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Letters

Comments on “On the design of feedforward neural networks for binary mapping [1]”

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Abstract

In the paper [1], the authors present a new design technique that builds a feedforward net for an arbitrary set of binary associations. The design method is a decomposition-based design technique and claim that the average number of intermediate operation is around $\frac{1}{2}nk$ for A and B of the same dimension $(k \times n)$. We will propose an alternative method, based on the augmented operation and the perceptron mapping, to reduce the number of layers to two.

Keywords: Binary mappings; Decomposition-based design; Perceptron neural networks

In the paper [1], the authors present a decomposition-based design technique that builds a feedforward net for an arbitrary set of binary associations. They claim that the average number of intermediate operations is around $\frac{1}{2}nk$ for A and B of the same dimension $(k \times n)$. In many complicated applications, the drawback of large number operations will make the overall realization impractical, which is obtained by cascading the simple feedforward subnets that realize the primitive operations in the decomposition. In this letter, we will propose an alternative method to reduce the number of layers to two. In our method, we don't need the entry-flipping primitive operation any more. But we introduce a special augmentation primitive operation. Denoted by E_{ij}, this operation appends one column in which the appended column will have zero entries except the ith one which uses 1 to indicate a flipping operation on the ith row.

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$$\begin{bmatrix}1&0\\1&1\\0&1\\0&0\end{bmatrix}\xrightarrow{XS_{11}} X=\begin{bmatrix}0&0&1\\1&1&0\\0&1&0\\0&0&0\end{bmatrix}\xrightarrow{XS_{41}} Y=\begin{bmatrix}0&0&1&0\\1&1&0&0\\0&1&0&0\\1&0&0&1\end{bmatrix}$$

$$\xrightarrow{D_{2,3,4}} B=\begin{bmatrix}0\\1\\0\\1\end{bmatrix}.$$

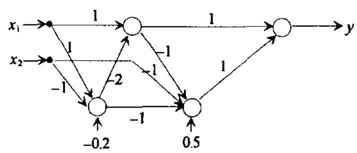

Fig. 1. The network structure for the *exclusive-or* problem [1].

As an example, the intermediate steps for decomposing the famous *exclusive-or* problem following [1] are shown in Fig. 1.

Fig. 1 [1] shows the final net configuration. For this extremely simple example, the intermediate steps for decomposing this problem following our method are shown in Fig. 2. Since the given classes Y and B are linearly separable, perceptrons will always find a set of solution weights in finite time [2]. Note that since the rank of Y is equal to the rank of Z, so the given classes Z and B are also linearly separable (means having perceptron solutions). The final net configuration is shown in Fig. 2. For this example, the geometrical design technique may build a two-layer net of 3 neurons, the method in [1] builds a feedforward net of 4 neurons, and our method results in a two-layer net of 3 neurons for the same problem.

Let's see Example 2 in [1]. It is easy to see that the decomposition consists of two primitive operations ($E_{11,21}$) and one perceptron operation. Each of these two

$$A=\begin{bmatrix}1&0\\1&1\\0&1\\0&0\end{bmatrix}\xrightarrow{E_{11,41}} Z=\begin{bmatrix}1&0&1&0\\1&1&0&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\xrightarrow{perceptron} B=\begin{bmatrix}0\\1\\0\\1\end{bmatrix}$$

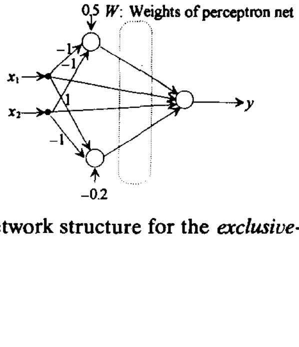

Fig. 2. The network structure for the *exclusive-or* problem.

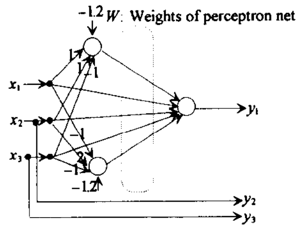

Fig. 3. The feedforward net built for Example 2 [1].

operations can be realized using one neuron. The final feedforward net structure is shown in Fig. 3.

The strategy adopted here is to replace the mapping

$$A \rightarrow \cdots \xrightarrow{XS_{i_1j_1}} X \xrightarrow{XS_{i_2j_2}} Y \xrightarrow{XS_{i_3j_3}} Z \rightarrow B$$

with the mapping

$$A \longrightarrow \cdots \xrightarrow{E_{i_1j_1,i_2j_2,i_3j_3}} Z' \xrightarrow{perceptron} B.$$

Now we show that these three primitive operations indeed form a complete set of operation.

Lemma 1. *The rank of Z is equal to the rank of Z'.*

Proof. The Lemma follows from the fact that the vector added by the entry-flipping operation is actually a linear combination of the proposed augmentation vector and an input vector. □

Lemma 2. *The given patterns (Z, B) are linearly separable and the given patterns (Z', B) are too.*

Proof. Since columns of B are a subset of those of Z, patterns (Z, B) are linearly separable. The Lemma follows from the above statement and the rank of Z is equal to that of Z'. □

Lemma 3. *Input Z' and output B have perceptron solutions.*

Proof. By Lemma 2. □

Theorem 1. *The mapping $A \rightarrow B$, where A and B are arbitrary matrices of dimensions $(k \times n)$, and $(k \times m)$, respectively, can be decomposed into a sequence of primitive operations* [1].

Proof. Let us assume $n = m$. If this is not the case, we can apply a sequence of $X_1(X_0)$ or D operations to make it so. Then the ijth entry of A is compared to the ijth entry of B for $i = 1,\ldots,k$ and $j = 1,\ldots,n$. If they are different, E_{ij} is applied to A to make Z'. Then the perceptron algorithm is applied to Z'. A is therefore brought to B by a sequence of primitive operations. The theorem is thus proved. □

Theorem 2. *The result net is a two-layer net.*

Proof. The resultant net is built by only two cascaded operations and each operation needs a layer of neurons to implement. □

One more important practical assumption underlying analog IC implementation of the scheme: the fan-in and fan-out rates are extremely limited. This limitation necessitates the use of buffer, cascading structure [1], or combining our structure and cascade structure [1]. Our approach will slightly increase the fan-in, but it required only one fan-out for each neuron when there is a single output neuron. However, the number of fan-outs of the net built in [1] was often more than one.

References

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[2] F. Rosenblatt, *Principles of Neurodynamics*, (New York, Spartan, 1962).

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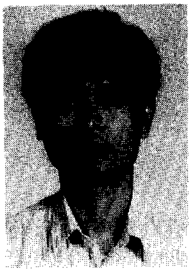

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