Epoch Distance of the Random Waypoint Model in Mobile Ad Hoc Networks

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Abstract. In this paper, we model the epoch distance of the random waypoint model in mobile ad hoc networks. In the random waypoint model, each node selects a target location (i.e., waypoint) to move at a speed selected from an interval. Once the target is reached, the node pauses for a random time and then selects another target with another speed to move again. The movement between two waypoints is referred to as an epoch. In this paper, we derive the probability distribution of the epoch distance for the random waypoint model. Such a study is important as the epoch length distribution may be required for the derivation of the link duration distribution or node spatial distribution for mobile ad hoc networks. The analytical result is then verified via simulation.

Keywords: Random waypoint model, epoch distance, ad hoc networks.

1 Introduction

Mobile ad hoc networks have received much attention in recent years. In such a network, no infrastructures such as base stations exist, and data are relayed by intermediate mobile hosts if the receiver is beyond the transmission range of the sender.

There have been many mobility models available for evaluating the performance of mobile ad hoc networks, including the random waypoint model [1], random walk [2], and group model [3]. In this paper, we focus on the random waypoint model. With this mobility model, each node selects a target location (i.e., waypoint) to move at a speed selected from a uniformly distributed interval $[V_{\min}, V_{\max}]$. Once the target is reached, the node pauses for a random time and then selects another target with another speed to move again.

In this paper, we model the epoch distance (length) of the random waypoint model, and derive the probability distribution of the distance between two waypoints in mobile ad hoc networks. Such a study is important as the epoch length distribution may be required for the derivation of the link duration distribution [4] or node spatial distribution [5] for mobile ad hoc networks. The analytical result is then verified by simulations.

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The rest of the paper is organized as follows. Sec. II gives the analytical model of epoch distance in the random waypoint model. Sec. III provides the simulation results to verify the analytical model. Finally, the paper is concluded in Sec. IV.

2 Epoch Distance in Random Waypoint Model

In our analysis, the movement area Q is a two-dimensional unit square area $[0, 1]^2$, and each node moves based on the random waypoint model with the same parameters. Nodes are assumed uniformly distributed in the area. Given two waypoints (x_1, y_1) and (x_2, y_2), there are four cases to consider, each with an equal probability: (i) $x_1 \le x_2$ and $y_1 \le y_2$, (ii) $x_1 \le x_2$ and $y_1 > y_2$, (iii) $x_1 > x_2$ and $y_1 \le y_2$, and (iv) $x_1 > x_2$ and $y_1 > y_2$. To save space, we will derive the epoch length distribution based on the conditions $x_1 \le x_2$ and $y_1 \le y_2$. The other cases can be obtained similarly.

Let D_E denote the random variable of an epoch distance for the random waypoint model. The probability of $D_E \le d$ is equal to the probability that the distance of two random points in a unit square less than or equal to d (i.e., $\le d$). The probability of $D_E \le d$ given the first point placed at (x_1, y_1) is equal to the probability that the second point falls inside the circle centered at (x_1, y_1) with radius d. We only consider the probability under the condition that (x_2, y_2) is at the upper right direction of (x_1, y_1), which is equivalent to the condition $x_1 \le x_2$ and $y_1 \le y_2$. Since the second point is uniformly placed on the movement area Q at random, the probability that the condition $D_E \le d$ is satisfied is equivalent to the area for the second point to locate on Q that satisfies the condition. To derive the probability of $D_E \le d$, we need to further consider two cases: (i) $d \le 1$ and (ii) $1 \le d \le \sqrt{2}$.

i) $d \le 1$

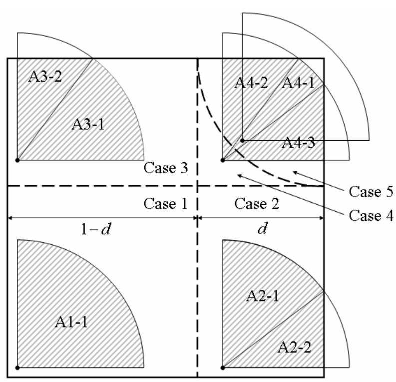

Fig. 1. The conditional probability of $D_E \le d$ given $d \le 1$

The probability of $D_E \le d$ given $d \le 1$ is illustrated in the shaded areas shown in Fig 1. The lowest-leftmost point of each shaded area is a possible location of (x_1, y_1), and the shaded area is the corresponding area for (x_2, y_2) to locate in Q, which satisfies $D_E \le d$ given $d \le 1$. There are five different sub-cases to derive the conditional probability of $D_E \le d$ given $d \le 1$, as shown in Fig. 1.

Case 1: $x_1 \le 1-d$ and $y_1 \le 1-d$

Case 1 states that as long as the first point (x_1, y_1) stays in the area with $x_1 \le 1-d$, and $y_1 \le 1-d$, where $d \le 1$, the possible area at which the second point (x_2, y_2) is located (see the shaded area in A1-1 in Fig. 1) will entirely fall in the movement area Q. Therefore, we have

$$\Pr\{D_E \le d \mid d \le 1, x_1 \le 1-d, y_1 \le 1-d\} = \frac{1}{4}\pi d^2 \quad (1)$$

Case 2: $x_1 \ge 1-d$ and $y_1 \le 1-d$

Case 2 states that once x_1 exceeds 1-d, x_2 must be limited to 1 to avoid the second point move out of the movement area Q. Consequently, the possible area in which the second point may be located is composed of a fan shape and a triangle as given by the shaded areas A2-1 and A2-2, respectively, in Fig. 1. Therefore, we have

$$\Pr\{D_E \le d \mid d \le 1, x_1 \ge 1-d, y_1 \le 1-d\} = \frac{d^2 \sin^{-1}(\frac{1-x}{d})}{2} + \frac{(1-x)\sqrt{d^2-(1-x)^2}}{2} \quad (2)$$

Case 3: $x_1 \le 1-d$ and $y_1 \ge 1-d$

Case 3 states that once y_1 exceeds 1-d, y_2 must be limited to 1 to avoid the second point move out of the movement area Q. Consequently, the possible area in which the second point may be located is also composed of a fan shape and a triangle as given by the shaded areas A3-1 and A3-2, respectively, in Fig. 1. Therefore, we have.

$$\Pr\{D_E \le d \mid d \le 1, x_1 \le 1-d, y_1 \ge 1-d\} = \frac{d^2 \sin^{-1}(\frac{1-y}{d})}{2} + \frac{(1-y)\sqrt{d^2-(1-y)^2}}{2} \quad (3)$$

Case 4: $x_1 \ge 1-d$ and $y_1 \ge 1-d$.

Case 4 states that since both x_1 and y_1 exceeds 1-d, both x_2 and y_2 must be limited to 1 to avoid the second point move out of the movement area Q. In Case 4, $\sqrt{(1-x_1)^2+(1-y_1)^2} \ge d$, so that the arc of the quarter circle still intersects the upper and right boundaries of the movement area Q. The possible location of the second point is composed of a fan shape and two triangles, as given by the shaded areas A4-1, A4-2, and A4-3, respectively, in Fig. 1. Hence, we have

$$\Pr\left\{D_E \le d \mid d \le 1, x_1 \ge 1-d, y_1 \ge 1-d, \sqrt{(1-x_1)^2+(1-y_1)^2} \ge d\right\}$$
$$= \frac{d^2\left(\frac{\pi}{2} - \cos^{-1}(\frac{1-x}{d}) - \cos^{-1}(\frac{1-y}{d})\right)}{2} + \frac{(1-y)\sqrt{d^2-(1-y)^2}}{2} + \frac{(1-x)\sqrt{d^2-(1-x)^2}}{2} \quad (4)$$

Case 5: $x_1 \geq 1-d$ and $y_1 \geq 1-d$

Similar to Case 4, but with $\sqrt{(1-x_1)^2+(1-y_1)^2} \leq d$, the arc of the quarter circle will not intersect any boundary of area Q. Thus, the possible location of the second point to stay is just a rectangular (i.e., the intersection the quarter circle and the unit square Q) as shown in Fig. 1. It yields

$$\Pr\left\{D_E \leq d \mid d \leq 1, \sqrt{(1-x_1)^2+(1-y_1)^2} \leq d\right\} = (1-x_1)(1-y_1) \tag{5}$$

From (1) to (5), we obtain

$$\Pr\{D_E \leq d \mid d \leq 1, x_1 \leq x_2, y_1 \leq y_2\} = \frac{\pi}{4}d^2 - \frac{2}{3}d^3 + \frac{1}{8}d^4 \tag{6}$$

Therefore,

$$\Pr\{D_E \leq d \mid d \leq 1\} = 4\Pr\{D_E \leq d \mid d \leq 1, x_1 \leq x_2, y_1 \leq y_2\} = \pi d^2 - \frac{8}{3}d^3 + \frac{1}{2}d^4 \tag{7}$$

ii) $1 \leq d \leq \sqrt{2}$

Given (x_1, y_1), the conditional probability of $D_E \leq d$ given $1 \leq d \leq \sqrt{2}$ is the shaded areas shown in Fig 2. Like in the case $d \leq 1$, the lowest-leftmost point of each shaded area is a possible location of (x_1, y_1), the shaded areas are the corresponding area for (x_2, y_2) to locate in Q, which satisfies $D_E \leq d$ given that $1 \leq d \leq \sqrt{2}$.

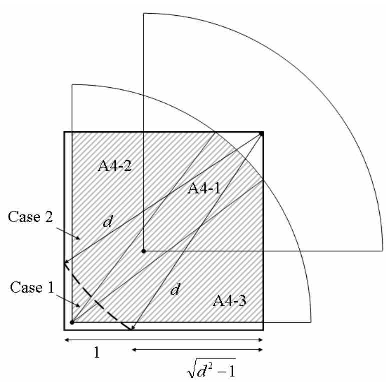

Fig. 2. The conditional probability of $D_E \leq d$ given $1 \leq d \leq \sqrt{2}$

There are two different sub-cases to derive the probability of $D_E \leq d$ given $1 \leq d \leq \sqrt{2}$, as shown in Fig. 2, which are similar to Cases 4 and 5 when $d \leq 1$.

Case 1: Since $\sqrt{(1-x_1)^2+(1-y_1)^2} \geq d$, the arc of the quarter circle will intersect the boundaries of the movement area *Q*. The possible location of the second point is composed of a fan shape and two triangles, as given by the shaded areas A4-1, A4-2, and A4-3, respectively, in Fig. 2. Thus,

$$\begin{aligned}&\Pr\left\{D_E \leq d \mid 1 \leq d \leq \sqrt{2}, \sqrt{(1-x_1)^2+(1-y_1)^2} \geq d\right\} \\ &= \frac{d^2\left(\frac{\pi}{2} - \cos^{-1}(\frac{1-x}{d}) - \cos^{-1}(\frac{1-y}{d})\right)}{2} + \frac{(1-y)\sqrt{d^2-(1-y)^2}}{2} + \frac{(1-x)\sqrt{d^2-(1-x)^2}}{2}\end{aligned} \tag{8}$$

Case 2: Since $\sqrt{(1-x_1)^2+(1-y_1)^2} \leq d$, the arc of the quarter circle will not intersect any boundary of area Q. The possible location of the second point is just a rectangular (i.e., the intersection the quarter circle and the unit square Q) as in Fig. 2. Hence, we have

$$\Pr\left\{D_E \leq d \mid 1 \leq d \leq \sqrt{2}, \sqrt{(1-x_1)^2+(1-y_1)^2} \leq d\right\} = (1-x)(1-y) \tag{9}$$

From (8) and (9), we obtain

$$\Pr\left\{D_E \leq d \mid 1 \leq d \leq \sqrt{2}, x_1 \leq x_2, y_1 \leq y_2\right\} = \frac{1}{12} + (\frac{\pi}{4} - \frac{1}{2} - \cos^{-1}(\frac{1}{d}))d^2 - \frac{1}{8}d^4 + \sqrt{d^2-1}(\frac{1}{3} + \frac{2}{3}d^2) \tag{10}$$

Therefore,

$$\begin{aligned}&\Pr\left\{D_E \leq d \mid 1 \leq d \leq \sqrt{2}\right\} \\ &= 4\Pr\left\{D_E \leq d \mid 1 \leq d \leq \sqrt{2}, x_1 \leq x_2, y_1 \leq y_2\right\} = \frac{1}{3} + (\pi - 2 - 4\cos^{-1}(\frac{1}{d}))d^2 - \frac{1}{2}d^4 + \sqrt{d^2-1}(\frac{4}{3} + \frac{8}{3}d^2)\end{aligned} \tag{11}$$

Based on (7) and (11), we can obtain the probability distribution of D_E, i.e. the distance between two destinations as follows.

$$\begin{aligned}&F_{D_E}(d) = \Pr\left\{D_E \leq d\right\} \\ &= \begin{cases} \pi d^2 - \frac{8}{3}d^3 + \frac{1}{2}d^4 & if\ d \leq 1 \\ \frac{1}{3} + (\pi - 2 - 4\cos^{-1}(\frac{1}{d}))d^2 - \frac{1}{2}d^4 + \sqrt{d^2-1}(\frac{4}{3} + \frac{8}{3}d^2) & otherwis \end{cases}\end{aligned} \tag{12}$$

$$\begin{aligned}&f_{D_E}(d) = \frac{dF_{D_E}(d)}{dd} \\ &= \begin{cases} 2\pi d - 8d^2 + 2d^3 & if\ d \leq 1 \\ (2\pi - 4 - 8\cos^{-1}(\frac{1}{d}))d - 2d^3 + 8d\sqrt{d^2-1} & otherwis \end{cases}\end{aligned} \tag{13}$$

3 Performance Evaluation

In this section, we verify our analytical model via simulation. In our simulation, there are 300 nodes initially distributed in a unit square as in [6,7]. The parameter settings are listed in Table I.

Table 1. Simulation parameters

Parameter	Value
number of nodes	300
transmission range *r*	0.15
nodal speed V_{fix}	0.01 (1/sec)
movement area *Q*	$[0, 1]^2$
simulation duration	600,000 (sec)

Fig. 3 plots the analytical probability density functions (pdf) of *DE* in comparison to the simulation. The figure shows that the analytical curves match the simulation results very well.

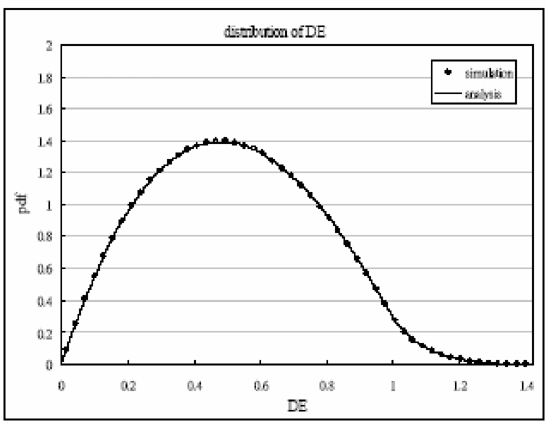

Fig. 3. The distribution of *DE*

4 Conclusion

In this paper, we model the epoch distance of the random waypoint model for mobile ad hoc networks. In particular, the probability distribution of the distance between two waypoints is derived. The analytical results are also verified via simulations. The results show that the analytical and simulation curves match well.

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