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非歐氏介面上板塊運動的理論推導

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行政院國家科學委員會專題研究計畫成果報告

非歐氏界面上板塊運動的理論推導

Theoretical derivation of the plate kinematics on a non-Euclidean surface

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一、中文摘要

近代地體板塊理論之所以能夠精確而簡明的描述相對板塊運動肇因於地球表面近似於理想球面之基本幾何架構。理想球面具有均勻不變的高斯曲度，因此容許藉助免於形變之理想旋轉流場來描述板塊運動。但是相對板塊運動進行的舞台並不侷限於地球表面。例如根據瓦班氏地震帶，我們知道海洋岩石圈在隱沒進入地幔過程之中維持的形態就是非歐氏二維空間(即具有隨位置變化的高斯曲度)。由於免於形變的剛體旋轉式流場在非歐氏二維空間之中已不可能，是以在這些發生於廣義曲面上的地體活動如何建構其運動流場是理論上必須理解的重要課題。而伴隨著特定流場所衍生的形變在空間分佈的形態、大小與特色以及其可能展現之地球物理觀測(例如地震活動)均是急待瞭解的研究對象。針對這些重要課題，我們在過去的努力主要在於推演在廣義非歐曲面上形變率張量的數值估算方法，以及根據形變率張量計算，對於特定曲面幾何之最佳流場(亦即形變最小流場)的預測。我們將發展的數值計算方法應用在對於特定隱沒帶幾何之流場建構以致於該隱沒過程中特定質點的傳輸途徑以及可能伴隨之形變估算，提供適當解釋不同隱沒帶若干難以理解觀測現象的理論基礎。最近的應用尚包括嘗試合理解釋九二一集集地震沿車籠埔斷層面上滑移向量的特殊側向變化形態。這些相關研究之能夠提供過去未見的新視野與思考固然令人鼓舞，但是數值計算之研究策略卻相對拙於辨明特定曲面上，內在幾何架構(主要極可能可以高斯曲度之側向變化來化約)與無可避免之形變分布形態在分析上的基本理論關係。此一基礎理論關係必須借助廣義曲面座標中二階張量之協變微分推導。我們打算從應變率張量之可積分條件在廣義曲面中的可能形式下手。 根據近來的思考，此一條件使得控管黏滯流體在廣義曲面上運動的動力平衡中，應力張量在空間的變化項中反應一個額外的由曲度控制的應變項。我們希望完成此一關乎正確理解及預測地體運動之基礎理論推導。

關鍵詞：板塊運動,非歐氏界面,高斯曲度,膜面形變

Abstract

Plate kinematics on the surface of the Earth has been described successfully by the Eulerian rotation. It is, however, difficult to specify the kinematics of the lithosphere subduction. Connected with the surface plate velocity across the pivot axis, the trench, the velocity vector field of the subducted slab had been conventionally defined by simply rotating the surface Eulerian kinematics with respect to the local strike onto the slab surface. It usually results in unrealistic in-plane deformation within the slab surface. Alternatively, the flow field as well as the observed slab geometry can be shown to be natural consequences of attaining the kinematic field with the minimum dissipation power. The dependence of the deformation derived for such flow field upon the intrinsic geometry of the non-Euclidean surface is, however, opaque and implicit. We derive, in this study, the fundamental compatibility equation of the strain-rate tensor for the specific flow field to highlight the fundamental dependency. There are two factors; one is associated with the variation of the product of the Gaussian curvature and the determinant of the metric tensor, the two characteristics of the slab geometry, along the stream lines. The other is the local compressibility amplified by the same product. We discuss the implications of these factors and point out that the argument based on mapping the Gaussian curvature variation of the subducted slab is not enough to delineating the potential membrane deformation of the subducted slab.

Keywords: Plate kinematics, non-Euclidean interface, Gaussian curvature, membrane deformation

二、緣由與目的

The theory of Plate Tectonics is fundamentally a kinematic description of the relative plate motion on the surface of the Earth. Due to the nearly perfect sphericity of the Earth that implies uniform Gaussian curvature, the product of curvatures measured along two orthogonal cross sections slicing normal to the local tangential plane, it is thus considered natural to have relative plate kinematics defined by rotation with respect to a single Euler pole such that there are no intraplate deformations. The most serious deviation from this scenario occurs within the subducted slabs. Retaining the basic two-dimensional surface configuration, as revealed from the hypocenter locations of intermediate as well as deep earthquakes and the fact that the few tens of kilometers thickness is relatively negligible as compared to hundreds or even thousands kilometers of lateral extent, the fundamental geometry is utterly different. Mainly, the Gaussian curvature not only deviates significantly away from the measurement obtained on the surface of the Earth (Bevis, 1986); it is also no longer uniform throughout the extent of the slab within the mantle. According to the enlightening wisdom from Gauss (1828), the free of in-plane, or membrane, deformation is possible only if the Gaussian curvature is preserved. It has long been a conventional theoretical cornerstone for the diagnoses of the presence of the in-plane deformation either with the study of fold structures (e.g., Lisle, 1994), or the characterization of the membrane deformation within the slab surface (Bevis, 1986; Cahill and Isacks, 1992; Nothrad et al, 1996). In essence, all these efforts tend to localize significant variations of Gaussian curvature in specific places and restate the original theorem due to Gauss to argue that there must be inevitable in-plane deformations. It is, however, not clear how the Gaussian curvature variation would quantitatively enforce the in-plane deformation within the surface.

三、Slab kinematics based on the flow field with the minimum dissipation power

Bevis (1986) argued that the subducted slab bears very different Gaussian curvature. This is obvious if we compare the measurement on Earth's surface, the reciprocal of the square of Earth's radius, with that obtained for the slab by multiplying the down-dip curvature with the along-trench curvature. Furthermore, inspection of the overall geometry of the subducted slab, as established by the spatial distribution of the Wadati-Benioff seismicity, also indicates that the Gaussian curvature is in fact not uniform anymore across the slab surface (e.g., Nothard et al., 1996). To characterize the subduction kinematics, one conventional practice is to generalize the surface Eulerian kinematics by rotating the surface Eulerian velocity vectors onto the slab surface with respect to the local strike. This is the obvious choice if the subduction takes place in a two-dimensional setup. In realistic situation, the along strike components tend to be important. The simply rotated flow field not only lacks of the virtue to avoid intraplate deformation but actually results in unrealistically high in-plane deformations.

The scenario is best illustrated with the specific cases where the trench shape are concave oceanward (e.g., Chiao and Creager, 2003), as oppose to the usual convex configurations (Frank, 1968). An obvious example is the subduction of the northwestern corner of the Pacific plate along the Kuril, Japan, Izu-Bonin and the Mariana trench systems (Figure 1). The subducted slab wraps around the Hokkaido and the Honshu corner that is convex toward the overriding Eurasia plate. Intermediate and deep seismicity associated with the slab indicates a shallower subduction dip underneath the Japan Sea and forms an arch structure. We speculate that the pronounced arch structure is a natural consequence for avoiding huge amount of in-plane deformation. To test on it, we design numerical experiment that starts with characterizing the trench shape by regression using simple polynomial (Figure 1). A synthetic model slab with uniform dip throughout the trench system is implemented (Figure 2a). For that particular geometry, we then rotate the Pacific versus Eurasia plate kinematics defined on the surface of the Earth onto the specified slab surface (Figure 2c). It is noted that this simply rotated flow field tends to converge around the oceanward concave corner underneath the Japan Sea and results in high along-strike compression strain rates there (Figure 2b, 2c). We further adopt a previously developed optimization scheme that seeks for the optimal slab geometry as well as the flow field on that surface that yields the least deformation rate, or the minimum dissipation power (Chiao, 1991; Chiao and Creager; 2002; Creager and Boyd, 1991; Creager et. al., 1995), a quantity defined by integrating the effective strain rates, the L_2 norm of each components of the strain rate tensor, throughout the extent of the slab. It is interesting to note that the result of minimizing the integrated in-plane deformation rate naturally requires an arch structure that is consistent with the observed slab geometry portrayed by the seismicity (Figure 2d). This geometry, along with the tuning of the subduction flow field reduces, as expected, the in-plane deformation by more than an order of magnitude (Figure 2e, 2f). In summary, both the arch geometry and the adjusted flow field are means of avoiding severe in-plane deformation rates. Comparison between the simply rotated, bearing large deformation rates, flow field (Figure 3a) and the properly adjusted flow field on the optimal slab geometry (Figure 3c) implies that the minimum deformation rate seems to be a reasonable criterion for the determination of the subduction kinematics. However, it is still not clear how does the particular, intrinsic geometry of the slab, manifested through the variation of its Gaussian curvature, affect the membrane deformation and consequently, the determination of a flow field that might avoid in-plane deformation as far as possible.

四、Compatibility equation for flow fields on a 2D non-Euclidean surface

For the flow field confined within a general non-Euclidean, two-dimensional surface, the associated strain-rate tensor field is defined to be the symmetric part of the covariant spatial derivative of the flow velocity vector field. That is,

$$\varepsilon_{ij} = \tfrac{1}{2}(D_i u_j + D_j u_i), \qquad (1)$$

where $u_i, i = 1, 2$, the i$^{\text{th}}$ component of the flow velocity vector field, is defined within a general curvilinear coordinate system; D_i stands for covariant derivative and ε_{ij} is then the strain-rate tensor. There will be exactly one compatibility equation to ensure that the strain-rate tensor field is in fact compatible with a consistent flow field. If we denote the metric tensor of the general surface by $\boldsymbol{g}$ with elements g_{ij} and the determinant $g = g_{11}g_{22} - g_{12}^{\ 2}$, whereas the Gaussian curvature is denoted by *K*, then it can be shown (see Appendix) that the compatibility equation is of the form,

$$D_1D_1\varepsilon_{22} + D_2D_2\varepsilon_{11} - D_1D_2\varepsilon_{12} - D_2D_1\varepsilon_{12} = -\boldsymbol{u}\bullet\nabla(K\mathrm{g}) - (K\mathrm{g})\nabla\bullet\boldsymbol{u} \quad (2)$$

To our knowledge, this general compatibility equation for kinematics defined for a general two-dimensional curvilinear coordinate, especially on a general non-Euclidean surface, had not been discussed in the past. Interesting implications of it on the subduction kinematics will be discussed in the following.

First of all, to be completely free from any deformation, that is to have null strain-rates, the right hand side of Equation (2) has to vanish. The trivial example is for flat plane with null Gaussian curvature, $K = 0$, where it is straightforward to setup a Cartesian coordinate system and the compatibility equation is reduced to the familiar Cartesian form

$$\partial^2 e_{xx} / \partial y - 2\partial^2 e_{xy} / \partial x\partial y + \partial^2 e_{yy} / \partial x = 0 . \quad (3)$$

It is still possible to have flow field with non-trivial deformation rates. However, vanishing right hand side of Equation (3) is indeed a necessary condition to make it to be completely free from deformation since otherwise it would not be possible.

Another example will be the scenario of the Plate Tectonics on Earth's surface, whereas all portions of a rigid plate are moving in a velocity field defined by the rotation with respect to the same Euler pole and the product of the Gaussian curvature, *K*, and the local determinant of the metric tensor, *g*, will be constant along the streamlines. With the additional requirement of incompressibility, the right hand forcing term of Equation (2) vanishes as the necessary condition leading to the absence of intraplate deformation rates. It is interesting to notice that the usual consensus states that the rigid plate kinematics is possible as long as the Gaussian curvature is preserved. We believe that this is a misinterpretation of the original lemma of Gauss. If it were true, then a finite portion of a plate can have arbitrary kinematics and still preserve its rigidity since the Gaussian curvature is stationary on the surface of the Earth. It should be pointed out that only when the kinematics is describable with the rotation around a single Euler pole does the spatial variation of *Kg*, not just *K*, vanishes and the rigid body kinematics becomes admissible.

五、Discussions

The compatibility condition (Equation 2) for the flow velocity vector field on a general non-Euclidean surface highlights the dependency of the embedded deformation rate tensor field upon the intrinsic geometry of the surface. It indicates that simply mapping the variation of the Gaussian curvature along the subducted slab (e.g., Nothard et. al., 1996) is not enough to specify the potential local in-plane deformation. The importance of the in-plane deformation with respect to the observed seismic activity within the slab remains to be resolved. The compatibility condition does not explicitly constrain the subduction flow field. It might be inevitable to invoke the implicit principle of minimum dissipation power for the definition of the subducting flow field. But the compatibility condition does highlight the dependency of the potential deformation associated with a given flow field upon the intrinsic geometry of the surface that the flow is embedded within, especially when the surface is not Euclidean and bears non-stationary Gaussian curvature.

Appendix 1. Derivation of the compatibility condition of flow field on a non-Euclidean surface

Starting from Equation (1) in the main text, $\varepsilon_{ij} = \frac{1}{2}(D_iu_j + D_ju_i)$, *i,j=1,2* on a general 2D surface, we would have

$$\begin{aligned} D_i\varepsilon_{ij} &= \tfrac{1}{2}(D_iD_iu_j + D_iD_ju_i) \\ &= \tfrac{1}{2}(D_iD_iu_j + D_iD_ju_i - D_jD_iu_i + D_jD_iu_i) \\ &= \tfrac{1}{2}(D_iD_iu_j + D_j\varepsilon_{ii} + R^l{}_{iji}u_l), \end{aligned} \quad \text{(A1)}$$

since $(D_iD_j - D_jD_i)u_k = R^l{}_{kji}u_l$ (e.g., Danielson, 1992); where $R^l{}_{kji} = g^{lm}R_{mkji}$, R_{lkji} is the Riemann curvature tensor, and g^{lm} is the contravariant metric tensor. Similarly, we have

$$\begin{aligned} D_j\varepsilon_{ij} &= \tfrac{1}{2}(D_jD_ju_i + D_i\varepsilon_{jj} + R^l{}_{jij}u_l), \\ D_i\varepsilon_{jj} &= D_iD_ju_j = D_iD_ju_j - D_jD_iu_j + D_jD_iu_j \\ &= R^l{}_{jji}u_l + D_jD_iu_j, \\ D_j\varepsilon_{ii} &= D_jD_iu_i = R^l{}_{iij}u_l + D_iD_ju_i . \end{aligned} \quad \text{(A2)}$$

Since $R^l{}_{ijk} = -R^l{}_{ikj}$, it is now straightforward to show that

$$\begin{aligned} & D_iD_i\varepsilon_{jj} + D_jD_j\varepsilon_{ii} - D_iD_j\varepsilon_{ij} - D_jD_i\varepsilon_{ij} \\ &= D_iD_i\varepsilon_{jj} + D_jD_j\varepsilon_{ii} \\ &\quad -\tfrac{1}{2}(D_iD_jD_ju_i + D_iD_i\varepsilon_{jj} + D_iR^l{}_{jij}u_l) \\ &\quad -\tfrac{1}{2}(D_jD_iD_iu_j + D_jD_j\varepsilon_{ii} + D_jR^l{}_{iji}u_l) \\ &= -D_iR^l{}_{jij}u_l - D_jR^l{}_{iji}u_l + R^l{}_{iji}\omega_{lj} + R^l{}_{jij}\omega_{li} . \end{aligned} \quad \text{(A3)}$$

But $D_i R^l{}_{jij} u_l = D_i g^{lm} R_{mjij} u_l = D_i R_{mjij} u^m = D_i R_{ljij} u^l$; and notice that there are only 4 nontrivial terms in the Riemann curvature tensor: $R_{1212} = R_{2121} = -R_{1221} = -R_{2112} = gK$,
where K is the Gaussian curvature and g is the determinant of the metric tensor **g** (e.g., Danielson, 1992), so (A3) becomes

$$\begin{aligned}
& D_1 D_1 \varepsilon_{22} + D_2 D_2 \varepsilon_{11} - D_1 D_2 \varepsilon_{12} - D_2 D_1 \varepsilon_{12} \\
&= -D_1 R_{l212} u^l - D_2 R_{l121} u^l + R^l{}_{121} \omega_{l2} + R^l{}_{212} \omega_{l1} \\
&= -D_1 R_{1212} u^1 - D_2 R_{2121} u^2 + R^2{}_{121} \omega_{22} + R^1{}_{212} \omega_{11} \qquad \text{(A4)} \\
&= -D_1 (R_{1212} u^1) - D_2 (R_{2121} u^2) \\
&= -\nabla \cdot (Kg\boldsymbol{u}) = -\boldsymbol{u} \cdot \nabla (Kg) - (Kg) \nabla \cdot \boldsymbol{u}.
\end{aligned}$$

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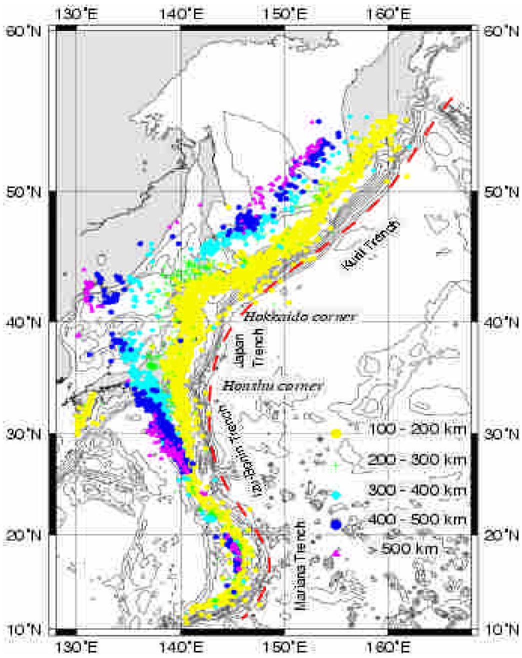

Figure 1. Subduction system around the northwestern Pacific where the Pacific plate is subducting underneath the Eurasia plate along the Kuril, Japan, Izu-Bonin and the Mariana trenches. Notice that around the Hokkaido corner and then the Honshu corner the trench exhibits a concave oceanward shape. Red dash lines are low order polynomial fit to the long wavelength shape of the trench that will be used in the following numerical experiments. Intermediate and deep seismicity are color coded to reveal a rough representation of the slab geometry. It is obvious that there is an anomalously shallow dip for slab subducting through the Japan trench underneath the Japan Sea, forming an arch structure across the subducted slab.

Figure 2. (a) Slab geometry portrayed by the 100 km depth contours. (b) In-plane strain-rate tensor field described by the compressional axes (bold bars) and tensional axes (thin line segments) and (c) the subduction flow field (paths originated from the trench and subduct down-dip) overlaid upon the magnitude variation of the effective strain-rates (the gray scale is determined by the logarithms of the effective strain-rate, i.e., gray scale -14 to -15 is for effective strain rate with magnitude between 10^{-14} and 10^{-15} per second). The lower panels (a,b,c) are for the synthetic slab geometry with uniform dips along the trench and the flow field is the velocity vector field constructed by simply rotating the surface Euler kinematics onto the local slab surface (notice that it converges underneath Japan sea, causing high in-plane strain-rate). The upper panels (d,e,f) are for presentations of the case that we held the prescribed dips only along selected bounding profiles (dash lines on (a)) and calculate the slab geometry and the appropriate flow field on that surface that minimizes the integrated in-plane deformation rates. Notice how the resulting geometry mimics the actual slab revealed by the Wadati-Benioff seismicity, and how the flow field has been adjusted to avoid significant in-plane deformation rates.

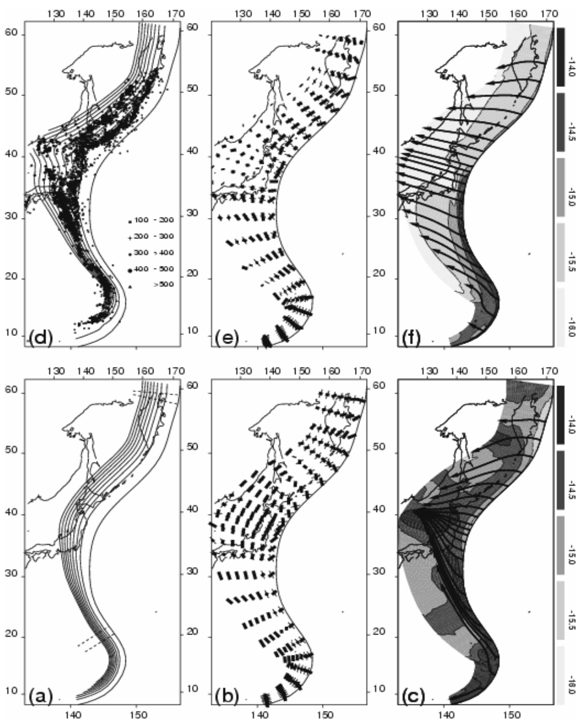

Figure 3. Subduction kinematics with green lines indicating particle paths. (a) Surface Eulerian kinematics rotated onto the slab surface for the synthetic model without the arch (Figure 2a). (b) The adjusted flow field obtained by minimizing the integrated effective strain-rates for the given fixed slab geometry, same as (a). (c) The subducting particle paths on the arched slab obtained by adjusting the flow field and the slab geometry simultaneously as the minimum dissipation power is pursued.