行政院國家科學委員會補助專題研究計畫成果報告

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計畫主持人：喬凌雲
共同主持人：

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行政院國家科學委員會專題研究計劃成果報告

多重尺度震波走時層析成像
Multiscale Seismic Tomography

計畫編號：NSC 89-2611-M-002-043
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主持人：喬凌雲　　執行機構及單位名稱：台灣大學海洋研究所

一、中文摘要

震波走時層析成像的本質是連續地球物理逆推問題,但是由於波線趨近之下,此一逆推問題之資料算核(data kernel)在垂直波線方向之頻譜寬度無法界定,因此使得 Gram 矩陣(由資料算核之內積組成)無法正常計算。為進行這類型的研究,勢必對於想要推估之速度模型預先執行有限參數化。傳統上這種有限參數化以球諧函數和箱型函數為主。近年來，由於震波走時資料之大量累積，各種容納高自由度之地球層析成像模型似乎具有愈益提高之解析度，也似乎提供愈益細緻的地球內部構造。但是由於這些層析成像模型彼此之間有相當程度的歧異，使得現階段之震波走時資料到底對於地球內部構造具有何種程度之解析能力成為需要釐清之課題。近年來，對於有限參數化所可能導致的映頻效應已經有較佳的掌握，但是基於不同基底函數之參數化以及不同型式的正規化(regularization)方式對於層析成像模型的影響雖為許多研究致力的方向，但目前累積的共識仍以經驗為主。我們相信參數化與正規化的一個核心問題未受應得之評估，即特定參數化所選擇基底函數之尺度-位置可同時標定的程度。利用近年來快速發展之複尺度分析，將球面小波基底函數成功應用於震波走時層析成像，我們發現這種參數化方式不但易於計算，而且所獲致結果較諸傳統的球諧函數和箱型函數具有低數十倍的模型變量(model variance)。且由於此一參數化方式本身即已具自然正規化之功能，不須借助額外無法驗證之先驗條件來設計正規化之運作，因此提供由資料本身所真正具有之解析能力。本計劃成功發展此一參數化方式之相關理論以及演算法則並將之利用於不同型式的層析成像問題以及其他的連續地球物理逆推問題。在利用 Sd-SKS **差異走時描繪核幔邊界的研究上有極佳之表現。**

關鍵詞：層析成像，複尺度分析，自然正規因子，尺度-位置之同時標定，球面小波。

二、英文摘要：

Seismic travel time tomography is commonly discretized by a truncated expansion of the pursued model in terms of chosen basis functions. The robustness of the resulting Earth model as well as whether parameterization affects the actual resolving power of a given data set have long been seriously debated. From the perspective of the model resolution, however, there is one important aspect of the parameterization issue of seismic tomography that has yet to be systematically explored, that is, the space-frequency localization of a chosen parameterization. In fact, the two most common parameterizations tend to enforce resolution in each of their own particular domains. Namely, the parameterization in terms of spherical harmonics with global support tends to emphasize spectral resolution while sacrificing the spatial resolution, whereas the compactly supported pixels tend to behave conversely. Some of the significant discrepancies among tomographic models are very likely to be manifestations of this effect, when dealing with data set with non-uniform sampling. With an example of the tomographic inversion for the lateral shear wave heterogeneity of the D" layer using *S-SKS* travel times, we demonstrate an alternative parameterization in terms of the multi-resolution representation of the pursued model function. Unlike previous attempts of multi-scale inversion that invoke pixels with variable sizes, or overlay several layers of tessellation with different grid intervals, our formulation invokes the biorthogonal generalized Harr wavelets on the sphere. We show that the multi-resolution representation can be very easily constructed from an existing blocks-based discretization. A natural scale hierarchy of the pursued model structure constrained by the resolving power of the given sampling is embedded within the obtained solution. It provides a natural regularization scheme based on the actual ray-paths sampling and is thus free from *a priori* prejudices intrinsic to most regularization schemes. Unlike solutions obtained through spherical harmonics or spherical blocks, that tend to collapse structures onto ray-paths, our parameterization imposes regionally varying Nyquist limits, that is, the robustly resolvable local wavelength bands within the obtained solution
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Keywords：continuous inverse problem, seismic tomography, multiresolution analysis, space-scale localization, spherical wavelets.

三、研究計畫之背景及目的

Ever since the early phase of the modern global tomographic study, the dichotomy among approaches that invoke different parameterizations has been obvious (Dziewonski, 1982,1984; Clayton and Comer, 1983). With the advance of the large amount of seismic travel time measurements available today, tomographic images of the Earth with more and more details have been published each year. However, inconsistencies among these recent models with high nominal resolutions have become a controversial issue that demands to be resolved (e.g., Dziewonski and Woodhouse, 1987; Morelli and Dziewonski, 1987a,b; Tanimoto, 1990; Woodward and Masters, 1991; Pulliam and Stark, 1993; Stark and Hengartner, 1993; Wang and Zhou, 1993; Su et al., 1994; Morelli and Dziewonski, 1995; Stark 1995; Masters et al., 1996; Zhou, 1996; Grand et al., 1997; Bijwaard et al., 1998; Boschi and Dizewonski, 1999). Among the major factors that these discrepancies might arise from, namely, different data sets, different numerical algorithms of inversion, different parameterizations and different regularization schemes, the latter two factors have been the central disputes that attracted considerable efforts. It is noted that although seismic tomography is in essence a continuous inverse problem, the data kernel based on ray theory is, however, not band-limited. This precludes the direct evaluation of the *Gram* matrix that consists of inner products of data kernels. Discretization through finite parameterization of the pursued model is thus inevitable. Rendering the continuous model function into a finite set of parameters, it is clear that any finite parameterization invokes an implicit regularization scheme that imposes selective weightings on different model components. The intertwined effects from parameterization and regularization further complicate the interpretation and comparison among earth models obtained by different groups. Clearly, to have a solution with the resolving power that is compatible with the actual sampling while avoiding either implicit or explicit extra unjustifiable prejudices should be the main concern of choosing a particular type of basis functions to execute the finite parameterization. In this study, we first review briefly some of the problems associated with the general finite parameterization. An alternative parameterization based on the spherical wavelets expansion is then introduced and invoked in a tomographic study of the lateral shear wave heterogeneity of the D" layer. Solutions obtained from parameterizations based on the three different types of basis functions, namely, spherical harmonics, spherical pixels and spherical wavelets are compared and discussed.

四、結果與討論

The resulting shear wave velocity perturbation is displayed and compared among parameterizations of spherical wavelets, spherical harmonics (degree 40) and spherical pixels (Fig. 1). We find that unlike the spherical wavelets solution that bears heterogeneous resolvable scales, the spherical pixels solution tends to gradually collapse significant structures along the ray-paths. Furthermore, magnitudes of long wavelength (low scale-level) components of the spherical pixels solution are considerably lower than the spherical wavelets solution.

The overall spatial patterns of the three solutions are similar, with remarkable clustering of the calculated plume roots (Steinberger and O’Connell, 1998) around the low velocity anomalies. However, there are significant discrepancies among these images. Note that the data set has been carefully sorted to ensure that the sampling coverage is as uniform as possible which explains the consistency between the spherical harmonics solution and the spherical pixels solution. Otherwise, with the presence of large data gaps or regionally redundant sampling, it is well acknowledged that considerable spurious artifacts will appear within data gaps for solutions parameterized by globally support basis functions (Pulliam and Stark, 1993), unless the inversion is heavily damped (Boschi and Dziewonski, 1999). The major difference among the three solutions, however, is that while both the spherical harmonics solution and the spherical pixels solution tend to collapse structures along the ray-paths, the grouping of local structures into longer wavelengths in the spherical wavelets solution is different.

Other than the overall spatial pattern, the level-wise contributions on the variance reduction, the root-mean-square model norm of the three solutions as well as the power spectrum when projected onto the spherical harmonics expansion are also compared. We notice that it is possible to project the spherical harmonics solution onto a representation in terms of spherical pixels and thus perform the level-wise decomposition. Inspecting the variance reduction of the spherical harmonics solution, projected and decomposed, from the root level and gradually incorporating higher scale-level details, it is found that contributions from the scale-level 5 actually deteriorate the data fitting. This is caused by the fact that the spherical harmonics parameterization was carried only up to the 40th degree, with the degrees of freedom less than the level 5 refinement of the spherical pixels discretization. The projection of the spherical harmonics solution onto the spherical pixels representation invokes components higher than the truncation level (degree 40) that is usually assumed negligible when constructing the final solution. Except for this complication, it is also noted that there are significant contributions on the variance reduction from fine structures in the scale-level 5 for both the spherical harmonics and the spherical pixels solution. In fact, if the discretization had been carried to even higher scale levels, this trend will still persist such that eventually all structures are collapsed onto the ray-paths. This is, however, not the case for the spherical wavelets solution, where the model contributions to comprehend the data essentially peaks at the scale-level 4, suggesting that a global maximum resolvable wavelength is no shorter than the characteristic wavelength of this scale-level.

五、計劃成果評估

Parameterization in terms of basis functions with global support tends to focus on frequency resolution and sacrifices spatial resolution. Parameterization in terms of basis functions with local support, on the other hand, does the opposite. Invoking curvature type and other smoothing regularization by way of quelling the data kernel with a finitely supported smoothing function might bring the two extremes closer together since a finite width will be imposed on the rays. However, *a priori* bandwidth has to be determined. Furthermore, this bandwidth is not flexible with respect to data sampling that varies regionally. The wavelet parameterization demonstrated in this study is data adaptive. The spatially varying bandwidth, that is robustly resolvable by the given data, is automatically adapted by the local hierarchy portrayed by the multi-resolution representation of the pursued model variation. The example of *S-SKS* travel time tomography utilizing the multi-scale parameterization has been shown to be very easily implemented. Based on an existing parameterization in terms of spherical pixels, straightforward reconfiguration of the Gram matrix yields robust solution that is less prone to the apparent pattern of the ray distribution but still faithfully reflect the sampling density.

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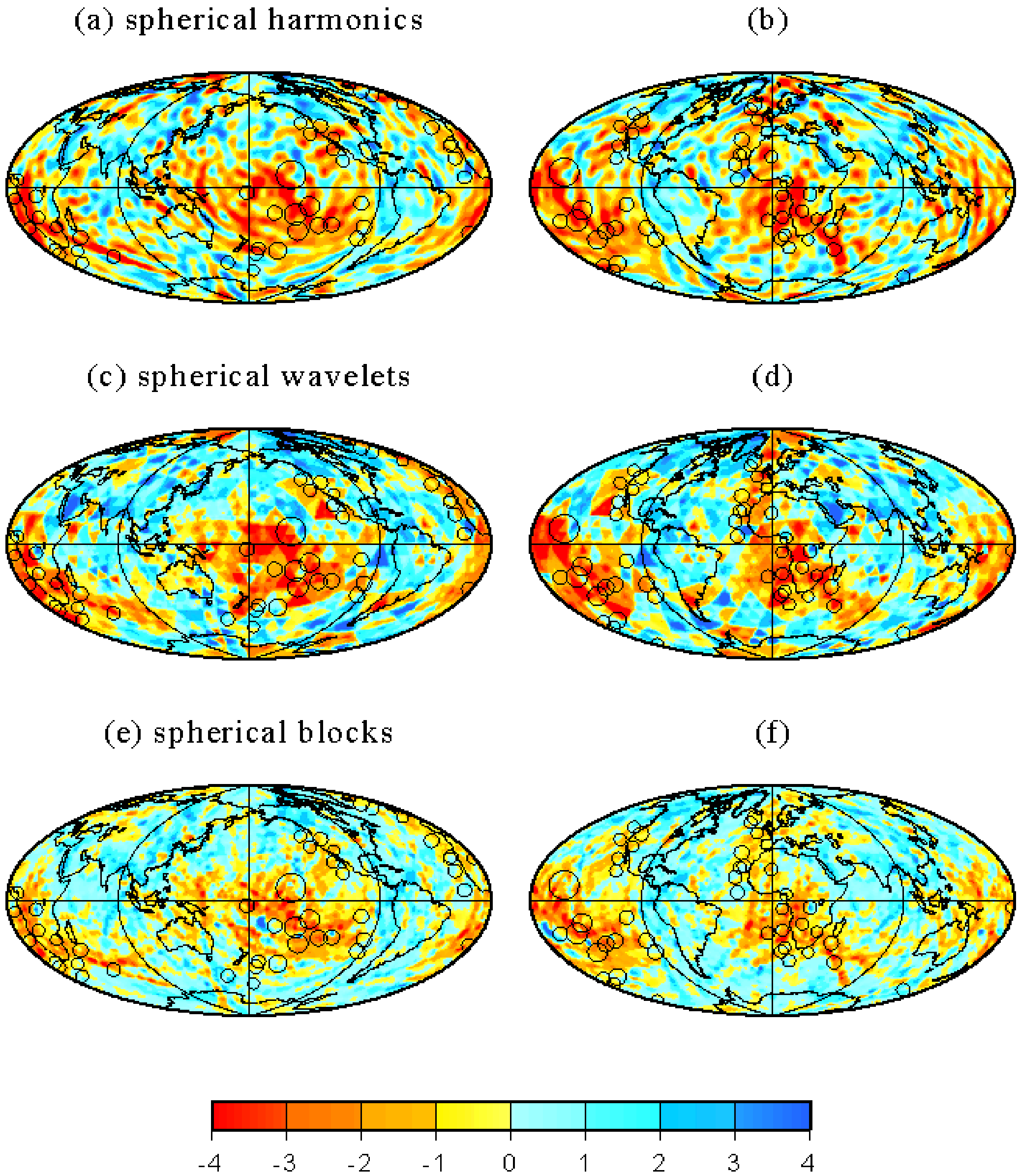

圖一、資料擬合程度均為 48%時, 不同參數化方式所得結果之比較 (a), (b) 截斷階數訂在 40^{th} degree 之球諧函數；(c), (d) 球面小波多重尺度參數化；(e), (f) 球面區塊參數化。