行政院國家科學委員會專題研究計畫成果報告

一些非微擾物理現象之探索
（三年期計畫）

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中　文　摘　要

本三年計畫對一些特定之非微擾物理現象從事理論研究以及其彼此間相關性之深入探索，研究題材雖以粒子物理與中能物理之範疇為主體，對其跨領域之特性，如宇宙學與統計物理，也希望能有重點性的突破。

嘗試深入探討之第一類題材係強作用物理或強子物理（含一些原子核物理）；雖然強作用應可由量子色動力學（Quantum Chormodynamics，即 QCD）加以描述，且於高能微擾預測得到佐證，強子之特性卻每每為其非微擾性質所主宰；如何定量處理 QCD 非微擾性質，以對強子之結構與反應（包括其非輕子弱衰變）建立第一原則出發之預測，計畫主持人希望藉著多年來物理界（以及自己）獲取之經驗，尋找重要突破點，以協助打開僵局。這方面之工作與成果，集中在 QCD 求和法則之各項應用及其 conceptual foundation 之探討。

相變（Phase Transitions）為擬從事探討之第二類題材，包括 QCD 中之手徵對稱破壞，產生質量之希格機制（Higgs Mechanisms），中子物質轉化為奇異夸克物質之相變，以及玻士愛因斯坦凝聚等，未來希望可以涵蓋這些相變之理論描述，以及其在其他領域，如早期宇宙、星球演化之末期等相關題材所扮演之角色能進一步釐清。

ABSTRACT

In this three-year project, we have attempted to conduct theoretical investigations of a few specfic nonperturbative physics phenomena, including possible correlations among them. Although the primary focus was on the topics in the area of particle and medium physics, it was hoped that some major progresses, or even some breakthrough, could be made in the related cross-disciplinary areas of cosmology and statistical physics.

The first major research topic had to do with strong interaction physics or hadron physics (including some nuclear physics problems). Although it is known, as already substantiated by perturbative QCD tests with high energy experiments, that strong interactions can be described by quantum chromodynamics or QCD, properties of hadrons or nuclei are dictated by the nonperturbative aspects of QCD. We had hoped to suitably exploit the experiences accumulated over years by our community (and by ourselves as well), to seek clues for breakthroughs, and to help unlock the present difficult situation, hoping that reliable methods to treat nonperturbative aspects of QCD can be obtained and first-principle predictions on hadron structure and properties (such as nonleptonic weak decays of hadrons) may be formulated accordingly.

The second major research topic has been focused on the general problem of phase transitions, including chiral symmetry breaking in QCD, Higgs mechanisms for mass generation in the standard model, phase transition from neutron matter to strange quark matter, and the recently discovered Bose-Einstein condensation. Our investigation of these phase transitions was also aimed to help clarify the role of phase transitions in the early universe, the end stage of stellar evolution, and other relevant cross-disciplinary topics. More progresses have yet to be made.

綜 合 說 明

非線性或非微擾物理現象已是許多物理不同領域研究之重要主題。強作用理論 QCD 係眾所周知之重要範例。

QCD 求和法則係針對難以正確解決之強作用理論 QCD 之一項攻堅方式；在執行本三年計畫前，主持人已與合作者及研究生等多方著墨，因此 1997 及 1998 仍屬 QCD 求和法則各項應用研究之量產階段。到 1997，主持人開始廣泛檢討 QCD 求和法則之觀念基礎，所寫之「QCD Sum Rules and Chiral Symmetry Breaking」，已開始對其觀念基礎（conceptual foundation）提出強烈質疑。緊接著，本人也開始針對 Large N_c QCD 進行類似求和法則之研究，發現求和法則與 Large N_c 之性質合併，其實可以將 QCD 解決到相當完整的地步。可惜主持人礙於身兼系主任之職，以及教育部卓越計畫總主持人兩項繁重職務，這方面研究成果之量產，也因之必須延後。

以 QCD 作為範例，進行非微擾物理現象之研究，大方向大致正確，也應該堅持下去。目前，我們也開始嚴謹地考慮 QCD 相變在早期宇宙演化中之確切角色。亦即夸克膠子相轉為強子相之相變，預期宇宙產生後 10^{-5} sec 至 10^{-4} sec 之間發生；相變如何發生，發生之際是否引致其他未曾預期之現象（如黑洞或拓樸弦之大量產生），以及是否留下可供偵測之痕跡，這些皆係亟待深入思考、分析以至解決之重要課題。

因此，我們這些研究將繼續深入，至少達到研究論文量產之收尾階段，同時也將開拓重要新方向，以期作出重大突破。

附錄：

1. 主持人 1997 至 2000 年之研究論文目錄。
2. 主持人一些研究論文之抽印本。

Research Publications During 1997-2000

Woei-Yann Pauchy Hwang

A. Refereed Journal Papers:

1. The QCD Sum Rule Approach for the Semileptonic Decay of the D or B Meson into a Light Meson and Leptons, Kwei-Chou Yang and W-Y. P. Hwang, Z. Phys. C - Particles and Fields **73**, 275 (1997).

2. QCD Sum Rules and Chiral Symmetry Breaking, W-Y. P. Hwang, Z. Phys. **C75**, 701 (1997), hep-ph/9601219.

3. Semi-inclusive Λ Production and Generalized Sullivan Processes, W-Y. P. Hwang and Chih-Yi Wen, Z. Phys. **A358**, 415 (1997).

4. QCD and Nuclear Physics, W-Y. P. Hwang, Chinese J. Phys. (Taipei) **35**, 444 (1997).

5. The Structure of Solitonic Quark Stars, J.-W. Chen, H.-Y. Chiu, and W-Y. P. Hwang, Chinese J. Phys. (Taipei) **35**, 543 (1997).

6. ρNN and ωNN Couplings in the External-Field QCD Sum Rule Method, Chih-Yi Wen and W-Y. P. Hwang, Phys. Rev. **C 56**, 3346 (1997).

7. Study of Fundamental Symmetries in Nuclei, W-Y. P. Hwang, Chinese J. Physics (Taipei) **35**, 847 (1997).

8. Σ_c and Λ_c Magnetic Moments from QCD Spectral Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Rev. **D56**, 7273 (1997).

9. $D^* \to D\gamma$ and $B^* \to B\gamma$ as derived from QCD Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Mod. Phys. Lett. **B12**, 3027 (1997).

10. Possible $\Sigma^0 - \Lambda$ Mixing in the QCD Sum Rule, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Rev. **D57**, 1524 (1998).

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13. Electromagnetic Decay of Vector Mesons as derived from QCD Sum Rules, Shi-lin Zhu, W-Y. P. Hwang, and Ze-sen Yang, Phys. Lett. **B420**, 8 (1998).

14. Addendum: The Weak Parity-Violating Pion-Nucleon Coupling, E. M. Henley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B 440**, 449-450 (1998).

15. Nucleon Spin Structure Functions in the Effective Chiral Quark Theory, Chun-Khiang Chua and W-Y. P. Hwang, Chinese J. Phys. (Taipei) **36**, 588 (1998).

16. Muon Capture in Deuterium, W-Y. P. Hwang and B.-J. Lin, International J. Mod. Phys. **E 8**, 101 (1999).

17. The Neutrino Magnetic Moment Induced by Leptoquarks, Chun-Khiang Chua and W-Y. P. Hwang, Phys. Rev. **D 60**, 073002 (1999).

18. Nucleon Spin Structure Functions in the Chiral Quark Model, Chun-Khiang Chua and W-Y. P. Hwang, Phys. Lett. **B451**, 283 (1999).

19. Neutrino Mass Induced Radiatively by Supersymmetric Leptoquarks, Chun-Khiang Chua, Xiao-Gang He, and W-Y. P. Hwang, Phys. Lett. **B479**, 224 (2000).

20. Nonleptonic Hyperon Decays with QCD Sum Rules, E.M. Heley, W-Y. P. Hwang, and L. S. Kisslinger, Phys. Lett. **B**, *resubmitted for publication* (2000).

21. Parity-violating Nuclear Force as derived from QCD Sum Rules, Chih-Yi Wen and W-Y. P. Hwang, Chinese J. Phys., *submitted for publication* (2000).

22. QCD Sum Rules and the Pseudoscalar Coupling, E.M. Henley, W-Y. P. Hwang, and L.S. Kisslinger, Phys. Lett. **B**, *re-submitted for publication* (2000).

23. Quark Propagator in Large N_c QCD, W-Y. P. Hwang, Euro. Phys. Journal **C**, *submitted for publication* (2000).

24. Green Functions in Large N_c QCD, W-Y. P. Hwang, Phys. Lett. **B**, *submitted for publication* (2000).

B. Conference Papers:

1. QCD Sum Rules and Parity-violating Nuclear Force, Proceedings of the First International Conference on Frontiers of Physics (OCPA), Shantou, China, August 5-9, 1995, Eds. L.-F. Li, K. K. Phua, S.S.M. Wong, and B.-L. Young (World Scientific, Singapore, 1997), p. 813.

2. QCD Sum Rules and Nonleptonic Weak Interactions, Proceedings of International Nuclear Physics Conference, Beijing, China, August 21-26, 1995, Eds. Sun Zuxun and Xu Jincheng (World Scientific, Singapore, 1996), p. 292.

3. Isospin Symmetry Breakings as from QCD Sum Rules, Proceedings of the 15th International Conference on Few-Body Problems in Physics, Groningen, Netherland, July 22-26, 1997, Nucl. Phys. **A631**, 487c (1998).

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5. Condensates in QCD, Proceedings of the 7th Asia Pacific Physics Conference, Beijing, China, August 19-23, 1997 (Science Press, Beijing,1998), p. 280 (1998).

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7. Few-Nucleon Systems and QCD Sum Rules, Proceedings of the 16th International Conference on Few-Body Problems in Physics, Taipei, Taiwan, March 6-10, 2000, *in press*.

8. Large N_c QCD Sum Rules, Proceedings of the 3rd International Symposium on Symmetries in Subatomic Physics, Adelaide, Australia, March 13-17, 2000, (A.I.P. Conference Proceedings No. 539, Eds. X.-H. Gao, A.W. Thomas, and A.G. Williams), p. 160 (2000).

Invited talks delivered at International Physics Conferences/Workshops:

1. "Chiral Symmetry Breaking and QCD Sum Rules", *The Second International Symposium on Symmetries in Subatomic Physics*, June 25-28, 1997, Seattle, Washington, U.S.A.

2. "Isospin Symmetry Breakings as from QCD Sum Rules", *Talk at the 15th International Conference on Few Body Problems in Physics*, July 22-26, 1997, Groningen, Netherlands.

3. "Nonlocal Condensates in QCD", *Talk at the Second Joint Meeting of Chinese Physical Societies*, August 11-15, 1997, Taipei, Taiwan.

4. Nuclei as the Laboratory to Study Fundamental Symmetries, *Talk at the International Conference on Physics Since Parity Symmetry Breaking – in Memory of Professor C.S. Wu*, August 16-19, 1997, Nanjing, China.

5. Condensates in QCD, *The 7th Asia Pacific Physics Conference*, August 19-23, 1997, Beijing, China.

6. The JHF Opportunities and Taiwan, *The OECD Megascience Forum: Woking Group Meeting on Hadron Physics*, March 3-8, 1998, KEK, Tsukuba, Japan.

7. Nonperturbative Effects in Parton Distributions, International Workshop on JHF Science (JHF98), March 4-7, 1998, KEK, Tsukuba, Japan.

8. Few-Nucleon Systems and QCD Sum Rules, *The 16th International Conference on Few-Body Problems in Physics*, March 6-10, 2000, Taipei, Taiwan.

9. Large N_c QCD Sum Rules, *The 3rd International Symposium on Symmetries in Subatomic Physics*, Adelaide, Australia, March 13-17, 2000.

10. The Early Universe and Taiwan COSPA Project, *The 3rd Cross-Strait Symposium on High and Medium Energy Physics*, Jinan, Shantong, China, October 23-27, 2000.

11. XS3: Summary and Concluding Remarks, *The 3rd Cross-Strait Symposium on High and Medium Energy Physics*, Jinan, Shantong, China, October 23-27, 2000.

Z. Phys. C 73, 275–292 (1997)

ZEITSCHRIFT FÜR PHYSIK C

The QCD sum rule approach for the semileptonic decay of the *D* or *B* meson into a light meson and leptons

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Abstract. We use the method of QCD sum rules to treat the semileptonic weak decay of the D or B meson into a light meson and leptons. To obtain the transition form factors, we adopt the two-point Green's function in the presence of an external vector or axial-vector field. We find that this method can be related approximately to the traditional three-point Green's function in the heavy quark limit ($m_Q \to \infty$). Unlike some existing QCD sum rule calculations, our results indicate that the form factors have simple dipole or monopole behavior. We obtain results on the various form factors of the semileptonic decay of D and B mesons into a light meson and investigate various decay processes such as $\bar{B}^0 \to \pi^+ \tau^- \bar{\nu}_\tau$ and $\bar{B}^0 \to \rho^+ \tau^- \bar{\nu}_\tau$. The method allows us to take into account nonperturbative strong interaction effects, thereby providing a more reliable determination of the Cabibbo-Kobayashi-Maskawa matrix elements from the experimental data.

I Introduction

Semileptonic decays are particularly important in exploring the V-A structure of weak interactions. In these decays, strong interaction effects are contained in the transition form factors as used for defining the hadronic matrix elements. Reliable determination of these transition form factors helps us to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from the relevant experimental data.

There exist several methods for studying the semileptonic decay of a heavy meson (D or B) into a light meson and leptons, in the framework of relativistic or nonrelativistic quark models [1–4]. Another method [5] has also been employed recently by incorporating chiral symmetry into the effective heavy quark theory to compute form factors near the maximum momentum transfer squared, q_m^2. Nevertheless, it remains highly desirable, if possible, to tackle these problems using directly QCD.

In contradistinction with the lattice QCD treatment [6–8] of heavy meson decays, the method of QCD sum rules [9] provides another powerful tool in studying hadron physics within the framework of QCD. In the method of QCD sum rules we use perturbative QCD augmented with non-perturbative effects, such as through introduction of quark and gluon condensates, to describe both short- and large-distance physics. In the context of the three-point Green's function approach, the pion formfactor was solved at intermediate momentum transfer squared $0.5\,\mathrm{GeV}^2 < -q^2 < 1.5\,\mathrm{GeV}^2$ [10, 11], but the approach may fail at smaller values of q^2 – since in general the three-point Green's function approach works well only in the deep Euclidean region $p^2, p'^2, q^2 \ll 0$ and it may fail due to the presence of the singular terms $1/q^2$, $1/q^4$, etc., which are not small for sufficiently small q^2. In an extension of the original QCD sum rule approach [9], Ioffe and Smilga [12] and, independently, Balitsky and Yung [13] considered that the quarks move in a constant external electromagnetic field. Instead of the traditional three-point Green's function approach, they evaluated a two-point Green's function in the presence of an external field and successfully obtained the nucleon magnetic moments at *zero* momentum transfer.

Some authors [14–21] treated weak semileptonic decays of charmed or bottomed mesons using the three-point Green's function approach. In our attempt to employ the two-point Green's function approach, we try to replace, by some suitable induced condensates, certain diagrams which appear in the three-point Green's function approach and for which the usual operator product expansion (OPE) approach may fail when $q^2 \approx 0$. Furthermore, we adopt axial vector currents for heavy mesons since usage of the pseudoscalar currents (such as $c\gamma_5\bar{d}$ for D^+) often leads to very large contributions from the quark condensate, sometime even larger than the leading perturbative contribution, and the convergence of the series would be in doubt. These aspects will be addressed in some detail later in Sects. II and III.

Specifically, we treat the weak semileptonic decay of D or B meson into a light meson and leptons. To ensure that the method of QCD sum rules can be used in the small momentum transfer region, we consider the two-point Green's function in a varying external field. A similar idea was proposed first by Eletsky and Kogan [22] in evaluating the electromagnetic decays of D mesons at $q^2 = 0$. We also generalize the method to the region $q^2 \neq 0$, since, as discussed in [18], the analysis of the relevant QCD sum rules can also be performed for positive values of q^2 (the time-like region). Adopting suitable axial-vector currents to describe heavy mesons, we find that the series converges sufficiently well at the quark level and that the approach using the two-point Green's function in the external field yields almost the same results as the three-point Green's function approach. Our numerical results suggest that the form factors $f_i(q^2)$ (which describe the weak matrix elements) possess the pole dependence:

$$f_i(q^2) = \frac{f_i(0)}{(1 - q^2/m_{\text{pole}}^2)^n},$$

where $n = 1$ or 2. Note that these results differ somewhat from some existing QCD sum rule calculations [18, 19]. For instance, in [18, 19], Ball et al. obtained unusual q^2-behaviors for both the form factors, $A_1(q^2)$ and $A_2(q^2)$ and the results are also quite sensitive to the chosen Borel range. (See below for the definition of these form factors.) Instead, we find that adoption of suitable axial-vector currents, instead of pseudoscalar currents, for describing heavy mesons leads to QCD sum rules which seem much better behaved.

The rest of this paper is organized as follows: In Sect. II, we derive the QCD sum rules, which allow an investigation of the various decay form factors of heavy mesons. In Sect. III, numerical results are described and discussed. Section IV contains a brief summary.

II The QCD sum rules

A. Form factors for the decay of a D or B meson into a light pseudoscalar meson and leptons

To treat the form factors for the semileptonic decay of a D or B meson into a light pseudoscalar meson and leptons, namely $M \to L + \ell + \nu_\ell$ [such as $\bar{B}^0 \to \pi^+ e^- \bar{\nu}_e$], we choose to consider the following Green's function in the external vector field ($\mathscr{V}$):

$$i(2\pi)^4 \delta^4(p - p' - q)\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = i^2 \iint d^4x d^4y\, e^{i(p'x - py)} \langle 0 | T\{j^{5(L)}_\mu(x), j^5_\nu(y)\} | 0 \rangle_{\mathscr{V}}, \quad (1)$$

where $j^{5(L)}_\mu = \bar{q}_1 \gamma_\mu \gamma_5 q_2$ and $j^5_\nu = \bar{Q} \gamma_\nu \gamma_5 q_1$ are axial-vector currents whose divergences carry the same quantum numbers as the pseudoscalar L (π or K) and M (D or B) mesons, respectively. We use Q as a generic notation for a heavy quark field operator (such as c or b), while q_1 or q_2 for a light quark field operator. In the following calculation, we neglect u and d quark masses, but keep terms linear in the s quark mass. The interaction with the external vector field is described by the additional term, $\Delta\mathscr{L}$, in the Lagrangian:

$$\Delta\mathscr{L}(x) = -\mathscr{V}^\alpha e^{iqx} J_\alpha(x), \quad (2)$$

where

$$J_\alpha = \bar{q}_2 \gamma_\alpha Q. \quad (3)$$

Here $\mathscr{V}^\alpha$ and q^α are the amplitude and momentum of the external vector field, respectively. Note that in the following calculation we consider only the amplitude of $\Pi^{\mathscr{V}}_{\mu\nu}$ linear in the external field $\mathscr{V}$. Generally speaking, the Green's function can be written via the dispersion relation as follows

$$\Pi_{\mu\nu}(p, p'; q^2) = \int ds \int ds' \frac{\rho_{\mu\nu}(p, p'; q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \quad (4)$$

where the subtraction terms have the form

$$\text{subtraction terms} = P_{1\mu\nu}(p^2) \int \frac{\Delta(s')ds'}{s' - p'^2} + P_{2\mu\nu}(p'^2) \int \frac{\Delta(s)ds}{s - p^2} + P_{3\mu\nu}(p^2, p'^2). \quad (5)$$

One finds that the contribution from the subtraction terms are of no importance since, after performing the Borel transform, all of the subtraction terms, $P_{1\mu\nu}$, $P_{2\mu\nu}$, and $P_{3\mu\nu}$, disappear. (For further discussions, see [23].)

At the hadron level (on the rhs), the imaginary part of $\Pi^{\mathscr{V}}_{\mu\nu}$ is

$$\begin{aligned} \rho^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) &= -\mathscr{V}^\alpha \langle 0 | j^{5(L)}_\mu(0) | L(p') \rangle \langle L(p') | J_\alpha(0) | M(p) \rangle \\ &\quad \times \langle M(p) | j^5_\nu(0) | 0 \rangle \delta(s' - m_L^2) \delta(s - m_M^2) + \dots \\ &= -\mathscr{V}^\alpha f_L f_M [F_+(q^2) p'_\mu (p + p')_\alpha p_\nu \\ &\quad + F_-(q^2) p'_\mu (p - p')_\alpha p_\nu] \delta(s' - m_L^2) \delta(s - m_M^2) + \dots, \end{aligned} \quad (6)$$

where we have adopted the conventional definitions:

$$\begin{aligned} &\langle 0 | j^{5(L)}_\mu(0) | L(p') \rangle = i f_L p'_\mu, \\ &\langle M(p) | j^5_\nu(0) | 0 \rangle = -i f_M p_\nu, \\ &\langle L(p') | J_\alpha(0) | M(p) \rangle \\ &\quad = [F_+(q^2)(p + p')_\alpha + F_-(q^2)(p - p')_\alpha]. \end{aligned} \quad (7)$$

The ellipses in (6) stand for the contributions from axial-vector mesons or from higher excited states (coming from either the nondiagonal transitions from the lowest state to some excited state, or diagonal transitions from an excited state to itself). These contributions give rise to additional Lorentz structures (and thus new QCD sum rules for other transitions), which are not relevant for the purpose of the present paper. They also modify the coefficients of the two relevant Lorentz structures but these nonleading terms can presumably be approximated as a continuum starting from some threshold (s_0, s'_0), making use of the quark level calculation. Note that the contributions from low-lying axial-vector mesons do not pose specific threat since their masses are in general fairly large and lie somewhere (deep) in the continuum. We have also investigated the additional Lorentz structures to obtain the QCD sum rules relevant for the determination of the transition of the heavy meson into the low-lying axial vector meson and such results, being peripheral to our present purpose, are to be reported elsewhere.

Note that the Green's function, in (1), contains 14 different tensor structures:

$$\Pi^{\mathscr{V}}_{\mu\nu} = \mathscr{V}^{\alpha}(\Pi_+ p'_\mu(p+p')_\alpha p_\nu + \Pi_- p'_\mu(p-p')_\alpha p_\nu + \ldots), \tag{8}$$

but, for the purpose of determining the form factors F_+ and F_-, we need only the two amplitudes Π_+, Π_-; and, in fact, no other Lorentz structures can arise if only the form factors $F_\pm(q^2)$ are included, i.e., if only the $M \to L$ transition (7) is included. To obtain the relevant QCD sum rules, we try to equate the coefficients, obtained at the quark level (on the lhs) and parametrized at the hadron level (rhs), of the same Lorentz structure $-p'_\mu(p+p')_\alpha p_\nu$ and $p'_\mu(p-p')_\alpha p_\nu$ for the determination of the form factors F_+ and F_-.

Equation (6) gives us the expression on the rhs (the result at the hadron level). By performing the double Borel transform [9, 24],

$$\mathbf{B}[f(p^2, p'^2)] = \lim_{\substack{m\to\infty \\ -p'^2\to\infty \\ \frac{-p'^2}{mM'^2}\ \text{fixed}}} \lim_{\substack{n\to\infty \\ -p^2\to\infty \\ \frac{-p^2}{nM^2}\ \text{fixed}}} \frac{1}{n!m!}(-p'^2)^{m+1} \times \left[\frac{d}{dp'^2}\right]^m (-p^2)^{n+1}\left[\frac{d}{dp^2}\right]^n f(p^2, p'^2), \tag{9}$$

we then obtain the r.h.s of the QCD sum rules:

$$\text{For } F_+(q^2): \quad -f_L f_M F_+(q^2) e^{-(m_M^2/M^2)} e^{-(m_L^2/M'^2)},$$

$$\text{For } F_-(q^2): \quad -f_L f_M F_-(q^2) e^{-(m_M^2/M^2)} e^{-(m_L^2/M'^2)}. \tag{10}$$

As indicated earlier, the contributions of higher excited states may be transferred to the left-hand side of the sum rules and, to a sufficient approximation, they may be represented by the perturbative continuum (evaluated at the quark level), which begins at certain thresholds, s_0 and s'_0.

At the quark level, on the other hand, the Green's function in the deep Euclidean region $[p^2, p'^2, q^2 \leqslant 0]$ can be evaluated as a sum of the perturbative and non-perturbative contributions (including nonperturbative contributions from induced condensates) by means of the (short-distance) operator product expansion (OPE). For the perturbative part, we write down the Feynman integral of the bare loop:

$$\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = 3i\mathscr{V}^{\alpha}\int \frac{d^4k}{(2\pi)^4}\frac{Tr[(\hat{p}'+\hat{k})\gamma_\alpha(\hat{p}+\hat{k})\gamma_\nu\hat{k}\gamma_\mu]}{k^2(k+p')^2[(k+p)^2 - m_Q^2]}. \tag{11}$$

The relevant diagram is shown in Fig. 1a [the very first diagram in Figs. 1], in which the heavy quark propagator is represented by the heavy line. According to Cutkosky's rule [25], the integral can be solved easily by taking the imaginary part of the quark propagators: $1/(p^2 - m_Q^2 + i\varepsilon) \to -2\pi i\delta(p^2 - m_Q^2)$. In this case we have ignored subtraction terms, (5). Thus this integral may be rewritten as a double dispersion relation

$$\Pi^{\mathscr{V}}_{\mu\nu}(p, p'; q^2) = -\frac{\mathscr{V}^{\alpha}}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho^{\mathscr{V}}_{\mu\nu\alpha}(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{12}$$

where

$$\rho^{\mathscr{V}}_{\mu\nu\alpha} = 3i\int\frac{d^4k}{(2\pi)^4}(-2\pi i)^3\delta(k^2)\delta[(k+p')^2]\delta[(k+p)^2 - m_Q^2]\ Tr[(\hat{p}'+\hat{k})\gamma_\alpha(\hat{p}+\hat{k})\gamma_\nu\hat{k}\gamma_\mu]. \tag{13}$$

Making use of the integration formulas listed in Appendix A, we obtain

$$\Pi^{\text{pert}}_{+}(p, p'; q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho_+(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{14}$$

for the coefficient of the structure $\mathscr{V}^{\alpha}p'_\mu(p+p')_\alpha p_\nu$, and

$$\Pi^{\text{pert}}_{-}(p, p'; q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds\,ds' \frac{\rho_-(s, s'; q^2)}{(s-p^2)(s'-p'^2)}, \tag{15}$$

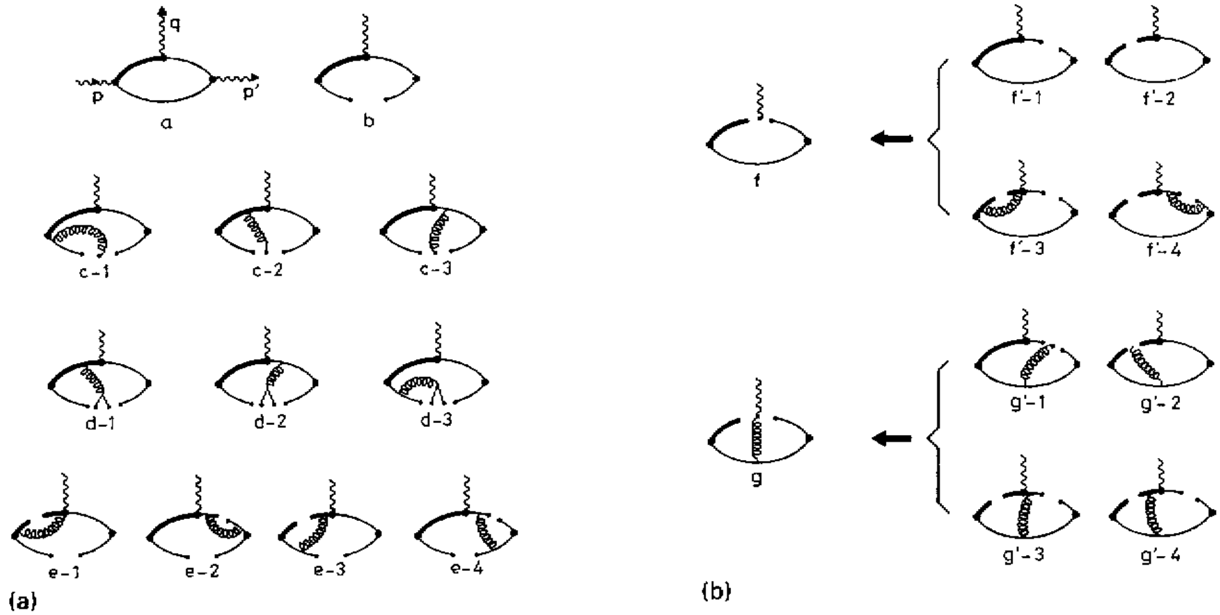
for the coefficient of the structure $\mathscr{V}^{\alpha}p'_\mu(p-p')_\alpha p_\nu$, where

$$\begin{aligned}\rho_+(s, s'; q^2) &= -3\Big\{-\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left(m_Q^2\frac{s'-s-q^2}{2} + sq^2\right) \\ &+ \frac{2s'}{\lambda^{5/2}}[3[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2](m_Q^2 - 2q^2) + [(s-m_Q^2)^2 - ss'](s-s'-m_Q^2)] \\ &+ \frac{1}{\lambda^{3/2}}[m_Q^2(m_Q^2 - q^2 + 3s' - s) - (2ss' - sq^2 - s'q^2)]\Big\},\end{aligned} \tag{16}$$

and

$$\begin{aligned}\rho_-(s, s'; q^2) &= -3\Big\{-\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left[m_Q^2\frac{3s+s'-q^2}{2} + s(s'-s)\right] \\ &+ \frac{2s'}{\lambda^{5/2}}[3[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times(2s - 2s' - m_Q^2) - [(s-m_Q^2)^2 - ss'](s+s'-m_Q^2)] \\ &+ \frac{1}{\lambda^{3/2}}[(s'-s)q^2 - m_Q^2(m_Q^2 - q^2 + 3s' - s)]\Big\},\end{aligned} \tag{17}$$

with $\lambda \equiv (s+s'-q^2)^2 - 4ss'$. Note that in this approach the integration region Ω is obtained via the Landau equation [26]. Namely, the corresponding integration region



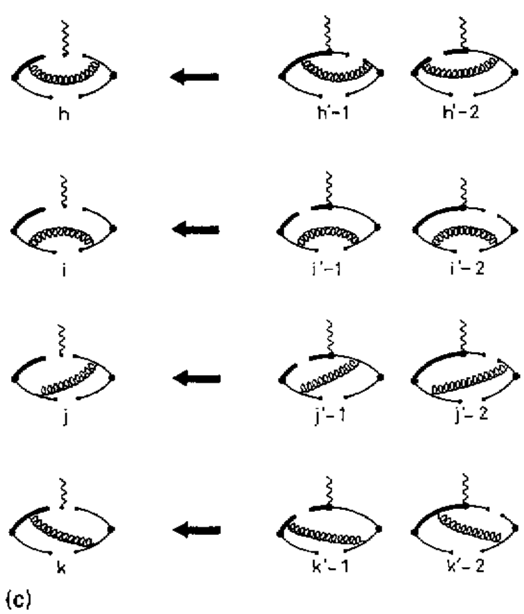


Fig. 1a–c. Diagrams for the two-point Green's function of (1) [or (33)] in the external vector field (or axial-vector field). The heavy quark propagator is represented by the heavy line. In this approach the right hand side of the heavy left-arrow, which corresponds to the three-point Green's function approach, is replaced by the left hand side of the heavy left-arrow, which corresponds to the two-point Green's function in the external field, respectively

Ω, which may be depicted as the shaded region in Fig. 2, is specified by the inequalities

$$\Omega\colon\quad s_0' > s' > 0, \quad s_0 > s > m_Q^2 + \frac{m_Q^2}{m_Q^2 - q^2}\, s', \tag{18}$$

where s_0 and s_0' are the thresholds of higher excited states, which can be decided by means of the relevant two-point sum rules [9,27]. In Fig. 2, the region I corresponds to diagonal higher excited state transitions, while the region II corresponds to nondiagonal transitions.

Next, we consider the contributions involving the quark and quark-gluon condensates which are represented pictorially by Fig. 1(*b*)–1(*e*), which can be evaluated

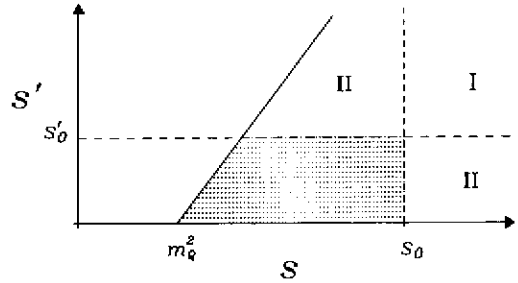


Fig. 2. Integration region in (18). The shaded region is denoted by Ω. The region I corresponds to diagonal higher excited state transitions, while the region II corresponds to nondiagonal transitions

in the standard manner. We only list the final results explicitly immediately below:

$$\Pi_{+}^{\text{nonpert}}(p, p'; q^2)$$

$$= m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q + m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}$$
$$- 2\frac{g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{4}{p'^4(p^2 - m_Q^2)^2}\right.$$
$$+ \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{2m_Q^2 - q^2}{p'^4(p^2 - m_Q^2)^3}$$
$$\left.- \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]$$
$$- \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)}\right.$$
$$\left.- \frac{1}{p'^2p^2} - \frac{m_Q^2}{p'^2p^4}\right]$$
$$- \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)}, \quad (19)$$

and

$$\Pi_{-}^{\text{nonpert}}(p, p'; q^2)$$

$$= - m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q - m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}$$
$$+ 2\frac{g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{6}{p'^4(p^2 - m_Q^2)^2}\right.$$
$$- \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{q^2}{p'^4(p^2 - m_Q^2)^3}$$
$$\left.+ \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]$$
$$- \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)} - \frac{1}{p'^2p^2} - \frac{m_Q^2}{p'^2p^4}\right]$$
$$+ \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)}, \quad (20)$$

where we have adopted the definition $\langle g_c\bar{q}\sigma Gq\rangle = -m_0^2\langle\bar{q}q\rangle$. Note that Figs. 1(c) have been evaluated with the aid of the heavy-quark propagator in an external field:

$$iS^A(x_2 - x_1) = \int\frac{d^4p}{(2\pi)^4}e^{-ip(x_2-x_1)}\left[\frac{i}{4}\frac{\lambda^a}{2}g_cG_{\mu\nu}^a(x_0)\right.$$
$$\times\frac{\sigma^{\mu\nu}(\hat{p} + m_Q) + (\hat{p} + m_Q)\sigma^{\mu\nu}}{(p^2 - m_Q^2)^2}$$
$$- \frac{iG_{\mu\nu}^a(x_0)\lambda^a}{4}g_c(x_1 - x_0)^\nu$$
$$\left.\times\left(\frac{1}{\hat{p} - m_Q}\gamma^\mu\frac{1}{\hat{p} - m_Q}\right)\right]. \quad (21)$$

Here we have used the fixed-point gauge, $(x - x_0)^\mu A_\mu^a = 0$, [9,24] for the gluon field, and adopted $\hat{p} \equiv \gamma_\mu p^\mu$. Note also that because both the contributions of the gluon condensate and the heavy quark condensates [9,28] are in fact rather small in the sum rules, we may neglect them here.

Finally, we need to consider the contributions arising from the induced condensates. To this end, we first caution that the short distance expansion (i.e., the usual OPE approach) is in fact not applicable in the evaluation of the contributions for the diagrams illustrated by Figs. 1(f')–(k') (albeit as done in the three-point Green's function approach), since the space-time point with q^2 entering is in general far from the other two points when $q^2 \approx 0$. To circumvent the problem, we choose to treat these diagrams by means of considering induced condensates in the presence of a suitable external field (as depicted in Figs. 1(f)–(k)). Such replacements are indicated by arrows in the figure.

To obtain the contributions corresponding to Figs. 1(f)–(k), we first write down the identity:

$$4\bar{Q}_\alpha q_{2\beta} = (1(\bar{Q}q_2) + \gamma_5(\bar{Q}\gamma_5 q_2) + \gamma^\rho(\bar{Q}\gamma_\rho q_2)$$
$$- \gamma^\rho\gamma_5(\bar{Q}\gamma_\rho\gamma_5 q_2) + \tfrac{1}{2}\sigma^{\rho\tau}(\bar{Q}\sigma_{\rho\tau}q_2))_{\alpha\beta}$$
$$= (1(\bar{Q}q_2) + \gamma_5(\bar{Q}\gamma_5 q_2) + \gamma^\rho(\bar{Q}\gamma_\rho q_2)$$
$$- \gamma^\rho\gamma_5(\bar{Q}\gamma_\rho\gamma_5 q_2) + \tfrac{1}{2}\sigma^{\rho\tau}\gamma_5(\bar{Q}\sigma_{\rho\tau}\gamma_5 q_2))_{\alpha\beta}. \quad (22)$$

Thus we may expand the quark condensate $\langle q_{2i}^a(x)\bar{Q}_j^b(y)\rangle_{\not{V}}$ in the approximation linear in $\not{V}^\mu$ (i.e. the weak-field approximation):

$$\langle q_{2i}^a(x)\bar{Q}_j^b(y)\rangle_{\not{V}} \approx -\frac{1}{12}\langle\bar{Q}(y)q_2(y)\rangle_{\not{V}}\delta^{ab}(\mathbf{1})_{ij}$$
$$-\frac{1}{12}\langle\bar{Q}(y)\gamma_\mu q_2(y)\rangle_{\not{V}}\delta^{ab}(\gamma^\mu)_{ij}$$
$$-\frac{1}{24}\langle\bar{Q}(y)\sigma_{\alpha\beta}q_2(y)\rangle_{\not{V}}\delta^{ab}(\sigma^{\alpha\beta})_{ij} + \ldots, \quad (23)$$

where a and b are color indices while i and j the Dirac indices. The induced condensates may be written as

$$\langle\bar{Q}(y)q_2(y)\rangle_{\not{V}}$$
$$= -i\not{V}^\mu\int d^4x e^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:, \quad (24a)$$

$$\langle\bar{Q}(y)\gamma_\alpha q_2(y)\rangle_{\not{V}}$$
$$= -i\not{V}^\mu\int d^4x e^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)\gamma_\alpha q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:, \quad (24b)$$

$$\langle\bar{Q}(y)\sigma_{\alpha\beta}q_2(y)\rangle_{\not{V}}$$
$$= -i\not{V}^\mu\int d^4x e^{iqx}:\langle 0|\mathrm{T}\{\bar{Q}(y)\sigma_{\alpha\beta}q_2(y), \bar{q}_2(x)\gamma_\mu Q(x)\}|0\rangle:. \quad (24c)$$

Here the notation ": :" has been used to denote subtraction of the perturbative contributions:

$$:\pi(q^2): = \pi(q^2) - \pi^{\text{pert}}(q^2)$$
$$= \frac{1}{\pi}\int_0^\infty\frac{ds}{s - q^2}(\mathrm{Im}\,\pi(s) - \mathrm{Im}\,\pi^{\text{pert}}(s)). \quad (25)$$

$B \to \rho\, e\, \nu_e$, we follow a similar line and consider the following Green's function in external vector ($\mathscr{V}$) and

with λ denoting the helicity state of the spin-1 particle (L^*).

Note that the calculation of perturbative parts in (1) with the aid of (23) and (24) will lead to (11), the bare loop of the three-point Green's function. This has led to the subtractions in (24a)–(24c). Note also that one can perform the expansion of (23), being a Taylor-series or short-distance expansion, only when the variable q^2 is in the deep Euclidean region.

To evaluate (25) (i.e., (24)), we may adopt the simple model

$$\mathrm{Im}\,\pi(s) = f\delta(s - m_f^2) + \mathrm{Im}\,\pi^{\mathrm{pert}}(s)\theta(s - s_0). \tag{26}$$

The various values of f could be determined by means of relevant 2-point Green's functions via the method of QCD sum rules. This is described briefly in Appendix B. (See [29, 30] for similar discussions.) Note that the contribution of $\langle \bar{Q}(y) q_2(y)\rangle_{\mathscr{V}}$ to the relevant sum rules (F_+ and F_-) vanishes identically while the total contribution of $\langle \bar{Q}(y)\gamma_\mu q_2(y)\rangle_{\mathscr{V}}$ is proportional to $m_{q_2}\langle \bar{q}_2 q_2\rangle$, or to the small light quark mass. Therefore it is sufficient to consider only the induced condensate $\langle \bar{Q}(y)\sigma_{\alpha\beta} q_2(y)\rangle_{\mathscr{V}}$, which may be characterized by

$$\langle \bar{Q}(y)\sigma_{\alpha\beta} q_2(y)\rangle_{\mathscr{V}} = -i\mathscr{V}^{\nu} e^{iqy} \chi^{\mathscr{V}}_{Qq_2}(q^2)(g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha). \tag{27}$$

The calculation described in Appendix B yields

$$\chi^{\mathscr{V}}_{Qq_2} = \frac{f^{\mathscr{V}}}{m_{f^{\mathscr{V}}}^2 - q^2} - \frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds \frac{m_Q}{s - q^2} \times \left[1 - 2\left(\frac{m_Q^2}{s}\right) + \left(\frac{m_Q^2}{s}\right)^2\right]. \tag{28}$$

In particular, we have calculated the induced condensates, $\chi^{\mathscr{V}}$, in the limit of the $SU(3)$ symmetry, i.e., m_u, m_d, and $m_s \to 0$. The results of $f^{\mathscr{V}}$ are listed in Table 1. It is of interest to note that, in the heavy quark limit, the following approximation appears to hold reasonably well:

$$\chi^{\mathscr{V}}_{Qq_2} = \frac{\langle \bar{q}_2 q_2\rangle}{q^2 - m_Q^2}.$$

Noting that the contribution of the induced condensate shown in Fig. 1(g) is proportional to the heavy quark condensate (so that we may neglect it here), the contributions of induced condensates are given by

$$\Pi_+^{\mathrm{induced}}(p, p'; q^2) = -\frac{2g_c^2\langle \bar{q}_1 q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2 - m_Q^2)p'^4},$$

$$\Pi_-^{\mathrm{induced}}(p, p'; q^2) = \frac{2g_c^2\langle \bar{q}_1 q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2 - m_Q^2)p'^4}. \tag{29}$$

Note that the results shown in Tables 1 and 2, together with some discussion in Appendix B, suggest that it is consistent to adopt, (1), axial-vector currents, instead of pseudoscalar currents, in the method with the two-point Green's function in the external field. The approach appears to yield, in the deep Euclidean region, the same results as in the three-point Green's function approach.

Table 1. Results of $f^{\mathscr{V}}$ and $s_0^{\mathscr{V}}$ obtained from relevant sum rules, (B.4), in Appendix B. The lowest-lying meson mass is used as an input parameter in the sum rule

Qq_2	$f^{\mathscr{V}}_{Qq_2}$(GeV^3)	$m_{f_{Qq_2}}$(GeV)	s_0 (GeV^2)
cd	0.22	2.01	6.0
bu	0.28	5.33	33

Table 2. Results of $f^{\mathscr{A}}$ and $s_0^{\mathscr{A}}$ obtained from relevant sum rules, (B.4), in Appendix B. The lowest-lying meson mass is used as an input parameter in the sum rule

Qq_2	$f^{\mathscr{A}}_{Qq_2}$(GeV^3)	$m_{f_{Qq_2}}$(GeV)	s_0 (GeV^2)
cd	0.48	2.42	9.3
bu	0.67	5.71	38.2

Carrying out the integrations in (16)–(17) and combining the results with (19), (20) and (29), we finally obtain

$$\begin{aligned}
\Pi_+^{\mathrm{lhs}}&(p, p'; q^2)\\
&= \Pi_+^{\mathrm{pert}}(p, p'; q^2) + \Pi_+^{\mathrm{induced}}(p, p'; q^2)\\
&\quad + m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q + m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{4}{p'^4(p^2 - m_Q^2)^2}\right.\\
&\quad + \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{2m_Q^2 - q^2}{p'^4(p^2 - m_Q^2)^3}\\
&\quad \left. - \frac{3m_Q^2}{p'^2(p^2 - m_Q^2)^4} - \frac{m_Q^2 - q^2}{p'^6(p^2 - m_Q^2)^2}\right]\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{Q}Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2 - m_Q^2)} - \frac{1}{p'^2 p^2} - \frac{m_Q^2}{p'^2 p^4}\right]\\
&\quad - \frac{2g_c^2\langle \bar{q}_1 q_1\rangle\langle \bar{q}_2 q_2\rangle}{9}\frac{1}{p'^6(p^2 - m_Q^2)},
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
\Pi_-^{\mathrm{lhs}}&(p, p'; q^2)\\
&= \Pi_-^{\mathrm{pert}}(p, p'; q^2) + \Pi_-^{\mathrm{induced}}(p, p'; q^2)\\
&\quad - m_0^2\langle \bar{q}_1 q_1\rangle \frac{m_Q - m_{q_2}}{6p'^4(p^2 - m_Q^2)^2}\\
&\quad + \frac{2g_c^2\langle \bar{q}_1 q_1\rangle^2}{81}\left[\frac{1}{p'^6(p^2 - m_Q^2)} + \frac{6}{p'^4(p^2 - m_Q^2)^2}\right.\\
&\quad - \frac{1}{p'^2(p^2 - m_Q^2)^3} - \frac{q^2}{p'^4(p^2 - m_Q^2)^3}
\end{aligned}$$

$$+\frac{3m_Q^2}{p'^2(p^2-m_Q^2)^4}-\frac{m_Q^2-q^2}{p'^6(p^2-m_Q^2)^2}\Bigg]$$
$$-\frac{2g_c^2\langle\bar q_1 q_1\rangle\langle\bar Q Q\rangle}{9m_Q^4}\left[\frac{1}{p'^2(p^2-m_Q^2)}-\frac{1}{p'^2p^2}-\frac{m_Q^2}{p'^2p^4}\right]$$
$$+\frac{2g_c^2\langle\bar q_1 q_1\rangle\langle\bar q_2 q_2\rangle}{9}\frac{1}{p'^6(p^2-m_Q^2)}. \quad (31)$$

After performing the Borel transform and comparing the r.h.s of QCD sum rules (10), we obtain the QCD sum rules for the determination of the form factors $F_\pm(q^2)$:

$$F_+(q^2)=-f_L^{-1}f_M^{-1}e^{(m_M^2/M^2)}e^{(m_L^2/M'^2)}\mathbf{B}[\Pi_+^{\rm lhs}]$$

and

$$F_-(q^2)=-f_L^{-1}f_M^{-1}e^{(m_M^2/M^2)}e^{(m_L^2/M'^2)}\mathbf{B}[\Pi_-^{\rm lhs}]. \quad (32)$$

B. Form factors for the decay of the D or B meson into a light vector meson and leptons

To treat the form factors for semileptonic decays of D and B mesons into a light vector meson and leptons such as $\bar B^0\to\rho^+e^-\bar\nu_e$, we follow a similar line and consider the following Green's function in external vector ($\mathscr{V}$) and external axial-vector ($\mathscr{A}$) fields:

$$i(2\pi)^4\delta^4(p-p'-q)\,\tilde\Pi_{\mu\nu}^{\mathscr{A}(\mathscr{V})}(p,p';q^2)$$
$$=i^2\int\int d^4x d^4y\, e^{i(p'x-py)}\langle 0|T\{j_\mu(x),j_\nu^5(y)\}|0\rangle_{\mathscr{A}(\mathscr{V})}, \quad (33)$$

with $j_\mu=\bar q_1\gamma_\mu q_2$ and $j_\nu^5=\bar Q\gamma_\nu\gamma_5 q_1$. Here the definitions of the notations are the same as before.

The interaction with the external field is described by the additional term in the Lagrangian:

$$\Delta\mathscr{L}^A(x)=-\mathscr{A}^\alpha e^{iqx}J_\alpha^5(x), \quad (34)$$

for the axial-vector field or

$$\Delta\mathscr{L}^V(x)=-\mathscr{V}^\alpha e^{iqx}J_\alpha(x), \quad (35)$$

for the vector field, where

$$J_\alpha^5=\bar q_2\gamma_\alpha\gamma_5 Q,$$
$$J_\alpha=\bar q_2\gamma_\alpha Q. \quad (36)$$

Here $\mathscr{A}(\mathscr{V})$ and q^α are the amplitude and momentum of the external axial-vector (or vector) field, respectively. Note that in the following calculation we consider only the amplitude of $\tilde\Pi_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$ linear in the external field $\mathscr{A}(\mathscr{V})$.

At the hadron level (r.h.s.), the imaginary parts of $\tilde\Pi_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$ are

$$\rho_{\mu\nu}^{\mathscr{A}}(p,p';q^2)$$
$$=-\mathscr{A}^\alpha\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle\langle L^*(p',\lambda)|J_\alpha^5(0)|M(p)\rangle$$
$$\times\langle M(p)|j_\nu^5(0)|0\rangle\delta(s'-m_{L^*}^2)\delta(s-m_M^2)+\ \dots, \quad (37)$$

and

$$\rho_{\mu\nu}^{\mathscr{V}}(p,p';q^2)$$
$$=-\mathscr{V}^\alpha\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle\langle L^*(p',\lambda)|J_\alpha(0)|M(p)\rangle$$
$$\times\langle M(p)|j_\nu^5(0)|0\rangle\delta(s'-m_{L^*}^2)\delta(s-m_M^2)+\ \dots, \quad (38)$$

respectively. In the calculation we adopt the definitions and Wirbel-Stech-Bauer (WSB) parametrization [1]:

$$\langle 0|j_\mu(0)|L^*(p',\lambda)\rangle=i\frac{m_{L^*}^2}{g_{L^*}}\varepsilon_{\mu,\lambda},$$
$$\langle L^*(p',\lambda)|J_\alpha^5(0)|M(p)\rangle$$
$$=(m_{L^*}+m_M)A_1(q^2)\varepsilon_{\alpha,\lambda}^*-\frac{(p+p')_\alpha}{m_{L^*}+m_M}(\varepsilon_\lambda^*\cdot p)A_2(q^2)$$
$$+(\varepsilon_\lambda^*\cdot p)2m_{L^*}A(q^2)q_\alpha,$$
$$\sum_{\lambda=1}^3\varepsilon_{\gamma,\lambda}\varepsilon_{\mu,\lambda}^*=-\left(g_{\gamma\mu}-\frac{p'_\gamma p'_\mu}{m_{L^*}^2}\right),$$
$$\langle L^*(p',\lambda)|J_\alpha(0)|M(p)\rangle=i\varepsilon_{\alpha\mu\nu\delta}p'^\mu p^\nu\varepsilon_\lambda^{*\delta}\frac{2V(q^2)}{m_M+m_{L^*}},$$
$$A(q^2)=\frac{A_0(q^2)-A_3(q^2)}{q^2},$$
$$A_3(q^2)=\frac{m_M+m_{L^*}}{2m_{L^*}}A_1(q^2)-\frac{m_M-m_{L^*}}{2m_{L^*}}A_2(q^2), \quad (39)$$

with λ denoting the helicity state of the spin-1 particle (L^*).

The Lorentz structures of $\tilde\Pi_{\mu\nu}^{\mathscr{A}(\mathscr{V})}$, in (33), may be written as follows,

$$\tilde\Pi_{\mu\nu}^{\mathscr{A}}=\mathscr{A}^\alpha(\tilde\Pi_{A_1}g_{\mu\alpha}p_\nu+\tilde\Pi_{A_2}p_\mu(p+p')_\alpha p_\nu$$
$$+\tilde\Pi_A p_\mu(p-p')_\alpha p_\nu+\ \cdots),$$
$$\tilde\Pi_{\mu\nu}^{\mathscr{V}}=i\mathscr{V}^\alpha(\tilde\Pi_V\varepsilon_{\alpha\mu\beta\rho}p_\nu p'^\beta p^\rho+\ \cdots), \quad (40)$$

where we obtain, at the hadronic level (rhs),

$$\tilde\Pi_{A_1}=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}(m_M+m_{L^*})A_1(q^2),$$
$$\tilde\Pi_{A_2}=-\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}\frac{A_2(q^2)}{(m_M+m_{L^*})},$$
$$\tilde\Pi_A=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}2m_{L^*}A(q^2),$$
$$\tilde\Pi_V=\frac{f_M m_{L^*}^2}{g_{L^*}}\frac{1}{(m_M^2-p^2)(m_{L^*}^2-p'^2)}\frac{2V(q^2)}{(m_M+m_{L^*})}. \quad (41)$$

Note that, as mentioned before, the contributions of higher excited states are transferred to the left hand side of the sum rules (via the continuum approximation). Note also that the amplitude $\tilde\Pi_{\mu\nu}^{\mathscr{V}}$ can in general be represented as

$$\tilde\Pi_{\mu\nu}^{\mathscr{V}}=i\mathscr{V}^\alpha(f_1p_\nu T_{\alpha\mu}+f_2p_\mu T_{\nu\alpha}+f_3p_\alpha T_{\mu\nu}+f_4S_{\alpha\mu\nu}$$
$$+f'_1p'_\nu T_{\alpha\mu}+f'_2p'_\mu T_{\nu\alpha}+f'_3p'_\alpha T_{\mu\nu}+f'_4S'_{\alpha\mu\nu}), \quad (42)$$

where $T_{\alpha\mu}=\varepsilon_{\alpha\mu\beta\rho}p'^\beta p^\rho$, $S_{\alpha\mu\nu}=\varepsilon_{\alpha\mu\nu\beta}p^\beta$, and $S'_{\alpha\mu\nu}=\varepsilon_{\alpha\mu\nu\beta}p'^\beta$. Among these tensors, there exist two relations [31, 32]

$$p_\nu T_{\alpha\mu}+p_\mu T_{\nu\alpha}+p_\alpha T_{\mu\nu}=(p\cdot p')S_{\mu\nu\alpha}-p^2S'_{\mu\nu\alpha},$$
$$p'_\nu T_{\alpha\mu}+p'_\mu T_{\nu\alpha}+p'_\alpha T_{\mu\nu}=-(p\cdot p')S'_{\mu\nu\alpha}+p'^2S_{\mu\nu\alpha}, \quad (43)$$

thereby reducing to only six linearly independent Lorentz structures. (For a proof of (43), consult Appendix C.) In this work, we choose $S_{\alpha\mu\nu}$, $S'_{\alpha\mu\nu}$, $p_\mu T_{\nu\alpha}-p_\alpha T_{\mu\nu}$,

$p'_\mu T_{\nu\alpha} - p'_\alpha T_{\mu\nu}$, $p_\nu T_{\alpha\mu}$, and $p'_\nu T_{\alpha\mu}$ as independent structures. We emphasize that the issue concerning linear independence does affect, to a minor extent, the final QCD sum rules and should be treated with care.

At the quark level (lhs), the derivation of the relevant QCD sum rules is carried out along the same line as before and it is sufficient to simply record the results. For the perturbative part, we obtain

$$\tilde{\Pi}^{\rm pert}_{A_1}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds ds' \frac{\rho_{A_1}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \quad (44)$$

$$\tilde{\Pi}^{\rm pert}_{A_2}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds ds' \frac{\rho_{A_2}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \quad (45)$$

$$\tilde{\Pi}^{\rm pert}_{A}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds ds' \frac{\rho_{A}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \quad (46)$$

$$\tilde{\Pi}^{\rm pert}_{V}(p,p';q^2) = -\frac{1}{4\pi^2}\int\int_\Omega ds ds' \frac{\rho_{V}(s,s';q^2)}{(s-p^2)(s'-p'^2)}, \quad (47)$$

where

$$\rho_{A_1}(s,s';q^2) = 3\left\{\frac{4s'^2}{\lambda^{5/2}}\left(s - m_Q^2 - \frac{s+s'-q^2}{2}\right)[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] - \frac{s'}{\lambda^{3/2}}[(m_Q^2-q^2)^2 - s'q^2] + \frac{s'}{\lambda^{3/2}}(s - s' + q^2 - 2m_Q^2)m_Q m_{q_2}\right\}, \quad (48)$$

$$\rho_{A_2}(s,s';q^2) = -3\left\{\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left[\left(\frac{s'+s-q^2}{2}\right)(s+s'-m_Q^2) - s'(2s-m_Q^2)\right] + \frac{s'}{\lambda^{5/2}}[3(s-7s'-q^2)[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] - 2s'[(s-s'-m_Q^2)^2 - s'q^2]] + \frac{s'}{\lambda^{3/2}}(2s'+q^2-m_Q^2)\right\}, \quad (49)$$

$$\rho_{A}(s,s';q^2) = -3\left\{\frac{40s'^2}{\lambda^{7/2}}[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] \times\left[s'm_Q^2 - \left(\frac{s'+s-q^2}{2}\right)(s-s'-m_Q^2)\right] + \frac{s'}{\lambda^{5/2}}[-3(s+s'-q^2)[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2] + 2s'[(s-m_Q^2)^2 - s'(s'+2m_Q^2-q^2)]] + \frac{s'}{\lambda^{3/2}}(2s'-q^2+m_Q^2)\right\}, \quad (50)$$

and

$$\rho_V(s,s';q^2) = -3\frac{s'}{\lambda^{5/2}}[\lambda(3m_Q^2 - 2q^2 - s) + 3(3s'-s+q^2) \times[(s-m_Q^2)(m_Q^2-q^2) - s'm_Q^2]]. \quad (51)$$

For the induced condensates, with the aid of (22), we could easily expand the quark condensate $\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{A}}$ in the approximation linear in $\mathscr{A}^\mu$:

$$\langle q^a_{2i}(x)\bar{Q}^b_j(y)\rangle_{\mathscr{A}} \approx -\frac{1}{12}\langle\bar{Q}(y)\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\gamma_5)_{ij} + \frac{1}{12}\langle\bar{Q}(y)\gamma_\mu\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\gamma^\mu\gamma_5)_{ij} - \frac{1}{24}\langle\bar{Q}(y)\sigma_{\alpha\beta}\gamma_5 q_2(y)\rangle_{\mathscr{A}}\delta^{ab}(\sigma^{\alpha\beta}\gamma_5)_{ij} + \dots \quad (52)$$

By the same token [as for (23)–(28)], we need to consider only the induced condensate $\langle\bar{Q}(y)\sigma_{\alpha\beta}\gamma_5 q_2(y)\rangle_{\mathscr{A}}$:

$$\langle\bar{Q}(y)\sigma^{\alpha\beta}\gamma_5 q(y)\rangle_{\mathscr{A}} = -i\mathscr{A}^\nu e^{iqy}\chi^{\mathscr{A}}_{Qq_2}(q^2)(g_{\nu\alpha}q_\beta - g_{\beta\nu}q_\alpha). \quad (53)$$

The result is

$$\chi^{\mathscr{A}}_{Qq_2} = \frac{f^{\mathscr{A}}}{m^2_{f\mathscr{A}} - q^2} - \frac{3}{8\pi}\int_{m_Q^2}^{s_0} ds \frac{m_Q}{s-q^2}\left[1 - 2\left(\frac{m_Q^2}{s}\right) + \left(\frac{m_Q^2}{s}\right)^2\right], \quad (54)$$

and the numerical values of $f^{\mathscr{A}}$ are listed in Table 2, in which the lowest-lying meson mass is used as the input parameter in the relevant sum rule. The detailed calculations of $f^{\mathscr{A}}$ are contained in Appendix B.

The contributions of induced condensates are then given by

$$\tilde{\Pi}^{\rm induced}_{A_1}(p,p';q^2) = -\frac{2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{p^2-p'^2-q^2}{(p^2-m_Q^2)p'^4},$$
$$\tilde{\Pi}^{\rm induced}_{A_2}(p,p';q^2) = \frac{2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4},$$
$$\tilde{\Pi}^{\rm induced}_{A}(p,p';q^2) = \frac{-2g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{A}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4},$$
$$\tilde{\Pi}^{\rm induced}_{V}(p,p';q^2) = \frac{4g_c^2\langle\bar{q}_1q_1\rangle}{9}\chi^{\mathscr{V}}_{Qq_2}(q^2)\frac{1}{(p^2-m_Q^2)p'^4}. \quad (55)$$

Combining (44)–(51) and (55) and considering non-perturbative contributions from quark condensates and quark-gluon condensates, we may record the results at the quark level:

$$\tilde{\Pi}^{\rm lhs}_{A_1}(p,p';q^2) = \tilde{\Pi}^{\rm pert}_{A_1}(p,p';q^2) + \tilde{\Pi}^{\rm induced}_{A_1}(p,p';q^2) - \frac{m_{q_2}\langle\bar{q}_1q_1\rangle}{p'^2(p^2-m_Q^2)} + \frac{m_0^2\langle\bar{q}_1q_1\rangle}{6}\left[\frac{m_{q_2}-2m_Q}{p'^2(p^2-m_Q^2)^2} + \frac{m_{q_2}}{p'^4(p^2-m_Q^2)}\right.$$