Proceedings of the 1991 IEEE
International Conference on Robotics and Automation
Sacramento, California - April 1991

Efficient Adaptive Hybrid Control Strategies for Robots in Constrained Manipulation

Jong-Hann Jean[1] and Li-Chen Fu[1,2]

Department of Electrical Engineering[1]
Department of Computer Sicence & Information Engineering[2]
National Taiwan University, Taiwan, R. O. C.

Abstract

This paper addresses the problem of adaptive hybrid controller design for constrained robots with the consideration of computational efficiency. Two efficient control schemes respectively based on Lagrange-Euler and Newton-Euler dynamics formulation are presented. Detailed analyses on tracking properties of joint positions, velocities, and constrained forces are derived for both the Lagrange-Euler approach and the Newton-Euler approach. Although control laws in these two approaches are developed independently, a tight connection between them is signified, which highlights a possible bridge over different general adaptive approaches respectively based on the two dyamics formulations.

1. Introduction

Many industrial applications of robotic manipulator systems may involve tasks in which the manipulator end-effectors are required to make contact with the environment. Typical examples of these tasks are such as contour following, grinding, scribing, writing, deburring, as well as all assembly related tasks. During the execution of such tasks, the motion of robot end-effectors can be viewed as being constrained by the environment and contact forces will build up to maintain satisfaction of the constraints. Owing to the nature of these tasks, accurate control of robot positions as well as contact forces should be taken into account for the success of task execution.

Recently, there have been considerable theorectical researches [1]–[6] which deal with such applications. Although these researches indeed provide a theoretical framework served as a basis for the study of robot performance of constrained tasks, all of those results are based on a critical assumption that full knowledge about the complex dynamics of the constrained robot is available. From a practical point of view, the inertial properties and gravitational loads vary from a task to another and, hence, may not be precisely known in advance. Care, therefore, should be taken in that if there is uncertainty about the robot dynamics, the controller so designed may give degraded performance and may even incur instability. This is the reason why adaptive control strategies have been proposed to handle the uncertainties in the robot dynamics.

Several adaptive control schemes for robot motion control have been developed to assure the stability of the overall system inspite of the nonlinear imprecise knowledge of the system dynamics. The methods are roughly classified into two categories: one is based on a decomposition of the robot dynamics into the product of a nonlinear implementable function matrix and a constant vector which consists of unknown system parameters [7]–[9] whereas the other directly deals with a form of the robot dynamics as a function vector [22]. In the former, estimates of those parameters are used to synthesize the control input and are updated on line based on a measure of tracking errors. In contrast, the latter introduced a concept of function learning instead of that of parameter learning.

Although these schemes can be shown to achieve trajectory tracking, the computational complexity for their implementation is considerably high. Therefore, more recently, several researchers addressed the problem of computational efficiency for realizing the abovementioned adaptive control schemes. Walker [10] utilized the method proposed in [8] to obtain a more efficient solution for manipulators containing closed kinematics loops via an adaptive control algorithm based on their Newton-Euler dynamics formulation. Sadegh and Horowitz [11] presented a modified version of their previous work [9] to allow off-line computation of the aforementioned nonlinear function matrix using the desired joint positions and velocities instead of the actual measurements. Both of these works lead the adaptive controller design toward a more practical direction.

For constrained robots, an adaptive control scheme has been proposed in the recent work [12], in which the dynamic model proposed in [1] and a parameter adaptation law similar to the one proposed in [8] were

This research was supproted by National Science Council under the grant NSC 79-0404-E002-03

used to derive conditions to ensure the asymtotic tracking of joint positions and the boundedness of force errors. However, no conditions for the asymtotic tracking of joint velocities, and constrained forces were concluded. In addition, the complexity for implementing this scheme still remains high.

This paper addresses the problem of adaptive hybrid controller design for constrained robots with the consideration of computational efficiency. Two efficient control schemes respectively based on Lagrange-Euler and Newton-Euler dynamics formulation are presented. Detailed analyses on tracking properties of joint positions, velocities, and constrained forces are derived for both the Lagrange-Euler approach and the Newton-Euler approach. Althougth control laws in these two approaches are developed independently, a tight connection between them is signified, which highlights a possible bridge over different general adaptive approaches respectively based on the two dyamics formulations. This facilitates robot control theorists to realize the spine of any adaptive control methodology irregardless of dynamics formulation of the target system.

The paper is organized as follows: In section 2, the dynamics formulation of a constrained robot is introduced. In section 3, the adaptive hybrid control scheme using Lagrange-Euler approach and the tracking properties of the overall system are presented. The Newton-Euler approach which is a counterpart of the previous one is given in section 4 and so is the connection between them. Finally, some conclusions as well as discussions are stated in section 5.

2. Dynamic Model

Consider a rigid robot whose end-effector is in contact with the environment modeled as a rigid frictionless surface. To restrict the robot end-effector to keep contact with the surface amounts to imposing kinematic constraints on the robot. Let $p \in R^l$ and $q \in R^n$ respectively denote the Cartesian coordinate of the end-effector and the joint coordinate of the robot arm. The constraint surface can be naturally represented in terms of the algebraic equation of the coordinate p, namely, $\pi(p) = 0$, where $\pi : R^l \mapsto R^r$. To relate the Cartesian coordinate of the end-effector with the joint coordinate of the arm, we assume the forward kinematics of the robot is known *a priori* and is expressed as $p = h(q)$, where $h : R^n \mapsto R^l$. Define the function $\phi(\cdot)$ by the relation $\phi(q) = \pi(h(q))$ so that the constraint equation can be rewritten as

$$\phi(q) = 0. \tag{2.1}$$

For convenience of our subsequent development, we will assume the constraints are nonredundant in the sense that $\partial\phi(q)/\partial q$ is assumed to have full row rank $r (r \leq n)$. The number of degrees of freedom is then equal to $m = n - r$. Furthermore, if we properly rearrange and partition the vector q into $q = [q_1^\top, q_2^\top]^\top$, where $q_1 \in R^r$ and $q_2 \in R^m$, then, by implicit function theorem [15], there is a unique function $\Omega : R^m \mapsto R^r$ and an open set $O \subset R^m$ such that

$$\phi(\Omega(q_2), q_2) = 0 \quad \text{for all } q_2 \in O. \tag{2.2}$$

Moreover, if the constraint surface, or, equivalently, the function ϕ is sufficiently smooth, then derivatives of Ω with respect to q_2 up to the second order will exist. This shows, locally in the open set O, the equation

$$q_1 = \Omega(q_2) \tag{2.3}$$

is a unique solution of (2.1). As a matter of fact, an n-tuple vector $q \in R^r \times O$ satisfies the constraint equation (2.1) if and only if it satisfies the equation (2.3). Thus, (2.1) and (2.3) can be identified as the same constraint equation for suitable choice of domain, and, in the rest of our paper, we will directly deal with constraints of the form in (2.3).

The constraints imposed on joint velocities and accelerations of the robot can be obtained by differentiating equation (2.3) into the form

$$\dot{q}_1 - E(q_2)\dot{q}_2 = A(q_2)\dot{q} = 0, \tag{2.4}$$

$$\ddot{q}_1 = E(q_2)\ddot{q}_2 + F(q_2, \dot{q}_2)\dot{q}_2, \tag{2.5}$$

$$\begin{aligned} A(q_2) &= [\, I_r \quad -E(q_2)\,] \\ E(q_2) &= \partial\Omega(q_2)/\partial q_2 \\ F(q_2, \dot{q}_2) &= dE(q_2)/dt \end{aligned} \tag{2.6}$$

According to the D'Alembert's principle and Lagrange's multiplier rule [16], the equation of motion for the constrained robot system has the following form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + A^\top\lambda, \tag{2.7}$$

where $M(q)$ denotes the generalized inertia matrix, $C(q, \dot{q})\dot{q}$ denotes the vector of centrifugal and Coriolis forces, $G(q)$ denotes the vector of gravitational forces, τ is the vector of nonconservative generalized forces, $\lambda \in R^r$ is the vector of Lagrange's multipliers associated with the constraints, and $A^\top\lambda$ is the constraint force vector.

To facilitate our subsequent analysis, we reformulate the equation of motion (2.7) as follows. First, decompose the equation of motion (2.7) into the following form

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1\dot{q} \\ C_2\dot{q} \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} + A^\top\lambda \tag{2.8}$$

After substitution of the constraint equations (2.3)-(2.5) into (2.8), it follows that

$$\begin{aligned} (M_{11}(q_2)E + M_{12}(q_2))\ddot{q}_2 + M_{11}(q_2)F\dot{q}_2 \\ + C_1X(q_2)\dot{q}_2 + G_1 = \tau_1 + \lambda \end{aligned} \tag{2.9}$$

$$(M_{21}(q_2)E + M_{22}(q_2))\ddot{q}_2 + M_{21}(q_2)F\dot{q}_2 + C_2X(q_2)\dot{q}_2 + G_2 = \tau_2 - E^\top\lambda \quad (2.10)$$

$$X(q_2) = \partial q/\partial q_2 = \begin{bmatrix} E(q_2) \\ I_m \end{bmatrix} \quad (2.11)$$

Note that $M_{ij}(q_2)$ is, in fact, $M_{ij}(\Omega(q_2), q_2)$, $i,j = 1,2$, in equations (2.9) and (2.10), which despite the abuse of notations emphasize the complete dependence of these submatrices upon q_2.

Premultiplying the matrix $E(q_2)^\top$ to (2.9) and adding the resultant equation to (2.10), we then obtain the following equation

$$\overline{M}(q_2)\ddot{q}_2 + \overline{C}(q_2,\dot{q}_2)\dot{q}_2 + \overline{G}(q_2) = \tau_c \quad (2.12)$$

where

$$\overline{M} = E^\top M_{11}E + E^\top M_{12} + M_{21}E + M_{22}$$
$$\overline{C} = E^\top M_{11}F + E^\top C_1X + M_{21}F + C_2X$$
$$\overline{G} = E^\top G_1 + G_2$$
$$\tau_c = E^\top(q_2)\tau_1 + \tau_2,$$

where we note that the constraint forces disappear. It is noteworthy the combinations of (2.12) and (2.9) are exactly the dynamical equations in reduced form derived in [1]. These two equations characterize respectively the motion of the robot and the evolution of constraint forces acting on the robot.

3. Adaptive Hybrid Controller: Lagrange-Euler Approach

Suppose the desired motion of robot can be described in joint coordinate by a vector function $q_d(t) = [q_{1d}(t)^\top, q_{2d}(t)^\top]^\top$, where $q_{1d}(t) = \Omega(q_{2d}(t))$ for consistency with the imposed constraints. Furthermore, the desired constrained forces $f_d(t)$ can be characterized by some multiplier function $\lambda_d(t)$ as $f_d(t) = A^\top(q_d(t))\lambda_d(t)$, where the multiplier function λ_d can be physically interpreted as the desired contact forces in the constraint coordinate [1]. Our objective is then to design an adaptive hybrid controller to achieve asymptotic tracking of both joint positions and constrained forces, that is, to yield

$$q(t) \to q_d(t) \text{ and } f(t) \to f_d(t) \quad \text{as } t \to \infty$$

Using the notations defined in (2.9) and (2.10), we can define

$$(M_{11}E + M_{12})\ddot{u} + (M_{11}F + C_1X)\dot{u} + G_1 = \omega_1^\top(q_2,\dot{q}_2,\dot{u},\ddot{u})\theta_1 \quad (3.1)$$

$$(M_{21}E + M_{22})\ddot{u} + (M_{21}F + C_2X)\dot{u} + G_2 = \omega_2^\top(q_2,\dot{q}_2,\dot{u},\ddot{u})\theta_2 \quad (3.2)$$

which lead to the following equation

$$\overline{M}(q_2)\ddot{u} + \overline{C}(q_2,\dot{q}_2)\dot{u} + \overline{G}(q_2) = [\,E^\top\omega_1^\top \quad \omega_2^\top\,]\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = \omega^\top(q_2,\dot{q}_2,\dot{u},\ddot{u})\theta, \quad (3.3)$$

where $\omega(q_2,\dot{q}_2,\dot{u},\ddot{u})$ is a known function matrix and θ consists of the dynamic parameters of the constrained manipulator.

Define the auxiliary signals q_{2r} and $\dot{\tilde{q}}_{2r}$ as

$$\dot{q}_{2r} = \dot{q}_{2d} - k_r\tilde{q}_2 \quad (3.4)$$
$$\dot{\tilde{q}}_{2r} = \dot{q}_2 - \dot{q}_{2r} \quad (3.5)$$

where $\tilde{q}_2 = q_2 - q_{2d}$ is the positional tracking error and k_r is a positive constant. Note that the signal $\dot{\tilde{q}}_{2r}$ is designed such that if $\dot{\tilde{q}}_{2r}$ is square-integrable, then $\tilde{q}_2$ approaches zero as time goes to infinity [17]. Design the control signals as

$$\tau_1 = \omega_1^\top(q_2,\dot{q}_2,\dot{q}_{2r},\ddot{q}_{2r})\hat{\theta}_1 + \kappa \quad (3.6)$$
$$\tau_2 = \omega_2^\top(q_2,\dot{q}_2,\dot{q}_{2r},\ddot{q}_{2r})\hat{\theta}_2 - E^\top\kappa - K_v\dot{\tilde{q}}_{2r} - K_p\tilde{q}_2 \quad (3.7)$$
$$\kappa = K_f(\lambda - \lambda_d) - \lambda,$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the estimates of θ_1 and θ_2, and K_p, K_v, and K_f are positive definite matrices. The parameter adaptation law is then designed as

$$\dot{\hat{\theta}} = -\Gamma^{-1}\omega(q_2,\dot{q}_2,\dot{q}_{2r},\ddot{q}_{2r})\dot{\tilde{q}}_{2r}, \qquad \Gamma > 0, \quad (3.8)$$

The main result of this section is stated in the following theorem. The detailed proof is provided in [23].

Theorem 3.1: Consider the constrained robot system whose dynamics are governed by (2.7). Given the bounded desired trajectory q_{2d} such that its first and second derivatives are also bounded almost everywhere. Then the following will be true provided the adaptive control laws (3.6)–(3.8) is used.

1. (a) Signals q_2, $\dot{q}_2$ and $\hat{\theta}$ are bounded and the asymptotic positional tracking will be ensured, i.e.,

$$\lim_{t\to\infty}(q - q_d) = 0$$

(b) Force errors are bounded and the bound is proportional to the norm of the inverse of the gain matrix K_f, i.e.,

$$\|\lambda - \lambda_d\| \le \alpha\,\|K_f^{-1}\|$$

for some $\alpha > 0$.

2. If the third order derivatives of q_{2d} is also bounded almost everywhere up to the third order, then
   (a) Joint velocities and accelerations track the desired ones asymptotically, i.e.,

$$\lim_{t\to\infty}(\dot{q}-\dot{q}_d)=0 \text{ and } \lim_{t\to\infty}(\ddot{q}-\ddot{q}_d)=0.$$

   (b) The parameter error vector $\tilde{\theta}=\hat{\theta}-\theta$ asymptotically falls into the null space of $\omega^{\mathsf{T}}(q_2,\dot{q}_2,\dot{q}_{2r},\ddot{q}_{2r})$, i.e.,

$$\lim_{t\to\infty}\omega^{\mathsf{T}}(q_2,\dot{q}_2,\dot{q}_{2r},\ddot{q}_{2r})\tilde{\theta}=0.$$

3. If $\omega(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})$, which is obtained by replacing q_2, $\dot{q}_2$, $\dot{u}$, and $\ddot{u}$ with q_{2d}, $\dot{q}_{2d}$, $\dot{q}_{2d}$, and $\ddot{q}_{2d}$ in (3.3) respectively, is persistently exciting(PE) [11][18][19], then
   (a) the parameter errors converge to zero, i.e.,

$$\lim_{t\to\infty}\tilde{\theta}(t)=0$$

   (b) force errors $\tilde{\lambda}=\lambda-\lambda_d$ converge to zero, i.e.,

$$\lim_{t\to\infty}\tilde{\lambda}(t)=0$$

For the consideration of computational efficiency, we modified the control law as

$$\tau_1=\omega_1^{\mathsf{T}}(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})\hat{\theta}_1+\kappa \quad (3.9)$$

$$\tau_2=\omega_2^{\mathsf{T}}(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})\hat{\theta}_2-E^{\mathsf{T}}\kappa - K_v\dot{\tilde{q}}_{2r}-K_p\tilde{q}_2-\sigma|\tilde{q}_2|^2\dot{\tilde{q}}_{2r} \quad (3.10)$$

and the adaptation law as

$$\dot{\hat{\theta}}=-\Gamma^{-1}\omega(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})\dot{\tilde{q}}_{2r}, \quad (3.11)$$

where the function matrix $\omega(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})$ is off-line computed prior to the control, then similar results can be obtained. The following corollary will summarize these results and the proof is provided in [23].

Corollary 3.2: Consider the constrained robot system whose dynamics are governed by (2.7). If the adaptive control laws (3.9),(3.10), and (3.11) are used, and K_v, K_p, and σ are sufficiently positive, then all the results in Theorem 3.1 hold.

4. ADAPTIVE HYBRID CONTROLLER: NEWTON-EULER APPROACH

Although, an efficient adaptive control strategy based on Lagrange-Euler dynamics formulation has been introduced in section 3, a drawback of this method is that a large volume of memory space will have to be allocated in order to save $\omega(q_{2d},\dot{q}_{2d},\ddot{q}_{2d})$ over the entire desired trajectory q_{2d}. Furthermore, the number of parameter estimates may be considerably large also when the robot dynamics become more complex. With this observation in mind, in this section we will start with the Newton-Euler formulation to speed up the calculation of the robot dynamics. Subsequently, we will show that the former Lagrange-Euler approach can be modified into a Newton-Euler approach but with a similar concept in designing the controller. The presentation to be given here will be based on the spatial notation introduced by Featherstone [20] and will borrow some conceptual development of the scheme proposed by Walker [10].

Denote the i-th joint variable and its associated actuator input respectively as ϑ_i and ρ_i, where the lower order joint is closer to the base and the higher order joint is closer to the end-effector. The spatial inertia matrix of link i, denoted as $\underline{I_i}$, with respect to the coordinate associated with link i is a constant 6×6 matrix of the form:

$$\underline{I_i}=\begin{bmatrix}(\underline{r_i}\times)^{\mathsf{T}} & m_{ci}I_3\\ \underline{J_i} & \underline{r_i}\times\end{bmatrix}$$

where m_{ci} is the total mass of link i, $\underline{r_i}$ is the total mass of link i times the position vector of center of mass of link i, $\underline{r_i}\times$ denotes a 3×3 skew symmetric matrix which satisfies $(\underline{r_i}\times)w=\underline{r_i}\times w$ for any constant vector $w\in R^3$, and $\underline{J_i}$ is the moment of inertia matrix of link i with respect to link i coordinate. The spatial inertia matrix of link i is uniquely determined by ten independent parameters; they are the six independent components of the matrix $\underline{J_i}$, the three components of the vector $\underline{r_i}$, and the total mass m_{ci}. For ease of reference, these parameters are grouped into a single 10×1 vector, m_i. It is easy to check that the matrix $\underline{I_i}$ can be represented as a linear combination of these parameters,

$$\underline{I_i}=\sum_{k=1}^{10}R_k m_{ik}$$

where R_k is a 6×6 matrix containing only zeros and ones.

Let the spatial transformation matrix from link i to the base be ${}_0X_i$, then the spatial inertia matrix of link i with respect to the base can be computed through the relation

$$I_i={}_0X_i\ \underline{I_i}\ {}_iX_0$$

In the Newton-Euler formulation, the equations of motion for a general closed kinematic mechanism can be described by a recursive inverse dynamics algorithm [10]. For our case, the inverse dynamics algorithm for dynamical equation (2.12) can be summarized as the following steps.

(S1) Given the desired positions, velocities, and accelerations of the independent joint variables, q_2, calculate the corresponding ones of the dependent joint variables, q_1, through equations (2.3)–(2.5).

(S2) For each link, determine ρ_i, the i-th actuator input needed to achieve the desired motion, from two recursions as follows.

Outward loop:

$$v_0 = 0, \quad i = 1, \ldots, n$$
$$v_i = v_{i-1} + s_i \dot{\vartheta}_i$$
$$\dot{v}_i = \dot{v}_{i-1} + s_i \ddot{\vartheta}_i + v_i \hat{\times} s_i \dot{\vartheta}_i$$

where s_i denotes the spatial vector representation of the i-th joint axis.

Inward loop:

$$f_{n+1} = 0, \quad j = n, \ldots, 1$$
$$F_j = I_j \dot{v}_j + v_j \hat{\times} I_j v_j + I_j g$$
$$f_j = F_j + f_{j+1}$$
$$\rho_j = s_j^* f_j$$

where $g = [0\ 0\ 9.8\ 0\ 0\ 0]^\mathsf{T}$ and the symbols $*$ and $\hat{\times}$ represent the spatial transpose and the spatial cross operator [20] respectively.

(S3) Determine the vectors τ_1 and τ_2 from the computation results of $\rho_i, i = 1, \ldots, n$. Then the vector τ_c can be obtained through (2.12).

The adaptive hybrid controller to be proposed here can then be given in terms of a algorithmic procedure involving three steps.

(C1) Given the desired position trajectory q_{2d}, the desired force trajectory λ_d, and the auxiliary signals q_{2r} and $\tilde{q}_{2r}$ which are defined in (3.4) and (3.5) respectively, we compute the following signals,

$$\dot{q}_{1r} = E(q_2)\dot{q}_{2r} \tag{4.1}$$
$$\ddot{q}_{1r} = E(q_2)\ddot{q}_{2r} + F(q_2, \dot{q}_2)\dot{q}_{2r}. \tag{4.2}$$

(C2) Generate the control signals $\rho_{i,r}, i = 1, \ldots, n$, as follows:

Outward loop:

$$v_{0,r} = 0, \quad i = 1, \ldots, n$$
$$v_{i,r} = v_{i-1,r} + s_i \dot{\vartheta}_{i,r}$$
$$\dot{v}_{i,r} = \dot{v}_{i-1,r} + s_i \ddot{\vartheta}_{i,r} + v_i \hat{\times} s_i \dot{\vartheta}_{i,r}$$

Inward loop:

$$f_{n+1,r} = 0, \quad j = n, \ldots, 1$$
$$F_{j,r} = \hat{I}_j(\dot{v}_{j,r} - \mu \tilde{v}_{j,r} + g) + v_j \hat{\times} \hat{I}_j v_{j,r}$$
$$f_{j,r} = F_{j,r} + f_{j+1,r}$$
$$\rho_{j,r} = s_j^* f_{j,r}$$

where $\hat{I}_j$ is an estimate of I_j, μ is a positive constant, and $\tilde{v}_{j,r} = v_j - v_{j,r}$.

Parameter adaptation law:

$$\dot{\hat{m}}_j = (\frac{-1}{\sigma_j}) w_j, \qquad \sigma_j > 0$$

where w_j is a 10×1 vector whose k-th component is,

$$w_{jk} = \underline{\tilde{v}_{j,r}^*}(R_k(\underline{\dot{v}_{j,r}} - \mu \underline{\tilde{v}_{j,r}}) + \underline{v_j} \hat{\times} R_k \underline{v_{j,r}} + R_k \underline{g_j})$$

where $\underline{v_j}$, $\underline{\tilde{v}_{j,r}}$, $\underline{v_{j,r}}$, and $\underline{g_j}$ are the vectors v_j, $\tilde{v}_{j,r}$, $v_{j,r}$, and g with respect to link j coordinate, respectively.

(C3) Determine the vectors $\tau_{1,r}$ and $\tau_{2,r}$ according to the computation results of $\rho_{i,r}$, $i = 1, \ldots, n$. Then the actuator inputs τ are implemented as follows.

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} \tau_{1,r} \\ \tau_{2,r} \end{bmatrix} + \begin{bmatrix} \kappa \\ -E^\mathsf{T} \kappa \end{bmatrix} \tag{4.3}$$

Although the form of this Newton-Euler approach is quite different from that of Lagrange-Euler approach given in section 3, the concept in designing respective controller are, in fact, quite similar and the following lemma will signify their relationship. For the proof of this lemma, please see [23].

Lemma 4.1: The control law (4.3) can be rewritten as

$$\tau_1 = (\hat{M}_{11}E + \hat{M}_{12})v + (\hat{M}_{11}F + \hat{C}_1 X)\dot{q}_{2r} + \hat{G}_1 + \kappa \tag{4.4}$$
$$\tau_2 = (\hat{M}_{21}E + \hat{M}_{22})v + (\hat{M}_{21}F + \hat{C}_2 X)\dot{q}_{2r} + \hat{G}_2 - E^\mathsf{T}\kappa \tag{4.5}$$

where

$$v = \ddot{q}_{2r} - \mu \dot{\tilde{q}}_{2r}.$$

Remark: Clearly, Lemma 4.1 points out the connection between the Lagrange-Euler approach and the Newton-Euler approach, which is useful in the subsequent analysis as well as for better understanding of the physical meaning of the underlying approach. As a matter of fact, several adaptive control schemes based on Lagrange-Euler dynamics have their counterparts in the domain of approaches based on Newton-Euler dynamics. The basic difference between them lies in the way to update the estimated parameters.

Using the symbols defined in (3.1) and (3.2), the positional error dynamics can be formulated from (4.4)–(4.5), (2.9)–(2.10) as the following:

$$\overline{M}(q_2)(\ddot{\tilde{q}}_{2r} + \mu \dot{\tilde{q}}_{2r}) + \overline{C}(q_2, \dot{q}_2)\dot{\tilde{q}}_{2r} = w^\mathsf{T}(q_2, \dot{q}_2, \dot{q}_{2r}, v)\tilde{\theta} \tag{4.6}$$

Note that the vector of parameter errors $\tilde{\theta}$ is derived in a way that is different from the one in section 3. The following theorem will summarize the properties of the constrained robot system after this Newton-Euler approach is applied. For the detailed proof of this theorem, please refer to [23].

Theorem 4.2: Consider the same constrained robot system as the one in Theorem 3.1. Then all the results (1a)–(3b) of Theorem 3.1 will still hold provided the adaptive hybrid control law (C1)–(C3) is applied.

5. Discussion and Conclusion

In this paper, we present efficient control strategies for adaptive hybrid control of a constrained robot system. Efficiency of the control law based on Lagrange-Euler approach is obtained by off-line calculation and prior storage of a complicated function matrix along the desired trajectory. The price, however, is that the complicated function matrix should be symbolically obtained in a closed form and, then, a large volume of memory space will have to be allocated to store these computation results especially when the number of joints is large. On the other hand, efficiency of the control law based on Newton-Euler approach lies in that the control law is formulated as an algorithm mainly consisting of two sets of recursive calculations subject to a law of dynamical parameter adaptation, and each recursion only takes generally smaller amount of time in comparison with the Lagrange-Euler approach. Furthermore, the number of unknown parameters to be estimated can be considerably reduced if partial knowledge of the robot is known. Nonetheless, if the number of joints is small or the overall dynamics of the constrained robot has a simplified form, then the Lagrange-Euler dynamics based approach can be more concise and may be more efficient.

A connection between the Lagrange-Euler approach and the Newton-Euler approach has also been pointed out. In the current literature, most research works on the adaptive control problem of a robot are based on the Lagrange-Euler dynamics formulation since it is more concise than the Newton-Euler one and, hence, more suitable for deriving the control law and for stability analysis. However, under the Newton-Euler dynamics formulation, we can have better understanding of the physical meaning of each component in the dynamics, which may lead to better intuition for designing control laws with better performance. However, no matter whichever dynamics formulation is under consideration for controller design, such a bridge suggested in this paper is always helpful to the robot control theorists to realize the spine of any adaptive control methodology.

References

[1] N. H. McClamroch and D. Wang, "Feedback Stabilization and Tracking of Constrained Robot," *IEEE Trans. Automat. Contr.*, vol. 33, no. 5, pp. 419-426, May 1988.

[2] N. H. McClamroch, "Singular Systems of Differential Equations as Dynamic Models for Constrained Robot Systems," in *Proc. IEEE Conf. Robotics Automat.*, San Francisco, CA, 1986, pp. 21-28.

[3] J. Mills and A. Goldenberg, "Force and Position Control of Manipulators During Constrained Motion Tasks," *IEEE Trans. Robotics Automat.*, vol. 5, no. 1, pp. 30-46, Feb. 1989.

[4] H. P. Huang, "Constrained Manipulators and Contact Force Control of Contour Following Problems," Ph.D. dissertation, Dep. Elect. Eng. Comput. Sci., Univ. Michigan, Ann Arbor, July 1986.

[5] T. Yoshikawa, "Dynamic Hybrid Position/Force Control of Robot Manipulators–Description of Hand Constraints and Calculation of Joint Driving Forces," in *Proc. IEEE Conf. Robotics Automat.*, San Francisco, CA, 1986, pp. 1393-1398.

[6] S. K. Lin, "Control of a Robot Constrained by the Environment," in *Proc. 13th Symp. Automat. Contr.*, Taipei, Taiwan, 1989, pp. 153-159.

[7] J. J. Craig, P. Hsu, and S. S. Sastry, "Adaptive Control of Mechanical Manipulators," *Int. J. Robotics Res.*, vol. 6, no. 2, 1986.

[8] J. Slotine and W. Li, "On the Adaptive Control of Robot Manipulators," *Int. J. Robotics Res.*, vol. 6, no. 2, 1986.

[9] N. Sadegh and R. Horowitz, "Stability Analysis of an Adaptive Controller for Robotic Manipulators,"in *Proc. IEEE Conf. Robotics Automat.*, vol. 3, pp. 1223-1229, 1987.

[10] M. W. Walker, "Adaptive Control of Manipulators Containing Closed Kinematic Loops," *IEEE Trans. Robotics Automat.*, vol. 6, no. 1, pp. 10-19, Feb. 1990.

[11] N. Sadegh and R. Horowitz, "Stability and Robustness Analysis of a Class of Adaptive Controllers," *Int. J. Robotics Res.*, vol. 9, no. 3, pp. 74-92, June 1990.

[12] R. Carelli and R. Kelly, "Adaptive Control of Constrained Robots Modeled by Singular Systems," in *Proc. IEEE Conf. Decision Contr.*, pp. 2635–2640, 1989.

[13] J.Y.S. Luh and Y. Zhang, "Computation of Input Generalized Forces for Robots with Closed Kinematic Chain Mechanisms," *IEEE J. of Robotics Automat.*, vol. RA-1, no. 2, pp. 95-103, June 1985.

[14] Y. Nakamura and M. Ghodoussi, "Dynamics Computation of Closed-Link Robot Mechanisms with Nonredundant and Redundant Actuators," *IEEE Trans. Robotics Automat.*, vol. 5, no. 3, pp.294-303, June 1989.

[15] Apostol, *Mathematical Analysis.*, Addison-Wesley, 1977

[16] R. M. Resenberg, *Analytical Dynamics of Discrete Systems.*, Plenum Press, New York, 1977.

[17] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. New York: Academic Press.

[18] R. Bitmead, "Persistence of Excitation Conditions and the Convergence of Adaptive System," *IEEE Trans. Inform. Theory*, vol. IT-30, no. 2, pp. 183-191, March 1984.

[19] K. S. Narendra and A. M. Annaswamy, "Persistence Excitation in Adaptive Systems," *INT. J. Contr.*, vol. 45, no. 1, pp. 127-160, 1987.

[20] R. Featherstone, *Robot Dynamics Algorithm*. Norwell, MA: Kluwer Academic Publishers, 1987.

[21] J. J. Craig, *Introduction to Robotics: Mechanics and Control*. MA: Addison-Wesley, 1986.

[22] W. Messner, R. Horowitz, W. Kao, and M.Boals, "A New Adaptive Learning Rule," RAMP 89–10/ESRC 89–12, University of California at Berkeley, April 1989.

[23] J. H. Jean and L. C. Fu, "Adaptive Hybrid Control Strategies for Constrained Robots," Technical Report, Department of Electrical Engineering, National Taiwan University, 1990.