechanisms. We are currently investigating a new response mechanism of the semiorder order model for thermometer data that relies on the Scott-Suppes representation of semiorders (cf. Theorem 3) but that circumvents the requirement that all respondents use the same threshold of utility discrimination. This approach is built on the idea that responses are generated in a fashion that is consistent with the Scott-Suppes representation, but where the values of the threshold ε follow some unknown probability distribution that does not need to be fully estimated from the data. Unlike the neighboring response mechanism, this new response mechanism can be readily generalized to the case of $n>3$ for the semiorder medium.

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隨機媒介理論在1992、1996及2000年美國總統選舉定群追蹤資料之應用

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本文將 Falmagne 等學者發展的一種以排比關係為基礎之媒介理論，應用到有關喜好程度轉變之數學模式研究。此類模式假設喜好程度之轉變是經由環境中某些信號傳遞的影響，而其主要課題即在於建立因時間改變而喜好程度轉變之機制。在此我們討論兩種有關喜好程度之排比：一為 weak order，另一為 semiorder。在此之前 Falmagne 等學者曾成功的將 weak order 模式應用於1992年美國總統選舉之資料分析上，但是對同一筆資料，其 semiorder 模式之應用並不成功。由於美國總統選舉之資料是以量數評定為依據，而且此評定與以semiorder 為基礎之排比關係並無一對一之轉換關係，本文的重點之一便在於此轉換關係之建立。據此，我們發展一套以鄰近狀態反應機制為基礎的 semiorder 模式。本文比較 weak order 模式，上述之 semiorder 模式，以及此二模式之轉入轉出延伸母模式，並將之應用於1992、1996及2000年美國總統選舉之資料分析上。本文最後並嘗試討論模式中一些重要參數之心理意函及其對負向選舉宣傳之解釋。

關鍵詞：定群追蹤資料、媒介理論、排比關係、量數評定、Semiorder、Weak order