The Optimal Pricing Strategy for Two-sided Platform Delivery in the Sharing Economy

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Abstract

Nowadays many platforms emerge to provide delivery services by having independent shoppers to deliver groceries from independent retailers to consumers. To understand how to price this service, we formulate a two-sided platform's profit maximization problem by considering network externality. We focus on three pricing strategies, membership-based pricing, transaction-based pricing, and cross subsidization. When time discounting is absent and consumers' order frequency is price-insensitive, it is shown that these three strategies are equivalent. As membership-based pricing collects money the earliest and maximize price-sensitive order frequency, our analysis explains some platforms' promotion of it.

Keywords: sharing economy, delivery service, two-sided platform, network externality, game theory.

1 Introduction

Traditionally, a delivery service provider delivers goods from self-owned warehouses to its consumers using its own trucks and employees. In the grocery delivery industry, companies like AmazonFresh adopt this operation model. Owing to the advances in technology, however, different types of delivery services spring up in recent years. In particular, some companies build

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Internet platforms for consumers to order groceries and food materials online. Instead of building a centralized logistics system, the platform assigns these consumer orders to independent contractors, often called shoppers in this business model, for them to buy the ordered goods from independent brick-and-mortar retailers and ship to consumers. As the central service enabler presents as a two-sided Internet platform connecting consumers and shoppers, we call it *platform delivery* in this study.

As of 2016, one of the most successful platform deliverer is Instacart, a San Francisco-based startup founded in 2012. Valued more than two billion dollars, Instacart was listed as top one in *Forbes America's most promising companies list* in 2015 (Soloman, 2015). Another famous platform deliverer is UberEATS. As one expansion service operated by the Uber group since 2014, it utilizes part-time deliverers to deliver meals made by partner restaurants to doors in more than twenty countries. Besides startups, big companies also enter this industry in the same way. For example, Google founded Google Express in 2013 to be another platform for grocery delivery service. An obvious advantage of this operation model is that the delivery service can be provided without owning any warehouse, trucks, and full-time shoppers. A huge initial investment can then be saved. Nevertheless, because the shoppers are not full-time employees, sufficient incentives must be provided to prevent shortage of shoppers. This is a key issue faced by all platforms in the sharing economy.

In general, the success of a platform delivery company (and most multi-sided Internet platforms) relies on its installed base. A higher number of shoppers attracts more consumers to join the platform, and vice versa. This feature is documented as the positive cross-side network externality. Obviously, more shoppers attract more consumers, as it will become easier and faster for a consumer to find a shopper to complete the delivery. Similarly, more consumers attract more shoppers, as it will be more likely to get assigned an order. Therefore, for a platform, the most critical problem is to provide sufficient incentives for both sides to be large enough. In the design of an incentive system, pricing is clearly crucial. The most challenging part of this problem is that, even though the platform faces two sides of users, the pricing problems for the two sides are never independent due to the cross-side network effect. The two pricing problems must be considered together to optimally provide incentives for shoppers and consumers to stay connected to the platform. This brings new challenges and a great potential values to the investigation of platform pricing.

In practice, almost all the platform deliverers allow one to place an order by paying a per-transaction fee. Instacart, Google Express, and UberEATS are all examples. This pay-by-transaction option is necessary to allow new consumers to try the service. Interestingly, some platform deliverers also offer subscription-based membership programs. As of December 2016, one may pay $149 to Instacart or $95 to Google Express to enjoy free shipping (by meeting some minimum order amount) for one year. Obviously, there must be some reasons behind this strategic decision. This motivates us to examine the economic value of a membership program and the general pricing problem for a delivery platform.

While in theory all kinds of pricing plans can be adopted, complicated policies are hard to execute and incurs implicit management costs. Therefore, in industry some simple strategies are popular. If a company adopts the *membership-based pricing* strategy, the platform sustains losses in every transaction but charges every consumer a fixed membership fee at the beginning of each membership period. On the opposite, the platform may charge a per-transaction fee but no fixed fee. This is the *transaction-based pricing* strategy. Note that, at least for the platform delivery business, it is less natural to subsidize a shopper a fixed fee before she/he provides any services. However, whether it is more profitable to charge membership fees, transaction fees, or both from consumers is not so clear. In either case, the platform needs to decide the amount paid to the shopper in each transaction. This introduces the third strategy, which we call it the *cross-subsidization* strategy, under which the platform simply subsidizes the shopper exactly the amount collected from the consumer in each transaction (and therefore earns revenues only from membership fees). It is worthwhile to investigate which pricing strategy may generate the highest profit for the platform.[1]

In this study, we construct a game-theoretic model featuring sharing economy and network externality to examine a grocery delivery platform's two-sided pricing strategy. There are three types of players in the market, a platform, a group of potential consumers, and a group of potential shoppers. The major purpose of our work is to study the profitability of the three pricing strategies mentioned above, whether any of them can be globally optimal, and figure out

[1]It is admittedly true that some platforms in practice adopt membership-based and transaction-based pricing simultaneously for a consumer to self-select. In this study, however, we do not study such a menu of offers for two reasons. First, if we can show that pure membership-based pricing may be better than pure transaction-based pricing and pure cross subsidization, we are able to provide at least one justification for the membership programs we observe in practice. Second, the analysis is difficult, if not impossible, as most platform studies in the economic literature does not consider a menu of contracts. Nevertheless, our analysis does provide a good foundation for future studies about menu design for a multi-sided platform.

factors that affect their profitability. On one hand, we aim to theoretically explain the economic rationale behind these pricing mechanisms popularly adopted in practice. On the other hand, we also hope to provide a good reference for platforms in industry to design their pricing plans to efficiently incentivize users to join the platform.

Our main findings are as follows. When time-discounting of revenues is absent and consumers' order frequency is insensitive to price, the three strategies are equivalent. They are equally good in incentivizing the players in this system, in a sense that all of them result in the same number of consumers, shoppers, and platform profits. This finding is valid regardless of the functional form of service quality, existence of negative same-side network externality among consumers, distributions of users' types, and magnitude of marginal cost. Nevertheless, when the platform is impatient in receiving revenues or consumers' order frequency is affected by the per-transaction fee, membership-based pricing is the most profitable strategy due to its capability of collecting money early and maximizing the price-sensitive order frequency. Our analysis provides an explanation of why in practice some platforms would promote its membership program.

In the next section, we review some related works with respect to sharing economy, delivery service, and two-sided platforms. In Section 3, we develop a game-theoretic model that describes the interaction among the platform, consumers, and shoppers. The analytical results of the basic model are then presented in Section 4. We examine extensions about time discounting and price sensitivity in Section 5 to draw more fruitful implications. Section 6 concludes. All proofs are in the appendix.

2 Literature review

In the transportation industry, many companies are eager to find out critical success factors making Uber a classic paradigm shift and follow similar business models. Many people attribute the success of Uber to "sharing economy," which emphasizes how to make good use of idle resources spread in the market. For instance, Santi et al. (2014) claim that the cumulative trip length could be reduced by roughly 40 percent when using ride sharing like Uber as opposed to using traditional taxis. Andersson et al. (2013) investigate ways ride sharing could improve the use of idle resources, and classify the business model of sharing economy into three kinds according to the properties of trade matching. Zervas et al. (2016) analyze the competition relationship between Airbnb and hotel chains. They conclude that Airbnb can expand their service coverage rapidly due to the little marginal cost and pose a threat to the traditional hotel chains. In general, it is argued that sharing economy has benefits including near-zero marginal cost from digitalization, high quality of trade matching through the Internet, and efficient utilization of available resources (Felländer et al., 2015).

In the delivery industry, sharing economy has also called the attentions from researchers. In two separate studies (Lam and Li, 2015; Rougés and Montreuil, 2014), the paradigm change of platform delivery (called crowdsourcing delivery in these studies) are discussed. Researchers of the two studies claim that platform delivery can eliminate the requirements of inventory management and is thus advantageous. While platform delivery seems to have more advantages over integrated delivery, it also has its drawbacks. In particular, Gurvich et al. (2016) argue that using a self-scheduling mechanism to let independent contractors provide services to customers can actually impose excess costs on a firm and hurt the service quality. Though we do not specifically address operational issues in this study, we contribute to the literature by investigating the limit of platform delivery.

In the economics literature, several past studies examine two-sided platforms and the impact of network externality. As a pioneer work, Katz and Shapiro (1985) start the analysis of network externality. Fudenberg and Tirole (2000) study the competition between a monopolistic incumbent and a potential entrant in a two-sided market. They explicitly develop an incumbent's pricing strategy to deter the threat of entrant. Armstrong (2006) develop an optimal pricing function similar to the Lerner index to depict how the price elasticity of demand and network externality jointly affect the platform's pricing strategy. Rochet and Tirole (2006) study a two-sided platform's pricing problem with pure membership prices and pure usage charges. Jing (2007) delves into how network externality impacts on the product line design. We follow this stream to adopt the same game-theoretic modeling approach to study the platform delivery business model.

Two-sided markets also appear in several transportation-related businesses. Zhang and Zhang (1997) consider an airport manager's pricing problem for aeronautical operations and concession operations. They show that a cross-subsidy from concession to aeronautical operations is needed for the airport to maximize its profit. Following this work, Gillen and Mantin (2014) show that, to minimize the welfare loss at the aeronautical side due to the privatization of an airport, the potential of concession revenue must be high enough. This allows the airport to share the concession revenues with airlines to incentivize them to supply more flights. In the taxi industry, Wang et al. (2016) study the pricing strategies for a taxi-hailing app through an aggregate and static approach. We contribute to the literature by studying how the same two-sided pricing problem can be solved by a platform delivery company for profit maximization.

3 Model

We consider a market with two groups of people, consumers (for each of them, she) and shoppers (for each of them, he), and a monopolistic platform (it) who provides platform delivery services. To join the platform, a consumer pays a membership fee F_{C} to the platform. She may then order on the platform and let the platform find a shopper for her. After matching a transaction successfully, a shopper buys groceries at retail stores and delivers the ordered goods to the consumer. For each delivery, the platform charges a transaction fee r_{C} from the consumer and compensates the shopper a per matching subsidy r_{S}.[2]

Consumers are heterogeneous on their type θ, the willingness to pay for high-quality services. We assumed that θ is uniformly distributed in $[0,1]$. Let $N > 0$ be the (expected) number of orders that a consumer will order in one membership period and q be the service quality, a type-θ consumer's utility is

$$u_{\mathrm{C}}(\theta) = N(\theta q - r_{\mathrm{C}}) - F_{\mathrm{C}}. \tag{1}$$

The service quality q perceived by a consumer is determined by the time one needs to wait before getting the ordered goods, which depends critically on the number of shoppers available on the platform. Therefore, suppose that q is the service quality and n_{S} is the number of shoppers on the platform, we assume that $q = \sqrt{n_{\mathrm{S}}}$. This setting, which is chosen to deliver clear-cut insights, captures the fact that the quality increases as the number of shoppers becomes larger but the marginal improvement is decreasing. Note that this implies that a consumer does not bring negative externality to other consumers, which is true when the number of consumers is relatively small compared to the number of shoppers. This assumption is also made by Armstrong (2006) to emphasize the impact of cross-side network externality on a platform's optimal pricing strategy.

[2]As a shopper is not a full-time employee of the platform, it would create a huge management burden to enforce a shopper to work if a fixed salary is paid in advance. Therefore, we assume that the platform cannot set up a "membership period" and make a fixed compensation to shoppers per period. While this is true in practice, this setting is also widely adopted in literature (see, e.g., Wang et al. (2016)).

To complete a transaction, a shopper incurs a per transaction cost η by spending his time. In this case, his net earning for completing one transaction is $r_{\mathrm{S}} - \eta$. If the platform has n_{C} members, there will be in total Nn_{C} orders in a membership period. If there are n_{S} shoppers on the platform, each shopper in expectation will get $\frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}$ orders. Therefore, a type-η shopper's utility in a membership period is

$$u_{\mathrm{S}}(\eta) = \frac{Nn_{\mathrm{C}}}{n_{\mathrm{S}}}(-\eta + r_{\mathrm{S}}). \tag{2}$$

As different people value their spare time differently, η is assumed to distribute uniformly within 0 and 1.

It is assumed that a consumer or shopper will join the platform if $u_{\mathrm{C}} \geq 0$ or $u_{\mathrm{S}} \geq 0$, respectively. This implies the existence of a critical value θ^* such that a consumer joins the platform if and only if $\theta > \theta^*$. Similarly, there exists a critical value η^* such that a shopper joins the platform if and only if $\eta < \eta^*$. In our notation, this means $n_{\mathrm{C}} = 1 - \theta^*$ and $n_{\mathrm{S}} = \eta^*$. The platform's problem is to maximize its profit

$$\pi = Nn_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + n_{\mathrm{C}}F_{\mathrm{C}} \tag{3}$$

by determining r_{C}, r_{S}, and F_{C}.

For the platform to maximize its profit, ideally it should search for the optimal combination of prices F_{C}, r_{C}, and r_{S} in a three-dimensional space. As this may be too complicated to solve or implement, many platforms in practice restrict themselves to simple pricing strategies. In this study, we investigate the profitability of three pricing strategies platforms adopt in practice. By adopting the *membership-based pricing* strategy, the platform only charges consumers a fixed membership fee, i.e., $r_{\mathrm{C}} = 0$. On the opposite, under the *transaction-based pricing* strategy, the platform allows consumers to join the platform freely, i.e., $F_{\mathrm{C}} = 0$, and rely on transaction fees to generate revenue. Under the third strategy, the *cross-subsidization* strategy, the platform collects both membership fees and transaction fees from consumers, while for each delivery subsidizing a shopper by the entire transaction fee, i.e., $r_{\mathrm{C}} = r_{\mathrm{S}}$. The main focus is to compare these three strategies and examine their optimality when possible.

The sequence of events is as follows. First, the platform decides the transaction fee r_{C}, the per delivery subsidy r_{S}, and the membership fees F_{C} simultaneously.[3] Second, potential con-

[3]In practice, when a platform launches its service, it must announce all the fees and subsidies for suppliers and consumers so that people have information to decide whether to join. This justifies why the three fees are determined simultaneously in this study.

sumers and shoppers observe the prices and decide whether to join the platform independently at the same time. The sizes of the two groups are then realized, and the platform earns profit.

A list of notations introduced so far is provided in Table 1.

Decision variables	
r_{C}	The per-transaction fee
r_{S}	The per-transaction subsidy
F_{C}	The membership fee
Parameters	
n_{C}	The number of consumers
n_{S}	The number of shoppers
θ	Consumers' valuation for per-quality service
η	Shoppers' per-transaction cost
N	Consumption of each consumer in one membership period
q	Quality of platform's service

Table 1: List of decision variables and parameters

4 Analysis

In this section, we analyze the optimization problems of the platform. We first derive the platform company's optimal prices respectively under the three pricing strategies. We then compare the equilibrium profits resulted from the three strategies and examine the optimality of these policies. Finally, we investigate the impact of several model variants.

4.1 Optimal pricing under each strategy

We first analyze the consumers' and shoppers' participation decisions. Recall that $n_{\mathrm{C}} = 1 - \theta^*$ and $n_{\mathrm{S}} = \eta^*$ denote the number of consumers and shoppers joining the platform, respectively. Given the announced prices r_{C}, r_{S}, and F_{C}, the utility functions (1) and (2) and the assumption $q = \sqrt{n_{\mathrm{S}}}$ together imply that

$$u_{\mathrm{C}}(\theta^*) = N(\theta^*\sqrt{\eta^*} - r_{\mathrm{C}}) - F_{\mathrm{C}} = 0 \quad \text{and} \quad u_{\mathrm{S}}(\eta^*) = N\left(\frac{1-\theta^*}{\eta^*}\right)(-\eta^* + r_{\mathrm{S}}) = 0, \tag{4}$$

where the former and latter are for the type-θ^* customer's and type-η^* shopper's utilities to be 0, respectively. It can indeed be observed that $u_{\text{C}}(\theta) > 0$ for all $\theta > \theta^*$ and $u_{\text{S}}(\eta) > 0$ for all $\eta < \eta^*$. By solving the system in (4), we obtain a unique solution $\theta^* = \frac{r_{\text{C}}N+F_{\text{C}}}{\sqrt{r_{\text{S}}}N}$ and $\eta^* = r_{\text{S}}$. Substituting them into (3), we have the platform's profit function as

$$\pi^* = \max_{r_{\text{C}},r_{\text{S}},F_{\text{C}}} \left(1 - \frac{r_{\text{C}}N + F_{\text{C}}}{\sqrt{r_{\text{S}}}N}\right)(F_{\text{C}} + N(r_{\text{C}} - r_{\text{S}})). \tag{5}$$

We now examine each of the three pricing strategies one by one. When the platform adopts membership-based pricing, it restricts itself to earn profits only from membership fees, i.e., $r_{\text{C}} = 0$. With this restriction, the platform's problem reduces to

$$\pi^{\text{M}} = \max_{F_{\text{C}},r_{\text{S}}} \left(1 - \frac{F_{\text{C}}}{\sqrt{r_{\text{S}}}N}\right)(F_{\text{C}} + N(-r_{\text{S}})). \tag{6}$$

If the platform chooses transaction-based pricing, it earns profits only from transaction fee, i.e., $F_{\text{C}} = 0$. Now, the platform solves

$$\pi^{\text{T}} = \max_{r_{\text{C}},r_{\text{S}}} \left(1 - \frac{r_{\text{C}}}{\sqrt{r_{\text{S}}}}\right)N(r_{\text{C}} - r_{\text{S}}). \tag{7}$$

Finally, under the cross-subsidization strategy, the platform requires itself to subsidize a shopper exactly the amount of transaction fee charged from a consumer in a delivery, i.e., $r_{\text{S}} = r_{\text{C}}$. This implies that the platform's problem can be reformulated as

$$\pi^{\text{X}} = \max_{F_{\text{C}},r_{\text{C}}} \left(1 - \frac{r_{\text{C}}N + F_{\text{C}}}{\sqrt{r_{\text{C}}}N}\right)F_{\text{C}}. \tag{8}$$

The optimal pricing plans under these three strategies are characterized in the Lemma 1.

Lemma 1. *Under membership-based pricing, the optimal prices are* $r_{\text{S}}^{\text{M}} = \frac{1}{9}$ *and* $F_{\text{C}}^{\text{M}} = \frac{2}{9}N$. *Under transaction-based pricing, the optimal prices are* $r_{\text{S}}^{\text{T}} = \frac{1}{9}$ *and* $r_{\text{C}}^{\text{T}} = \frac{2}{9}$. *Finally, under cross subsidization, the optimal prices are* $r_{\text{S}}^{\text{X}} = r_{\text{C}}^{\text{X}} = \frac{1}{9}$ *and* $F_{\text{C}}^{\text{X}} = \frac{1}{9}N$.

Obviously, relaxing the restrictions imposed under the three strategies leaves the platform more flexibility in designing the prices (e.g., it may choose to offer a negative membership fee but create a gap between the transaction fee and per-transaction subsidy to generate profit). The platform's profit may therefore becomes higher. Nevertheless, before we examine this issue, we would first compare the equilibrium outcomes under the three strategies. The optimality of these strategies are left to the end of this section.

4.2 Comparisons

To understand which out of the three strategies is the best for the platform, we should compare the equilibrium profit under these strategies. The result, which is presented in Proposition 1, is somewhat surprising: All the three policies are equally good.

Proposition 1. *The delivery platform's equilibrium profits under membership-based pricing, transaction-based pricing, and cross subsidization are all the same:* $\pi^{\mathrm{M}} = \pi^{\mathrm{T}} = \pi^{\mathrm{X}} = \frac{N}{27}$.

Proposition 1 shows that the three pricing strategies are equally profitable for the platform to maximize its profit. To understand this result, it helps to look at the optimal prices under these strategies. In particular, we may observe that $r_{\mathrm{S}}^{\mathrm{X}} = r_{\mathrm{S}}^{\mathrm{M}} = r_{\mathrm{S}}^{\mathrm{T}}$, which means that to implement each strategy, the platform will choose to give the same amount of per-transaction subsidy to shoppers. According to the shoppers' utility function defined in (2), this then induces the same number of shoppers to join the platform and leads to the same service level to consumers. Now consider the consumers' utility function defined in (1), which can be rewritten as $u_{\mathrm{C}}(\theta) = N\theta q - (Nr_{\mathrm{C}} + F_{\mathrm{C}})$. As q is the same across all three policies, and $Nr_{\mathrm{C}} + F_{\mathrm{C}}$ is also identical (which can be easily verified using the result of Lemma 1), the number of consumers joining the platform are also the same. In summary, the three pricing strategies are not just equally profitable; they are actually equivalent.

This equivalence result may be explained from a different perspective. Each of the three strategies is obtained by adding a restriction onto the most general strategy. However, none of these restrictions ($r_{\mathrm{C}} = 0$, $F_{\mathrm{C}} = 0$, and $r_{\mathrm{S}} = r_{\mathrm{C}}$) prevents the platform from charging a consumer the same total amount $Nr_{\mathrm{C}} + F_{\mathrm{C}}$ in a membership period. From the platform's perspective, if it wants to increase r_{C} by 1, all it needs to do to maintain the attractiveness of the offer is to cut down F_{C} by N. In other words, the three policies make no difference at the consumer side. The platform is then able to provide any degree of incentives to attract consumers to join. As the same number of consumers gives the shoppers the same degree of incentives to do the business, it remains for the platform to do is to offer the same amount of per-transaction subsidy to acquire the same number of shoppers. The three strategies are therefore equally effective for the platform to incentivize the two groups of users.

4.3 Optimality of the three strategies

The only question remains unsolved is whether the three strategies are indeed optimal among all possibilities. To answer this question, in the next proposition we derive a necessary and sufficient condition for a solution $(r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}})$ to be optimal. It turns out that a family of pricing strategies are all optimal, and membership-based pricing, transaction-based pricing, and cross subsidization all satisfy the condition. Therefore, they are all optimal.

Proposition 2. *A pricing plan $(r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}})$ is optimal to the platform's problem in (5) if and only if $r_{\mathrm{S}} = \frac{1}{9}$ and $r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N$.*

The platform's pricing problem in general is to search for an optimal point in a three-dimensional space. Given Proposition 2, we find that the optimal solutions form a line, not just a point. There exists an optimal number of shoppers, which should be induced by setting the per-transaction subsidy to $r_{\mathrm{S}} = \frac{1}{9}$. There also exists an optimal number of consumers, which can be incentivized to join the platform by charging them in total $r_{\mathrm{C}}N + F_{\mathrm{C}} = \frac{2}{9}N$ in a single membership period. Combining this with the discussions below Proposition 1, it is clear that all the three strategies of interest are optimal. Even if we take away one pricing variable (by following any of the three strategies), the platform may still perfectly match supply and demand to maximize its profit.

Proposition 2 provides us two important managerial implications. First, it demonstrates the optimality of those simple pricing plans. Platforms in practice may therefore choose to limit themselves to restricted pricing plans, which is much easier to obtain and implement. Even if the real-world business environment is more complicated than our stylized model, those simple pricing plans are not too restricted to provide adequate incentives for shoppers and consumers to join the platform.

Moreover, it is shown that in all optimal solutions, the number of shoppers (and thus consumers) are all the same. Even though people today face such an intertwined interaction and dynamics between the two sides of a platform, there are still some simple structures to follow. In particular, as shoppers are heterogeneous in their costs of providing services, the platform should only induce those with the lowest costs to join, otherwise too much compensation will be needed, and the platform's profit will decrease. Our analysis reveals the existence of such a unique optimal number of shoppers. Platforms in industry should first identify that optimal supply level (by taking the consumer side into account simultaneously), set an appropriate amount of subsidy to achieve that, and then determine the prices charged from consumers to keep the market equilibrium in the most profitable way.

4.4 Discussions

In this section, we demonstrate that the equivalence and optimality of the three strategies remain valid when some technical assumptions are relaxed. In particular, we examine the general settings of the service quality function, distribution of shoppers' and consumers' types, and marginal transaction cost. The main objective of these examinations is to show that the three strategies' equal ability of incentivizing users to join the platform is robust. If there are factors (e.g., those identified in Section 5) for the platform to prefer one strategy to the others, the difference is due to other reasons, not from the efficiency of attracting users' participation.

4.4.1 Service quality

In this section, we generalize the platform's service quality q to be jointly affected by the numbers of shoppers and consumers in a general way. Specifically, we assume that $q = f(n_{\rm S}, n_{\rm C})$, where the only assumptions we impose on $f(n_{\rm S}, n_{\rm C})$ are $f(0, n_{\rm C}) = 0$, $f_1(n_{\rm S}, n_{\rm C}) > 0$, $f_{11}(n_{\rm S}, n_{\rm C}) < 0$, and $f_2(n_{\rm S}, n_{\rm C}) \leq 0$. The first assumption means that the service quality is 0 when there is no shopper; the second and third assumptions mean that the service quality is an increasing and concave function of the number of shoppers $n_{\rm S}$; the last assumption implies that the service quality may go down as the number of consumers $n_{\rm C}$ increases. This function $f(n_{\rm S}, n_{\rm C})$ is clearly a generalization of $\sqrt{n_{\rm S}}$ in our basic model. First, it allows $n_{\rm S}$ to affect the service quality in a more general way. Moreover, it incorporates the potential negative network externality among consumers if they need to compete for a limited supply of shoppers.

Under this setting, the type-θ consumer's utility becomes $u_{\rm C}(\theta) = N(\theta f(n_{\rm S}, n_{\rm C}) - r_{\rm C}) - F_{\rm C}$. Given any pricing plan $(r_{\rm C}, r_{\rm S}, F_{\rm C})$, in equilibrium the number of shoppers is still $r_{\rm S}$, and all consumers whose $\theta > \theta^*$ will join the platform, where θ^* satisfies

$$N(\theta^* f(r_{\rm S}, 1 - \theta^*) - r_{\rm C}) - F_{\rm C} = 0. \tag{9}$$

We may rigorously confirm that θ^* exists uniquely as long as the platform's pricing plan may induce at least the highest-valuation consumer (whose $\theta = 1$) to use the service.

Lemma 2. *If the pricing plan $(r_{\rm C}, r_{\rm S}, F_{\rm C})$ satisfies $N(f(r_{\rm S}, 0) - r_{\rm C}) - F_{\rm C} > 0$, there exists a unique $\theta^* \in (0, 1)$ satisfying (9).*

Given the existence and uniqueness of θ^*, we are able to write down the platform's optimization problem as

$$\begin{aligned} \max_{r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}}, \theta^*} \quad & (1-\theta^*)(F_{\mathrm{C}} + N(r_{\mathrm{C}} - r_{\mathrm{S}})) \\ \text{s.t.} \quad & N(\theta^* f(r_{\mathrm{S}}, 1-\theta^*) - r_{\mathrm{C}}) - F_{\mathrm{C}} = 0, \end{aligned} \tag{10}$$

where the constraint is the condition for θ^* to satisfy in equilibrium based on (9). Our next proposition demonstrates that the three pricing strategies remain equivalent (and optimal).

Proposition 3. *When $q = f(n_{\mathrm{S}}, n_{\mathrm{C}})$, all the three strategies achieve the maximum attainable profit. There exists an optimal value of r_{S} that should be offered under all three strategies. While the three strategies differ in their r_{C} and F_{C}, their $Nr_{\mathrm{C}} + F_{\mathrm{C}}$ are identical.*

Proposition 3 indicates that our main insights remain valid for any functional form of $f(n_{\mathrm{S}}, n_{\mathrm{C}})$. The three strategies remain equivalent, as the key to maximize the platform's profit is still to first identify the optimal number of shoppers, set the per-transaction subsidy accordingly, then find the optimal number of consumers, and finally set the membership fee and/or transaction fee to achieve that number of consumers. None of the restrictions imposed by these strategies makes the last step infeasible. While the change in the shape of $f(n_{\mathrm{S}}, n_{\mathrm{C}})$ changes the optimal amounts of the subsidies and fees quantitatively, it does not change the incentive system qualitatively. Though there is no closed-form expression for an optimal solution, knowing its existence is sufficiently good. The platform may do a numerical search to solve its profit maximization problem.

4.4.2 Distributions of users' types

In our basic model, we assume that the consumers' willingness-to-pay θ and shoppers' delivery cost η are both uniformly distributed in 0 and 1. To consider a more general setting, we now assume that the cumulative distribution functions of θ and η are $G(\cdot)$ and $H(\cdot)$, respectively, where $G(\cdot)$ and $H(\cdot)$ are two general distribution functions. Our goal is to examine whether the equivalence and optimality result is a consequence of our previous assumption on type distributions.

Given an announced pricing plan $(r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}})$, it is still true that a type-η shopper will join the platform if and only if $\eta < r_{\mathrm{S}}$. However, now the number of participating shoppers becomes $n_{\mathrm{S}} = H(r_{\mathrm{S}})$. This implies that the number of participating consumers becomes $n_{\mathrm{C}} =$

$1 - G(\frac{r_C N + F_C}{N\sqrt{H(r_S)}})$. We may then formulate the platform's optimization problem as

$$\max_{r_C, r_S, F_C} \left[1 - G\left(\frac{r_C N + F_C}{N\sqrt{H(r_S)}}\right)\right](F_C + N(r_C - r_S)). \tag{11}$$

The optimization problems under membership-based pricing, transaction-based pricing, and cross subsidization are obtained by adding constraints $r_C = 0$, $F_C = 0$, and $r_S = r_C$, respectively.

Obviously, without knowledge about the functional forms of $G(\cdot)$ and $H(\cdot)$, there is no way to solve the pricing problems. Nevertheless, the next proposition shows that the three strategies are still equivalent, and all of them achieves optimality.

Proposition 4. *When the distribution functions of θ and η are $G(\cdot)$ and $H(\cdot)$, respectively, all the three strategies achieve the maximum attainable profit. There exists an optimal value of r_S that should be offered under all three strategies. While the three strategies differ in their r_C and F_C, their $Nr_C + F_C$ are identical.*

Our analysis here reveals that the distributions of users' types have no impact on the equivalence and optimality of the three strategies. By implementing the three strategies in the optimal ways, the equilibrium numbers of shoppers and consumers remain identical. This confirms again that the three strategies' equal efficiency in attracting users to join the platform is regardless of the shapes of the type distributions.

4.4.3 Marginal transaction cost

As most Internet-enabled services, the marginal cost for completing a delivery transaction is extremely low, if not zero. Nevertheless, we may add a marginal cost $c > 0$ into our model and make the platform's optimization problem become

$$\max_{r_C, r_S, F_C} \left(1 - \frac{r_C N + F_C}{\sqrt{r_S} N}\right)(F_C + N(r_C - r_S - c)),$$

where the per-transaction net income becomes $r_C - r_S - c$. Again, all the three strategies can be shown to be equivalent and optimal.[4]

Proposition 5. *For any marginal cost $c > 0$, the platform's equilibrium profits under the three pricing strategies are all identical.*

[4]For the closed-form expressions of the solutions and profits, see the proof of Proposition 5 in the appendix.

To understand this result, note that the positive transaction cost uniformly reduces the per-transaction net incomes under all strategies. Therefore, the per-transaction subsidies are affected in the same way. As we report in the proof of Proposition 5, the per-transaction subsidies are indeed the same in all cases. It then follows that the equilibrium numbers of consumers, numbers of shoppers, and numbers of transactions are all the same under the three strategies. A positive marginal cost then uniformly reduces the equilibrium profits under the three strategies. Our major finding is thus not altered by the presence of a marginal cost.

5 Impact of time discounting and price sensitivity

In Section 4, we show that the three strategies are equivalent. More importantly, as the equilibrium numbers of shoppers and consumers are all the same under the optimal implementation of all strategies, our analysis identifies the main reason of their equivalence: they all have the same efficiency of incentivizing users to join. Nevertheless, in practice we do observe some platforms' efforts in promoting their membership programs. Below we discuss two potential reasons driving the promotion: time discounting and price-sensitive order frequency. When any of these two factors presents, membership-based pricing outperforms transaction-based pricing and cross subsidization.

Before we start the discussion, it is important to highlight the value of our previous analysis. First, without the equivalence result established in the basic model, we are unable to understand or even demonstrate the better performance of membership-based pricing. In addition, we may be certain about the true reasons of its superiority only after the equal efficiency in providing incentives is revealed. The analysis in Section 4 are thus necessary to deliver our messages in this section.

5.1 Time discounting

In practice, time discounting may be an important issue for business owners. For startups running platforms in the sharing economy, it can be even more desirable to earn a dollar today than tomorrow. Benefits of receiving cash early include lower financial risks, more investment chances, higher R&D possibilities, among others (Bates et al., 2009). To model the patience level, we add a parameter $a \in (0, 1)$ as the discount factor into the platform's profit function to modify it to $Nan_{\mathrm{C}}(r_{\mathrm{C}} - r_{\mathrm{S}}) + n_{\mathrm{C}}F_{\mathrm{C}}$. For each member who pays the membership fee F_{C}

up front, that membership fee revenue is not discounted. However, for the revenues generated by completing orders during the membership periods, those future cash flows are in average discounted by a. The lower the value of a is, the more impatient the platform is.[5]

Substituting n_{C} and n_{S} by the solution of (4), we obtain the platform's general problem

$$\pi_{\mathrm{dis}}(a) = \max_{r_{\mathrm{C}}, r_{\mathrm{S}}, F_{\mathrm{C}}} \left(1 - \frac{r_{\mathrm{C}}N + F_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}N}\right)(F_{\mathrm{C}} + Na(r_{\mathrm{C}} - r_{\mathrm{S}})),$$

and we are interested in understand how the discount factor a affects the optimal prices under the three strategies. Moreover, let $\pi^{\mathrm{M}}_{\mathrm{dis}}(a)$, $\pi^{\mathrm{T}}_{\mathrm{dis}}(a)$, and $\pi^{\mathrm{X}}_{\mathrm{dis}}(a)$ be the equilibrium profits under membership-based pricing, transaction-based pricing, and cross subsidization when the discount factor is a, respectively. We also aim to compare the relative magnitudes of $\pi^{\mathrm{M}}_{\mathrm{dis}}(a)$, $\pi^{\mathrm{T}}_{\mathrm{dis}}(a)$, and $\pi^{\mathrm{X}}_{\mathrm{dis}}(a)$ to compare the profitability of the three strategies.

Interestingly, it turns out that a has no impact on the platform's decision under transaction-based pricing and cross subsidization. Consider transaction-based pricing first. With the constraint $F_{\mathrm{C}} = 0$, the objective function becomes $(1 - \frac{\sqrt{r_{\mathrm{S}}} r_{\mathrm{C}}}{r_{\mathrm{S}}}) Na(r_{\mathrm{C}} - r_{\mathrm{S}})$, which deviate from that in our basic model only by a multiple of a. It then follows that for any value of a, the optimal prices are all the same. Similarly, under cross subsidization, we have the additional constraint $r_{\mathrm{S}} = r_{\mathrm{C}}$, which reduces the objective function to $(1 - \frac{\sqrt{r_{\mathrm{C}}}(r_{\mathrm{C}}N + F_{\mathrm{C}})}{r_{\mathrm{C}}N})F_{\mathrm{C}}$, which is not related to a at all. Therefore, it remains to derive the optimal prices under membership-based pricing to complete our analysis. The results are summarized below.

Lemma 3. *Suppose that there is a time discount factor $a \in [0, 1]$. Under membership-based pricing, the optimal prices are $r^{\mathrm{M}}_{\mathrm{S}} = \frac{1}{9a^2}$ and $F^{\mathrm{M}}_{\mathrm{C}} = \frac{2}{9a}N$ if $a \geq \frac{1}{3}$ or $r^{\mathrm{M}}_{\mathrm{S}} = 1$ and $F^{\mathrm{M}}_{\mathrm{C}} = \frac{2}{3}N$ otherwise. Under transaction-based pricing, the optimal prices are $r^{\mathrm{T}}_{\mathrm{S}} = \frac{1}{9}$ and $r^{\mathrm{T}}_{\mathrm{C}} = \frac{2}{9}$. Finally, under cross subsidization, the optimal prices are $r^{\mathrm{X}}_{\mathrm{S}} = r^{\mathrm{X}}_{\mathrm{C}} = \frac{1}{9}$ and $F^{\mathrm{X}}_{\mathrm{C}} = \frac{1}{9}N$.*

Though time discounting does not affect the platform's pricing decisions under transaction-based pricing and cross subsidization, it still introduces significant changes to the comparison of the three policies. In particular, while the per-transaction subsidies are all the same when $a = 1$, when $a < 1$ we have $r^{\mathrm{T}}_{\mathrm{S}} = r^{\mathrm{X}}_{\mathrm{S}} = \frac{1}{9} < r^{\mathrm{M}}_{\mathrm{S}} = \min\{\frac{1}{9a^2}, 1\}$. This means that the platform will offer a more generous subsidy to shoppers to include more shoppers into the platform. The

[5] Indeed, some consumers' and shoppers' are also aware of time discounting. Nevertheless, it is arguably true that a company typically care much more than an individual about discounting. To avoid messy case-by-case comparisons and to focus on our main research question, in this study we assume that consumers' and shoppers' do not discount their future payments/subsidies.

reason is two-fold. First, as a decreases, the platform views future payments as less significant. This pushes it to increase the per-transaction subsidy. Furthermore, the platform also weighs the membership fees more and needs to find a way to increase the consumers' willingness-to-pay to charge a higher membership fee. Offering a more generous compensation plan for shoppers to increase the number of shoppers and thus the service level is then a natural choice. As a keeps going down, the per-transaction subsidy will keep go up until all shoppers join the platform (when $r_{\mathrm{S}} = 1$, which happens when $a = \frac{1}{3}$). The per-transaction subsidy will then stop at 1 even if a reduces further.

Being impatient also makes the platform value the three strategies differently, as the next proposition indicates.

Proposition 6. *For any time discount factor $a \in (0,1)$, the delivery platform's equilibrium profit under membership-base pricing is strictly higher than that under cross subsidization, which is strictly higher than that under transaction-based pricing:*

$$\pi_{\mathrm{dis}}^{\mathrm{M}}(a) = \min\left\{\frac{1}{27a}N, \frac{(1-a)^2}{2}N\right\} > \pi_{\mathrm{dis}}^{\mathrm{X}}(a) = \frac{1}{27}N > \pi_{\mathrm{dis}}^{\mathrm{T}}(a) = \frac{a}{27}N.$$

Proposition 6 is an interesting discovery. It shows that the three pricing strategies of interest are no longer equivalent when the platform is impatient (i.e., $a < 1$). In this case, membership-based pricing outperforms cross-subsidization, which outperforms transaction-based pricing. While this seems to be intuitive, as membership-based pricing collects money the earliest, the dominance result can be intuitively understood only after we recognize that the three policies are equally good without time discounting. The early timing of collecting money is indeed a reason for membership-based pricing to be the most promising strategy.

5.2 Price-sensitive order frequency

In some situations, a consumer's expected number of orders N in a membership period increases as the transaction fee r_{C} is reduced. When this is true, a consumer tends to use the service the most frequently under membership-based pricing due to the absence of a transaction fee. As this requires the platform to subsidize shoppers the most, one may suspect that membership-based pricing would become less profitable than the other two strategies.

To examine the impact of price-sensitive order frequency, we set the expected number of order in a membership period to $N(r_{\mathrm{C}})$, i.e., a function of r_{C}, which decreases in r_{C}. To facilitate discussion, let $N(0) = N$ be the number of orders one may place when $r_{\rm C} = 0$. In this case, the platform's profit maximization problem becomes

$$\pi^*_{\rm sen} = \max_{r_{\rm C}, r_{\rm S}, F_{\rm C}} \left(1 - \frac{r_{\rm C} N(r_{\rm C}) + F_{\rm C}}{\sqrt{r_{\rm S}} N(r_{\rm C})}\right)(F_{\rm C} + N(r_{\rm C})(r_{\rm C} - r_{\rm S})),$$

which is obtained by replacing N by $N(r_{\rm C})$ in (5). By adding constraints $r_{\rm C} = 0$, $F_{\rm C} = 0$, and $r_{\rm C} = r_{\rm S}$, we may obtain the pricing problems under membership-based pricing, transaction-based pricing, and cross subsidization, respectively. Let the equilibrium profits under the three strategies be $\pi^{\rm M}_{\rm sen}$, $\pi^{\rm X}_{\rm sen}$, and $\pi^{\rm T}_{\rm sen}$. To compare the profitability of these strategies, we should solve the three problems and compare $\pi^{\rm M}_{\rm sen}$, $\pi^{\rm X}_{\rm sen}$, and $\pi^{\rm T}_{\rm sen}$. While closed-form solutions are not possible due to the price-sensitive setting, it can be shown that membership-based pricing is unambiguously the best out of the three strategies.

Proposition 7. *When the expected number of orders in a membership period $N(r_{\rm C})$ decreases in $r_{\rm C}$, the delivery platform's equilibrium profit under membership-base pricing is strictly higher than those under cross subsidization and transaction-based pricing:*

$$\pi^{\rm M}_{\rm sen} > \max\{\pi^{\rm X}_{\rm sen}, \pi^{\rm T}_{\rm sen}\}.$$

The result seems to be counterintuitive, as the platform may earn the most when it subsidizes the most. Nevertheless, note that as consumers order the most under membership-based pricing, they are also willing to pay the highest amount of membership fees. The benefit of increasing consumers' willingness to pay for the service turns out to cover the costs required to subsidize shoppers. Because membership-based pricing is the only way allowing a zero transaction cost, it is the only way to achieve the maximum attainable number of transactions on the platform. It follows that membership-based pricing outperforms the other two.

Just like Proposition 6, Proposition 7 also demonstrates the benefit of adopting membership-based pricing. In practice, many platform deliverers prefer their consumers to subscribe as members. Instacart, who tries hard to promote its membership program Instacart Express, provides a good illustration. If a consumer pays \$149 to purchase the one-year membership, she will then enjoy unlimited free two-hour shipping and scheduled deliveries for orders over \$35 before her membership expires (Instacart, 2016). For a platform deliverer who does not have self-owned logistics system and full-time shoppers, guaranteed delivery time obviously creates operational challenges and requires a generous compensation for shoppers. As Instacart must allow non-member consumers to order so that they get chances to try the service, if the membership plan is of no economic value, Instacart should not promote that. Our result may partly explain why delivery platforms like Instacart would make this strategic choice.

6 Conclusions

In this study, we present a game-theoretic study featuring network externality and sharing economy to investigate three pricing strategies in platform delivery, i.e., membership-based pricing, transaction-based pricing, and cross-subsidization. We analytically derive the optimal prices under these three strategies and show that all the three strategies result in the same numbers of shoppers, consumers, and profits in equilibrium. This implies that they are equally efficient in incentivizing shoppers and consumers to join the platform. This insight is not prone to the functional form of service quality, existence of negative same-side network externality among consumers, distributions of users' types, and magnitude of marginal cost. We then identify two features of membership-based pricing making it outperform the others: earliness of collecting money and maximizing the price-sensitive order frequency. Our results provide one explanation of platform deliverers' efforts in promoting membership programs.

Our study certainly has its limitations. First, it would be interesting to compare this delivery model with the traditional approach, i.e., shipping from one's own warehouse by one's own full-time employees (like AmazonFresh). Conditions under which platform delivery is a better model helps us understand the applicability of the platform business model. The platform's pricing plan can also be expanded further. For example, we have not considered menu pricing, i.e., designing a menu of offers for users to select from, and dynamic pricing (e.g., surge pricing by UberEATS and busy pricing by Instacart) based on the supply and demand volumes. Finally, those retailers who play critical roles in the ecosystem are ignored in the current study. In practice, they may ask a platform to share revenues, compete in offering discounts to have the platform promote their products/stores, or even leverage platform deliverers' capability to provide delivery services.[6] How the retailers' strategic decisions affect the whole system calls for future research.

[6] In June, 2016, Walmart announced its experimental program, which outsources their groceries delivery business to Uber, Lyft and Deliv beside their self-owned trucks (Bender, 2016).

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Appendix

Proof of Lemma 1. Consider membership-based pricing first. Let $\pi^{\mathrm{M}}(F_{\mathrm{C}}, r_{\mathrm{S}})$ be the platform's profit as a function of the two prices, we know an optimal solution must satisfy

$$\frac{\partial \pi^{\mathrm{M}}(F_{\mathrm{C}}, r_{\mathrm{S}})}{\partial F_{\mathrm{C}}} = 1 + \sqrt{r_{\mathrm{S}}} - \frac{2F_{\mathrm{C}}}{\sqrt{r_{\mathrm{S}}}N} = 0 \quad \text{and} \quad \frac{\partial \pi^{\mathrm{M}}(F_{\mathrm{C}}, r_{\mathrm{S}})}{\partial r_{\mathrm{S}}} = \frac{F_{\mathrm{C}}^2}{2r_{\mathrm{S}}^{3/2}N} - N + \frac{F_{\mathrm{C}}}{2\sqrt{r_{\mathrm{S}}}} = 0.$$

The first equation requires $F_{\mathrm{C}} = \frac{(1+\sqrt{r_{\mathrm{S}}})\sqrt{r_{\mathrm{S}}}N}{2}$. Plugging this into the second equation, we may find two candidate values of the per-transaction subsidy r_{S} as $\frac{1}{9}$ and 1. The two corresponding candidate values of the membership fee F_{C} are $\frac{2}{9}N$ and N, respectively. It can be easily verified that $(r_{\mathrm{S}}, F_{\mathrm{C}}) = (\frac{1}{9}, \frac{2}{9}N)$ is optimal.

Next we consider transaction-based pricing. Let $r_{\mathrm{P}} = r_{\mathrm{C}} - r_{\mathrm{S}}$ be the platform's per-transaction net income and $\pi^{\mathrm{T}}(r_{\mathrm{C}}, r_{\mathrm{P}}) = (1 - \frac{\sqrt{r_{\mathrm{C}} - r_{\mathrm{P}}} r_{\mathrm{C}}}{r_{\mathrm{C}} - r_{\mathrm{P}}}) N r_{\mathrm{P}}$ be the platform's profit as a function of the two prices, we know an optimal solution must satisfy

$$\frac{\partial \pi^{\mathrm{T}}(r_{\mathrm{C}}, r_{\mathrm{S}})}{\partial r_{\mathrm{C}}} = r_{\mathrm{P}} \left(\frac{r_{\mathrm{C}}}{2(r_{\mathrm{C}} - r_{\mathrm{P}})^{3/2}} - \frac{1}{\sqrt{r_{\mathrm{C}} - r_{\mathrm{P}}}} \right) N = 0,$$

which implies $r_{\mathrm{C}} = 2r_{\mathrm{P}}$. With this, the other first-order condition is

$$\frac{\partial \pi^{\mathrm{T}}(r_{\mathrm{C}}, r_{\mathrm{S}})}{\partial r_{\mathrm{P}}} = \frac{\left(2(r_{\mathrm{C}} - r_{\mathrm{P}})^{3/2} - 2r_{\mathrm{C}}^2 + r_{\mathrm{P}} r_{\mathrm{C}}\right) N}{2(r_{\mathrm{C}} - r_{\mathrm{P}})^{3/2}} = \frac{2r_{\mathrm{P}}^{3/2} - 6r_{\mathrm{P}}^2}{2r_{\mathrm{P}}^{3/2}} = 1 - 3\sqrt{r_{\mathrm{P}}} = 0.$$

The only solution satisfying this equation is $r_{\mathrm{P}} = \frac{1}{9}$, which then implies that at optimality $r_{\mathrm{S}} = \frac{1}{9}$ and $r_{\mathrm{C}} = \frac{2}{9}$.

Lastly, we consider cross subsidization. Let $\pi^{\mathrm{X}}(F_{\mathrm{C}}, r_{\mathrm{C}})$ be the platform's profit as a function of the two prices, we know an optimal solution must satisfy

$$\frac{\partial \pi^{\mathrm{X}}(F_{\mathrm{C}}, r_{\mathrm{C}})}{\partial F_{\mathrm{C}}} = 1 - \sqrt{r_{\mathrm{C}}} - \frac{2F_{\mathrm{C}}}{\sqrt{r_{\mathrm{C}}}N} = 0 \quad \text{and} \quad \frac{\partial \pi^{\mathrm{X}}(F_{\mathrm{C}}, r_{\mathrm{C}})}{\partial r_{\mathrm{S}}} = F_{\mathrm{C}}\left(\frac{r_{\mathrm{C}}N + F_{\mathrm{C}}}{2r_{\mathrm{C}}^{3/2}N} - \frac{1}{\sqrt{r_{\mathrm{C}}}}\right) = 0.$$

The first equation requires $F_{\mathrm{C}} = \frac{(1-\sqrt{r_{\mathrm{C}}})\sqrt{r_{\mathrm{C}}}N}{2}$. Plugging this into the second equation, we may find three candidate values of the transaction fee r_{C} as 0, $\frac{1}{9}$, or 1. It can be easily verified that $r_{\mathrm{C}} = \frac{1}{9}$ is the optimal one and thus $F_{\mathrm{C}} = \frac{1}{9}N$ is the optimal membership fee.

In all the three cases, the Hessian at the unique reported solution can be easily verified to be negative definite. The derivation is omitted. □

Proof of Proposition 1. When we plug in the optimal prices under the three pricing strategies into their objective functions, it can be calculated that platform's profits are all $\frac{N}{27}$. □

Proof of Proposition 2. Let $y = r_{\mathrm{C}}N + F_{\mathrm{C}}$, the optimization problem in (5) can then be rewritten as

$$\max_{r_{\mathrm{S}}, y} \left(1 - \frac{y}{\sqrt{r_{\mathrm{S}}}N}\right)(y - r_{\mathrm{S}}N).$$

Let $\pi(r_{\mathrm{S}}, y)$ be the objective function, an optimal solution must satisfy

$$\frac{\partial \pi(r_{\mathrm{S}}, y)}{\partial y} = -\frac{2y - r_{\mathrm{S}}N}{\sqrt{r_{\mathrm{S}}}N} + 1 = 0 \quad \text{and} \quad \frac{\partial \pi}{\partial r_{\mathrm{S}}} = \frac{y(y - r_{\mathrm{S}}N)}{2r_{\mathrm{S}}^{3/2}N} - (1 - \frac{y}{\sqrt{r_{\mathrm{S}}}N})N = 0.$$

The first one requires $y = \frac{r_{\mathrm{S}}+\sqrt{r_{\mathrm{S}}}}{2}N$. Plugging this into the second one results in the equality $(\sqrt{r_{\mathrm{S}}} - r_{\mathrm{S}})(\sqrt{r_{\mathrm{S}}} - 3r_{\mathrm{S}}) = 0$, which has two candidate solutions $r_{\mathrm{S}} = 1$ and $\frac{1}{9}$. The two corresponding candidate values of y are N and $\frac{2}{9}N$, respectively. It can be easily verified that $(r_{\mathrm{S}}, y) = (\frac{1}{9}, \frac{2}{9}N)$ is optimal. The Hessian at $(r_{\mathrm{S}}, y) = (\frac{1}{9}, \frac{2}{9}N)$ can be easily verified to be negative definite. □

Proof of Lemma 2. Let $h(\theta|r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}}) = N(\theta f(r_{\mathrm{S}}, 1-\theta) - r_{\mathrm{C}}) - F_{\mathrm{C}} = 0$. To see that a $\theta^* \in (0,1)$ exists to satisfy $h(\theta^*|r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}})$, note that $h(0|r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}}) = -Nr_{\mathrm{C}} - F_{\mathrm{C}} < 0$ and $h(1|r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}}) = N(f(r_{\mathrm{S}}, 0) - r_{\mathrm{C}}) - F_{\mathrm{C}} > 0$. The continuity of h then implies the existence of θ^*. To see that such a θ^* is unique, all we need to show is that h increases in θ. This is clearly true as $h'(\theta|r_{\mathrm{S}}, r_{\mathrm{C}}, F_{\mathrm{C}}) = N[f(r_{\mathrm{S}}, 1-\theta) - \theta f_2(r_{\mathrm{S}}, 1-\theta)]$, which is positive due to the assumption $f_2(r_{\mathrm{S}}, r_{\mathrm{C}}) < 0$. □

Proof of Proposition 3. Let $x = \theta^*$ and $y = r_{\mathrm{C}}N + F_{\mathrm{C}}$, the optimization problem in (10) can then be rewritten as maximize $(1-x)(y - Nr_{\mathrm{S}})$ over x, y, and r_{S} subject to $Nxf(r_{\mathrm{S}}, 1-x) = y$. This reduces to

$$\max_{r_{\mathrm{S}}, x} (1-x)(Nxf(r_{\mathrm{S}}, 1-x) - Nr_{\mathrm{S}}).$$

For any optimal solution (r_S^*, x^*), the optimal value of $y^* = Nx^* f(r_S^*, 1 - x^*)$. Therefore, if the platform adopts any of the three strategies, as long as it sets r_S to r_S^* and makes r_C and F_C satisfy $Nr_C + F_C = Nx^* f(r_S^*, 1 - x^*)$, the resulting profit will all be identical. The membership fee, transaction fee, and membership fee under membership-based pricing, transaction-based pricing, and cross subsidization are $Nx^* f(r_S^*, 1-x^*)$, $x^* f(r_S^*, 1-x^*)$, and $N(x^* f(r_S^*, 1-x^*) - r_S^*)$, respectively. □

Proof of Proposition 4. Let $x = r_S$ and $y = r_C N + F_C$, the optimization problem in (11) can then be rewritten as

$$\max_{x,y} \left[1 - G\left(\frac{y}{N\sqrt{H(x)}}\right)\right](y - Nx).$$

For any optimal solution (x^*, y^*), the platform adopting any of the three strategies may induce the same number of shoppers to join by setting $r_S = x^*$. Then it may set $F_C = y^*$ under membership-based pricing, $r_C = \frac{y^*}{N}$ under transaction-based pricing, or $F_C = y^* - Nx^*$ under cross subsidization. The three strategies are thus equivalent and all optimal. □

Proof of Proposition 5. We first need to find the optimal prices under each strategy. As the derivations are similar to those in the proof of Lemma 1, we omit the details here. For membership-based pricing, the optimal prices are

$$r_S^M = \frac{6c + 1 + \sqrt{12c+1}}{18} \quad \text{and} \quad F_C^M = \left(\frac{\sqrt{12c+1} + 24c + 1}{36} + \frac{\sqrt{2(\sqrt{12c+1} + 12c + 2)}}{12}\right)N.$$

For transaction-based pricing, the optimal prices are

$$r_S^T = \frac{6c + 1 + \sqrt{12c+1}}{18} \quad \text{and} \quad r_C^T = \frac{6c + 1 + \sqrt{12c+1}}{9}.$$

Finally, for cross subsidization, the optimal prices are

$$r_S^X = r_C^X = \frac{6c + 1 + \sqrt{12c+1}}{18} \quad \text{and} \quad F_C^X = \frac{6c + 1 + \sqrt{12c+1}}{18} N.$$

It is obvious that $r_S^M = r_S^X = r_S^T$. Therefore, for the three strategies to result in the same equilibrium profits, all we need is for $F_C^M = r_C^T N = r_C^X N + F_C^X$, which can indeed be verified by some arithmetic arrangements. This completes the proof. □

Proof of Lemma 3. Let

$$\pi^M(F_C, r_S) = \left(1 - \frac{\sqrt{r_S} F_C}{r_S N}\right)(F_C + Na(-r_S))$$

be the platform's profit as a function of the two prices, we know an optimal solution must satisfy

$$\frac{\partial \pi^M(F_C, r_S)}{\partial F_C} = 1 + a\sqrt{r_S} - \frac{2F_C}{\sqrt{r_S} N} = 0 \quad \text{and} \quad \frac{\partial \pi^M(F_C, r_S)}{\partial r_S} = \frac{F_C^2}{2r_S^{3/2} N} - aN + \frac{aF_C}{2\sqrt{r_S}} = 0.$$

The first equation requires $F_C = \frac{(1+a\sqrt{r_S})\sqrt{r_S}N}{2}$. Plugging this into the second equation, we may find two candidate values of the per-transaction subsidy r_S as $\frac{1}{9a^2}$ and $\frac{1}{a^2}$. The two corresponding candidate values of the membership fee F_C are $\frac{2}{9a}N$ and $\frac{1}{a}N$, respectively. Because $(r_S, F_C) = (\frac{1}{a^2}, \frac{1}{a}N)$ results in a null profit, $(r_S, F_C) = (\frac{1}{9a^2}, \frac{2}{9a}N)$ is optimal as long as the implicit constraint $r_S \leq 1$ is satisfied (to make the number of shoppers $n_S \leq 1$). This is true if $a \geq \frac{1}{3}$. For $a < \frac{1}{3}$, the boundary solution $(r_S, F_C) = (1, \frac{(1+a)N}{2})$ is optimal, where $F_C = \frac{(1+a)N}{2}$ maximizes $(1 - \frac{F_C}{N})(F_C - Na)$, the profit function when $r_S = 1$. □

Proof of Proposition 6. According to the optimal prices obtained in Lemma 3, the equilibrium profits are $\pi^M_{dis}(a) = \min\{\frac{1}{27a}N, (\frac{1-a}{2})^2 N\}$ under membership-based pricing, $\pi^T_{dis}(a) = \frac{1}{27}aN$ under transaction-based pricing, and $\pi^X_{dis}(a) = \frac{1}{27}N$ under cross subsidization. It is clear that $\pi^M_{dis}(a) > \pi^X_{dis}(a) > \pi^T_{dis}(a)$ if $a < 1$. □

Proof of Proposition 7. Consider π^M_{sen} first. As $N(0) = 0$, the problem under membership-based pricing is exactly the same as (6) when the order frequency is not price-sensitive. Therefore, we have $\pi^M_{sen} = \pi^M$. As it is shown in Proposition 1 that $\pi^M = \pi^T = \pi^X$, it suffices to show that $\pi^T_{sen} < \pi^T$ and $\pi^X_{sen} < \pi^X$. Consider π^T_{sen}, which is defined as

$$\pi^T_{sen} = \max_{r_C, r_S} \left(1 - \frac{r_C}{\sqrt{r_S}}\right) N(r_C)(r_C - r_S).$$

As $N(r_C) < N$ for all $r_C > 0$ and having $r_C = 0$ is never optimal, a direct comparison between the above problem and (7) indicates that $\pi^T_{sen} < \pi^T$. Finally, under cross subsidization, the platform solves

$$\pi^X_{sen} = \max_{r_C, F_C} \left(1 - \frac{r_C N(r_C) + F_C}{\sqrt{r_C} N(r_C)}\right) F_C.$$

By comparing this problem and (8), we may derive $\pi^X_{sen} < \pi^X$ again by using $N(r_C) < N$ for all $r_C > 0$ and understanding that r_C cannot be 0. This completes the proof. □

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